Reflection and transmission of magnons through an exchange-coupled biferromagnetic interface

Avishay Yaniv
Department of Physics and Astronomy, Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel
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The transmission and reflection of magnons from an exchange-coupled biferromagnetic interface is analyzed for both normal and off-normal incidence. It is shown that a measurement of the transmission coefficient determines, almost uniquely, the value of the interface exchange constant.

The rapid development in vacuum evaporation techniques has led to an increased interest in the properties of the solid-solid interface. One of these interesting interfaces is the biferromagnetic interface. The properties of such an interface, formed between two dipolar ferromagnets, as well as the transmission of magnons through a planar barrier were studied by us in an earlier work.[1,2] More recently, we have studied the theory of magnons in a system of an exchange-coupled biferromagnetic interface.[3] In this report we apply the results derived in Ref. 3 to analyze the reflection and transmission of magnons through such an interface. The method we use is similar to the one used by Arnold and Menon[4] to determine the transmission coefficient for phonons incident upon an interface between two different crystals, and by Arnold[5] in his analysis of electron transmission through a bimetallic interface using the model developed by Yaniv.[6]

The system considered here is an ideal (100) interface formed between two simple cubic Heisenberg ferromagnets. The two crystals forming the interface are assumed to have the same lattice constant, so that the crystal momentum parallel to the interface $\vec{k}_{\parallel}$, is a good quantum number. The atomic layers $n=0,1,2,\ldots$ and $n=-1,-2,-3,\ldots$ are occupied by S_1 spins interacting via a nearest-neighbor exchange constant J_1, and by S_2 spins interacting via J_2, respectively. The interface is thus formed between the spin layers $n=0$ and $n=-1$, and is characterized by the interface nearest-neighbor exchange constant $J_{12}>0$. The properties of this interface were analyzed in Ref. 3 and the corresponding Green's function was determined within the mixed Bloch-Wannier representation. For lattice planes of index m located to the right of the interface, the diagonal Green's function can be written as

$$G(m,m)=G^0(m,m)[1+r(E,\vec{k}_{\parallel})\times e^{2im\phi(E,\vec{k}_{\parallel})}] , \tag{1}$$

where G^0 is the corresponding bulk Green's function for the S_1 spins. ϕ is a real angle for energies inside the $\vec{k}_{\parallel}$ subband of the right-hand-side ferromagnet. The quantity $r(E,\vec{k}_{\parallel})$ in (1) is given by

$$r=-\frac{\{2-\Lambda+\alpha\psi-\varepsilon-i[1-(3-\Lambda-\varepsilon)^2]^{1/2}\}\{2\Gamma-\Lambda\Gamma+\psi-\varepsilon+i\Gamma[1-(3-\Lambda-\varepsilon/\Gamma)^2]^{1/2}\}-\alpha\psi^2}{\{2-\Lambda+\alpha\psi-\varepsilon+i[1-(3-\Lambda-\varepsilon)^2]^{1/2}\}\{2\Gamma-\Lambda\Gamma+\psi-\varepsilon+i\Gamma[1-(3-\Lambda-\varepsilon/\Gamma)^2]^{1/2}\}-\alpha\psi^2} , \tag{2}$$

where

$$\Lambda(\vec{k}_{\parallel})=\cos(k_ya)+\cos(k_za) , \tag{3}$$

a being the common lattice constant, and

$$\alpha=S_2/S_1 , \tag{4}$$

$$\Gamma=J_2S_2/J_1S_1 , \tag{5}$$

$$\psi=J_{12}/J_1 . \tag{6}$$

ε is the energy measured in units of half the subband width of the right-hand-side ferromagnet,

$$\varepsilon=E/4J_1S_1 . \tag{7}$$

The second term in the square brackets of (1) is due to reflection of magnons from the interface. The corresponding reflection coefficient is[5] $|r|^2$. An immediate result that follows from expression (2) is that for energies that are inside the bulk subband of the right-hand-side ferromagnet, but outside the corresponding subband of the left-hand-side ferromagnet $|r|^2=1$, so that magnons of this energy are totally reflected from the interface.

