Josephson tunneling and the proximity effect

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Josephson junctions containing one or two proximity systems (e.g., S_α-M_β-I-S_γ, S_α-M_β-I-M_γ-S_δ; $M_{\beta(\gamma)}$ is a normal metal or superconductor) are studied. An analysis of the thickness and temperature dependences of the maximum dc Josephson current I_M is carried out. The curvature of the function $I_M(L_\beta)$ is found to depend strongly on the ratio of energy gaps $\epsilon_\gamma/\epsilon_\alpha$ and on the temperature. The analysis is based on the thermodynamic Green's-function method, and the effect of strong coupling is also taken into account. If the proximity system contains two superconductors S_α and S_β, then it appears that the electron-phonon interaction described by the constant λ_β contributes noticeably even if $T > T_{c\beta}$.

I. INTRODUCTION

The present paper is concerned with properties of the Josephson junction containing one or two proximity systems. As is well known (see Refs. 1–12) the presence of a proximity system affects very noticeably the properties of the junction and, first of all, the behavior of the maximum dc Josephson current. We are going to consider junctions of the type S_α-M_β-I-S_γ containing the proximity sandwich S_α-M_β, where S_α and S_γ are superconductors, I is an insulator, and M_β is a normal metal or a superconductor. We will also consider a junction containing two proximity sandwiches. Lately, these systems have attracted a lot of interest, particularly in connection with the investigation of Josephson junctions with an artificial barrier, such as Nb-Al-Al0$_x$-Pb. Even usual niobium-based Josephson junctions are characterized (see, e.g., Ref. 8) by the existence of the proximity layer at the Nb and oxide interface, and the proximity effect has to be taken into account. Hence it is important to develop a theoretical approach which allows one to describe Josephson tunneling into the proximity sandwich. Moreover, the Josephson current is very sensitive to the properties of the proximity system and, in principle, one can change its behavior in the desired direction.

Consideration of the Josephson tunneling into a proximity system is interesting also from the point of view of investigation of the proximity effect. The corresponding experimental data contain very interesting information about the proximity effect.

The Josephson tunneling in proximity systems has been studied, e.g., in Refs. 1–11, and interesting experimental data describing thickness and temperature dependence of the maximum Josephson current I_M have been obtained. Theoretical consideration[1–3] is based on the Ginsburg-Landau theory and results in a good agreement. As is known, this approach is applicable in the region $T \sim T_c$. Gilabert *et al.*[7] carried out a numerical calculation of I_M based on the McMillan tunneling model[13] for the systems Nb-Nb_xO_y-Cu-Pb and Nb-Nb_xO_y-Al-Pb. This model gives a good description of experimental data (see also the experimental study of the proximity effect[14–16]). As is known, lead is a superconductor with strong coupling and the authors[7] used the correction $\sim$20% which was obtained[17] for the usual Josephson contact. The calculations for the pure specular tunneling ($\kappa_{\|}=0$) were carried out by Gallagher.[18] He used a method different from[13] assuming a spatially constant order parameter. Matsuda *et al.*[8] used the McMillan approach in order to describe the properties of a Nb-based Josephson junction.

In this paper we analyze the behavior of the Josephson current in the presence of proximity systems. We evaluate the thickness and temperature of the maximum current I_M. It turns out that it is possible to develop a theoretical approach based on the thermodynamic Green's-function method (see, e.g, Ref. 19) which allows one to describe the Josephson tunneling for different systems (S_α-S_β-I-S_γ, S_α-N_β-I-S_γ, S_α-N_β-I-N_γ-S_δ, and so on). One can introduce the universal functions and investigate the effect of different factors on the thickness and temperature dependence of I_M. The effects of strong coupling are also taken into account. It turns out that the correction caused by strong electron-phonon interactions depends not only on the properties of the superconductor S_α but also on the thickness L_β. The thermodynamic Green's-function method has been used by the present author[20] in order to evaluate T_c of the proximity system.

We also evaluate the Josephson current in the S_α-S_β-I-S_γ junction containing a proximity system S_α-S_β with two superconductors. It appears that this contribution is very noticeable even if $T > T_{c\beta}$ ($T_{c\beta}$ is the critical temperature of the isolated β film).

The plan of the present paper is as follows. Section II addresses the problem of obtaining the main equations. As was mentioned, we use the thermodynamic Green's-function method and, moreover, we take into account the electron-phonon interaction directly. We consider the thickness and temperature dependence of I_M in Secs. III and IV, respectively. The junction containing the S_α-S_β proximity system will be considered in Sec. V.

II. MAIN EQUATIONS

Let us consider the system S_α-M_β-I-S_γ (see Fig. 1), where S_α and S_β are superconducting films, I is an insulator, and M_β is a normal metal (or semiconductor), e.g.,

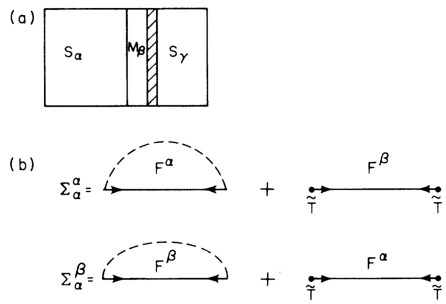

FIG. 1. (a) S_α-M_β-I-S_γ system, and (b) self-energy parts.