Before we analyze the general expression (2), con-

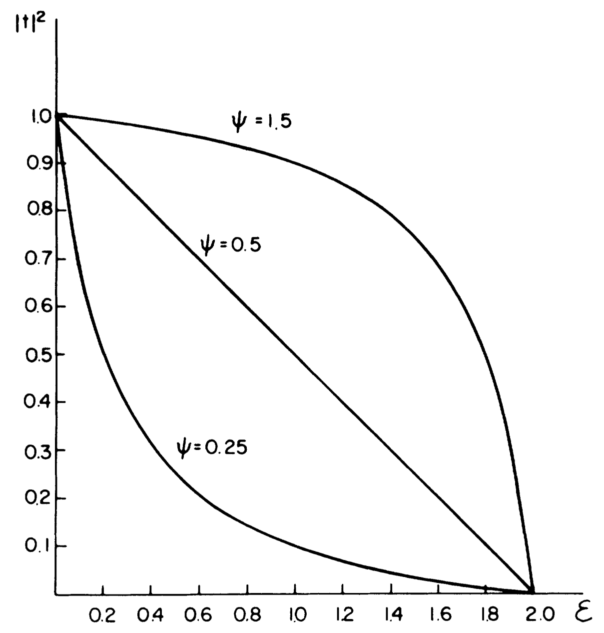

FIG. 1. The transmission coefficient of magnons, at normal incidence to a planar defect, as a function of the energy, for $\psi=0.25$, 0.5, and 1.5.

sider first the simpler system with $S_1=S_2$ and $J_1=J_2$ (i.e., $\alpha=\Gamma=1$). For this system the $\vec{k}_{\parallel}$ subbands on the two sides of the interface coincide for all $\vec{k}_{\parallel}$. Such an interface can be realized in principle, in the laboratory, by the absorption of an insulating nonmagnetic monolayer on a ferromagnetic substrate, and a subsequent epitaxial growing of the same ferromagnetic material, as in the substrate, on top of this monolayer. Thus, we refer to this interference as the planar defect.

It follows from the general expression (2) that in the present case the reflection coefficient is given by

$$|r|^2=\frac{(\varepsilon-2+\Lambda)(1-\psi)^2}{(\varepsilon-2+\Lambda)(1-2\psi)+2\psi^2} . \qquad (8)$$

This reflection coefficient has the following properties:

(a) It is a function of the energy relative to the bottom of the $\vec{k}_{\parallel}$ subband only, $\varepsilon-2+\Lambda(\vec{k}_{\parallel})$.

(b) It vanishes for $\psi=1$ since no real interface exists in this case.

(c) It is equal to 1 for $\psi=0$ since all magnons are reflected from a free surface.

(d) $|r|^2$ vanishes for magnons at the bottom of the $\vec{k}_{\parallel}$ subband ($\varepsilon=2-\Lambda$).

Figure 1 describes the transmission coefficient $|t|^2=1-|r|^2$ as a function of the energy relative to the bottom of the $\vec{k}_{\parallel}$ subband, or as a function of the energy for magnons striking the interface at normal incidence. As the interface coupling increases from 0 to 1, $|t|^2$ grows monotonically. At $\psi=0.5$, $|t|^2$ becomes a linear function of the energy. At $\psi=1$, $|t|^2=1$ and as ψ grows beyond this value, $|t|^2$ decreases monotonically and reaches at $\psi=\infty$ the same function as for $\psi=0.5$. Thus, a measurement of the reflection or the transmission coefficient of bulk magnons can be used to determine, almost uniquely, the value of the interface coupling $J_{12}=J_1\psi$. If one plots, for example, $|r|^{-2}$ as a function of the energy relative to the bottom of the subband, one should obtain a straight line having a positive slope of $2\psi^2/(1-\psi)^2$, and an intercept of $(1-2\psi)/(1-\psi)^2$. This intercept is positive for $\psi<0.5$, but negative for $\psi>0.5$. When ψ is less than 0.5, it can be determined uniquely from the slope of the straight line. If we denote this slope by s, then ψ is given by

$$\psi=\frac{(2s)^{1/2}}{2+(2s)^{1/2}} . \qquad (9)$$

As noted before, if $\psi>0.5$ there are two values of ψ that give the same reflection and transmission coefficients. Of these two values one is larger than 1 and the other smaller than 1. The two possible values of the interface coupling are given in terms of the slope by

$$\psi=\frac{\pm(2s)^{1/2}}{2\pm(2s)^{1/2}} . \qquad (10)$$

In this case one needs an additional measurement in order to determine the correct interface exchange coupling out of the two values given by Eq. (10). As we have shown in Ref. (3), there exists an interface magnon branch for $\psi>1$. If such a branch exists, then ψ is given by (10) with the minus signs.

Consider now the scattering of magnons from the planar defect at off-normal incidence. For simplici-

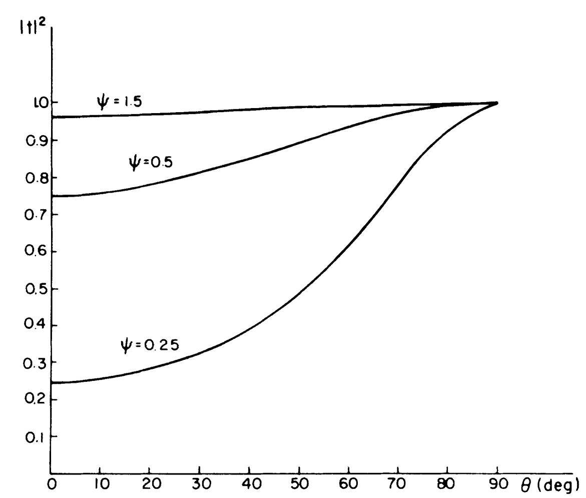

FIG. 2. The transmission coefficient for the planar defect as a function of the incidence angle, for $\varepsilon=0.5$ and $\psi=0.25$, 0.5, and 1.5.

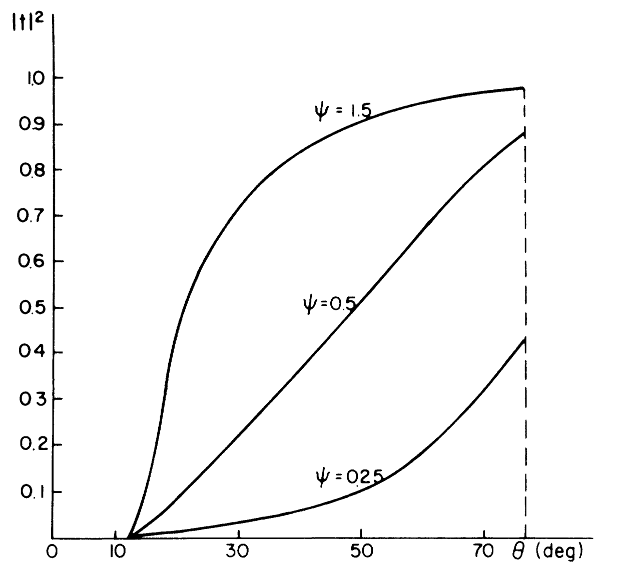

FIG. 3. The transmission coefficient for the planar defect as a function of the incidence angle, for $\varepsilon=2.25$ and $\psi=0.25$, 0.5, and 1.5.

ty we assume that the scattering process takes place in the xz plane, where the x direction is normal to the interface. The magnon wave vector forms an angle θ with the normal to the interface plane, where $\tan\theta=k_z/k_x$. For a given incidence angle θ and a given magnon energy ε, k_z is a solution of the equation

$$\varepsilon+\cos(k_z a)+\cos(k_z a/\tan\theta)-2=0 . \tag{11}$$

Solving this equation for $k_z a$ we can determine, from Eq. (8), the dependence of the reflection and the transmission coefficient on the incidence angle. Figure 2 shows the variation of $|t|^2$ for magnons with $\varepsilon=0.5$, as a function of θ for $\psi=0.25$, 0.5, and 1.5, respectively. $|t|^2$ grows monotonically from

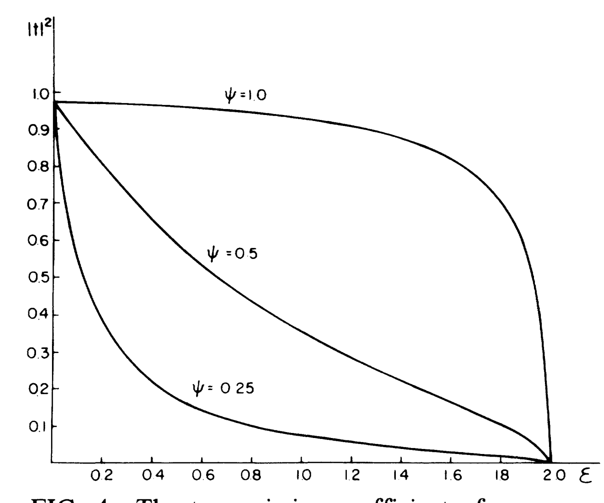

FIG. 4. The transmission coefficient of magnons at normal incidence to an interface with $\alpha=1$ and $\Gamma=2$ as a function of the energy, for $\psi=0.25$, 0.5, and 1.0.

its value of

$$|t|^2=\frac{(2-\varepsilon)\psi^2}{\varepsilon(1-2\psi)+2\psi^2} , \tag{12}$$

at normal incidence to the value $|t|^2=1$ at $\theta=90°$. For a given incidence angle, $|t|^2$ grows monotonically as ψ grows from 0 to 1. As ψ increases further, $|t|^2$ decreases and approaches asymptotically from above the curve corresponding to $\psi=0.5$.

For energies outside the $\vec{k}_{\parallel}=0$ subband the situation is quite different. If we consider scattering in the xz plane as before, then outside the $\vec{k}_{\parallel}=0$ subband the energy varies in the range $2\le\varepsilon\le 4$. In this region the bulk magnons exist within a limited angular range

$$\theta_1\le\theta\le\tfrac{1}{2}\pi-\theta_1 , \tag{13}$$

where

$$\tan\theta_1=\frac{1}{\pi}\arccos(3-\varepsilon) . \tag{14}$$

Figure 3 shows a typical behavior of $|t|^2$ as a function of θ for an energy in this region. $|t|^2$ vanishes at $\theta=\theta_1$ and grows monotonically to the value of

$$|t|^2=\frac{(4-\varepsilon)\psi^2}{(\varepsilon-2)(1-2\psi)+2\psi^2} , \tag{15}$$

at $\theta=\tfrac{1}{2}\pi-\theta_1$.

We turn now to the case of a general interface characterized by bulk parameters α and Γ and by an interface coupling constant ψ. Consider first the transmission of magnons perpendicular to the interface, i.e., $\Lambda=2$. Transmission of magnons across the interface is possible only in the energy region which is the intersection of the two $\vec{k}_{\parallel}=0$ subbands, i.e., $0\le\varepsilon\le\min(2,2\Gamma)$. At $\varepsilon=0$ one has a finite

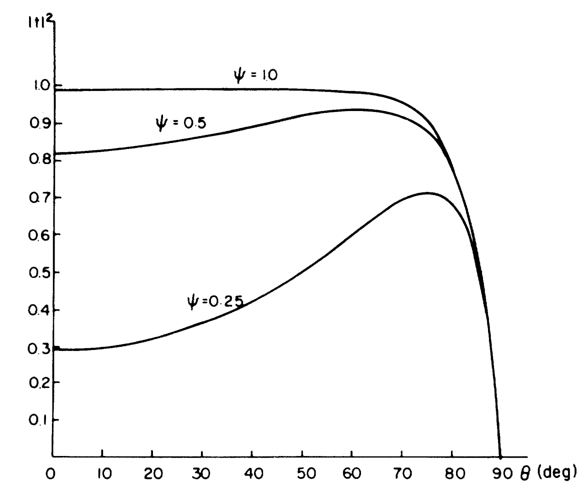

FIG. 5. The transmission coefficient for an interface with $\alpha=1$ and $\Gamma=\frac{3}{4}$ as a function of the incidence angle θ, for $\varepsilon=0.5$ and $\psi=0.25$, 0.5, and 1.0.

transmission coefficient, independent of the interface coupling, which is given by

$$|t|^2=\frac{4\alpha\sqrt{\Gamma}}{(\alpha\sqrt{\Gamma}+1)^2} . \tag{16}$$

At the other edge of the transmission region, i.e., at $\varepsilon=2$ for $\Gamma\geq 1$ or $\varepsilon=2\Gamma$ for $\Gamma<1$, the magnons are totally reflected, and $|t|^2$ vanishes. Figure 4 shows the variation of $|t|^2$ for magnons at normal incidence in the case of an interface with $\alpha=1$, $\Gamma=2$, and several values of ψ, as a function of the magnon energy. The general behavior is similar to the one in the case of the planar defect, Fig. 1, except that the value of $|t|^2$ at $\varepsilon=0$ is reduced from the value of 1 in the case of the planar defect, to the value (16) in the case of the interface. Also the region of energies over which $|t|^2$ does not vanish is reduced to the region $0\leq\varepsilon\leq 2\Gamma$ if $\Gamma<1$.

We consider next the off-normal transmission of magnons through the interface. As before we discuss the case where the scattering takes place in the xz plane. We study first the case with $\Gamma<1$. Figure 5 shows a typical behavior of $|t|^2$ as a function of θ for $\varepsilon\leq 2\Gamma$. This is quite different from the corresponding planar-defect result shown in Fig. 2. For a true interface $|t|^2$ vanishes for $\theta=90°$, whereas it is equal to 1 for the planar defect.

In the energy region $2\Gamma\leq\varepsilon\leq 2\Gamma/(1-\Gamma)$ for $\Gamma<\frac{1}{2}$ or $2\Gamma\leq\varepsilon\leq 2$ for $\frac{1}{2}<\Gamma<1$, transmission of magnons through the interface is possible in a limited angular range,

$$\theta_2\leq\theta\leq\tfrac{1}{2}\pi , \tag{17}$$

where

$$\tan\theta_2=\frac{\arccos(3-\varepsilon/\Gamma)}{\arccos[\varepsilon(1-\Gamma)/\Gamma-1]} . \tag{18}$$

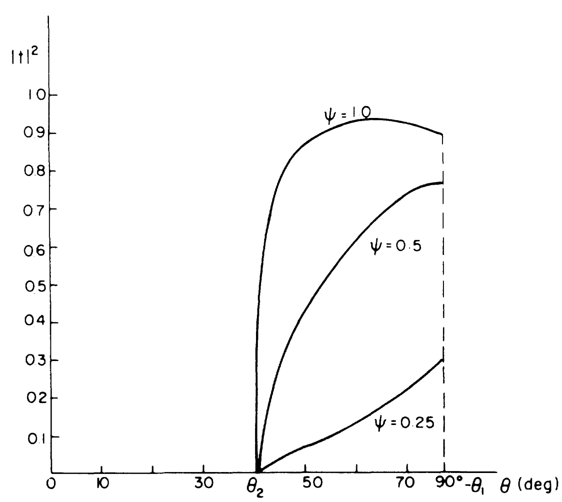

FIG. 6. The transmission coefficient for an interface with $\alpha=1$ and $\Gamma=\frac{3}{4}$ as a function of the incidence angle, for $\varepsilon=2.25$ and $\psi=0.25$, 0.5, and 1.0.

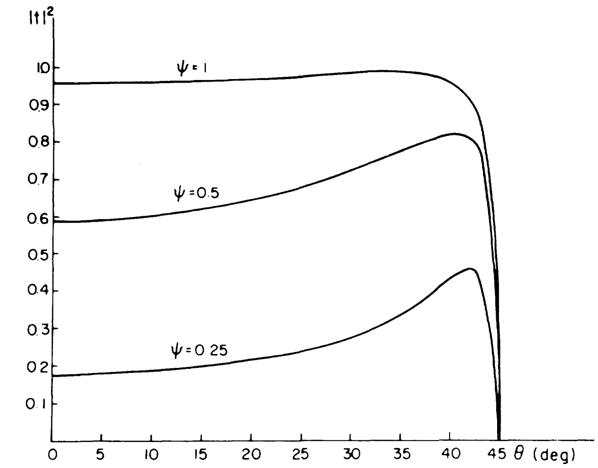

FIG. 7. The transmission coefficient for an interface with $\alpha=1$ and $\Gamma=2$ as a function of the incidence angle, for $\varepsilon=0.5$ and $\psi=0.25$, 0.5, and 1.0.

For energies in the region $2\leq\varepsilon\leq 2\Gamma/(1-\Gamma)$ for $\frac{1}{2}\leq\Gamma\leq\frac{2}{3}$ or in the region $2\leq\varepsilon\leq 4$ for $\frac{2}{3}\leq\Gamma\leq 1$ transmission occurs in a more limited angular range

$$\theta_2\leq\theta\leq\tfrac{1}{2}\pi-\theta_1 , \tag{19}$$

where θ_1 and θ_2 are given by Eqs. (14) and (18), respectively. $|t|^2$ vanishes at $\theta=\theta_2$ and reaches a finite value at $\theta=\frac{1}{2}\pi-\theta_1$. A typical behavior of $|t|^2$ in this energy region is shown in Fig. 6 for an interface with $\alpha=1$ and $\Gamma=\frac{3}{4}$.

We turn now to off-normal magnon scattering, in the xz plane, for interfaces having $\Gamma>1$. In this case there exists an upper critical scattering angle θ_c, above which magnons are totally reflected from the interface. This critical angle is given by

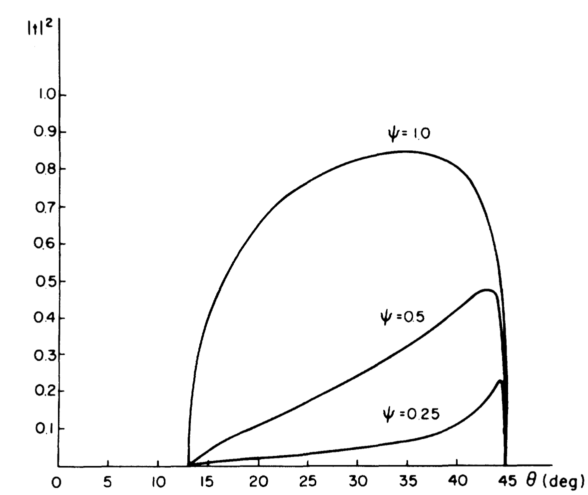

FIG. 8. The transmission coefficient for an interface with $\alpha=1$ and $\Gamma=2$ as a function of the incidence angle, for $\varepsilon=2.25$ and $\psi=0.25$, 0.5, and 1.0.

$$\tan\theta_c = \frac{\arccos(1-\varepsilon/\Gamma)}{\arccos[1-\varepsilon(\Gamma-1)/\Gamma]} . \tag{20}$$

In the energy range $\varepsilon < 2$ transmission occurs for

$$0 \le \theta \le \theta_c . \tag{21}$$

A typical behavior in this situation is shown in Fig. 7 for an interface with $\alpha = 1$ and $\Gamma = 2$.

For energies above the $k_{\|} = 0$ subband, in the region

$$2 \le \varepsilon \le \min[4, 2\Gamma/(\Gamma-1)] ,$$

transmission occurs in the angular region

$$\theta_1 \le \theta \le \theta_c , \tag{22}$$

where θ_1 is given by expression (14). $|t|^2$ vanishes at both ends of this transmission region. An example of this behavior is shown in Fig. 8 for an interface with bulk parameters $\alpha = 1$ and $\Gamma = 2$ and several values of the interface coupling ψ.

Finally we note that as in the case of the planar defect, a measurement of the transmission coefficient determines almost uniquely the value of the interface exchange coupling.

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