Pb-Cu-PbO-Pb, or a superconductor (e.g., Nb-Al-AlO_x-Pb). We assume that $T_c^\beta < T_c^\alpha$. The contact S_α-M_β forms a proximity sandwich and this sandwich is a part of the Josephson junction. One can consider a more general case of the system S_α-M_β-I-M_γ-S_δ containing two proximity sandwiches. The maximum Josephson current is equal to (see, e.g., Ref. 21)

$$I_M=\frac{T}{\pi eR}\sum_{\omega_n}\int d\xi_p^\beta d\xi_q^\gamma F_\beta^+(p,\omega_n)F_\gamma(q,-\omega_n) . \tag{1}$$

Here $\omega_n=(2n+1)\pi T$ and F_β and F_γ are the anomalous thermodynamic Green's functions describing the Cooper pairing:

$$\begin{aligned}F_{\beta(\gamma)}=&\Sigma_{2\beta(\gamma)}(p,\omega_n)\\&\times[\omega_n^2 Z^2_{\beta(\gamma)}(p,\omega_n)\\&+\xi^2_{\beta(\gamma)}(p)+\Sigma^2_{2\beta(\gamma)}(p,\omega_n)]^{-1} ,\end{aligned} \tag{2}$$

where $\xi_{\beta(\gamma)}$ is the energy of an ordinary electron in the film $\beta(\gamma)$ referred to in the Fermi level, $Z_{\beta(\gamma)}$ is the renormalization function, $\Sigma_{2\beta(\gamma)}$ is the self-energy part describing the Cooper pairing, and R is the normal resistance of the barrier I. The equation of continuity allows one to calculate the Josephson current through any section of the junction, and we have chosen the current flowing through the insulator I.

It is worth noting that expressions (1) and (3) (see below) are valid for systems with strong electron-phonon interaction. Equation (1) can be reduced to the form

$$I_M eR=\pi T\sum_{\omega_n}\frac{\Delta_\beta(\omega_n)\Delta_\gamma(\omega_n)}{[\omega_n^2+\Delta_\beta^2(\omega_n)]^{1/2}[\omega_n^2+\Delta_\gamma^2(\omega_n)]^{1/2}} . \tag{3}$$

Here $\Delta_\beta=\Sigma_\beta(\omega_n)/Z_\beta$ and $\Delta_\gamma=\Sigma_\gamma(\omega_n)/Z_\gamma$ ($p=p_F$) are the renormalized self-energy parts (order parameters).

Hence in order to evaluate the maximum current I_M, we should calculate the order parameter $\Delta_\beta(\omega_n)$ and $\Delta_\gamma(\omega_n)$. In this chapter we restrict ourselves to the consideration of the system S_α-M_β-I-S_γ [see Fig. 1(a)]. The generalization for the system containing two proximity sandwiches is straightforward (see Sec. III B). We consider a general case S_α-S_β with $\lambda_\beta \neq 0$ (λ_β describes the electron-phonon coupling in the β film).

The contact S_α-S_β forms a proximity system. The superconducting state in the β film is caused by the electron-phonon interaction in the film and by the proximity effect. Hence the quantities $\Delta_\alpha(\omega_n)$ and $\Delta_\beta(\omega_n)$ should be evaluated on the basis of the theory of the proximity effect.

Suppose that the thicknesses L_α and L_β satisfy the conditions $L_\alpha \gg L_\beta$, $L_\beta \ll \xi_\beta$, where ξ_β is the coherence length. Moreover, suppose that the α film is "dirty" in the Anderson sense.[22] Under these conditions we can use the well-known McMillan tunneling approach to the proximity effect.[13] The electron-phonon interaction can be included in the McMillan model (see Ref. 23). The description of a proximity system based on the use of the thermodynamic Green's function was given by the present author.[20] As is well known (see, e.g., Refs. 7–9 and 14–16), the results obtained on the basis of the McMillan approach are in very good agreement with experimental data.

The equations for the order parameters are seen in Fig. 1(b) or in analytical form:

$$\begin{aligned}\Delta_\alpha(\omega_n)=&Z_\alpha^{-1}\pi T\sum_{\omega_{n'}}\int d\Omega\, g_\alpha(\Omega)D(\Omega,\omega_n-\omega_{n'})\\&\times\kappa_\alpha^{-1}(\omega_{n'})\Delta_\alpha(\omega_{n'})\\&+Z_\alpha^{-1}\Gamma^{\alpha\beta}\kappa_\beta^{-1}(\omega_n)\Delta_\beta(\omega_n) ,\end{aligned} \tag{4}$$

$$\begin{aligned}\Delta_\beta(\omega_n)=&Z_\beta^{-1}\pi T\sum_{\omega_{n'}}\int d\Omega\, g_\beta(\Omega)D(\Omega,\omega_n-\omega_{n'})\\&\times\kappa_\beta^{-1}(\omega_{n'})\Delta_\beta(\omega_{n'})\\&+Z_\beta^{-1}\Gamma^{\beta\alpha}\kappa_\alpha^{-1}(\omega_n)\Delta_\alpha(\omega_n) .\end{aligned} \tag{5}$$

Here

$$D=\Omega^2/[\Omega^2+(\omega_n-\omega_{n'})^2]$$

is the phonon thermodynamic Green's function,

$$g_{\alpha(\beta)}(\Omega)=a^2_{\alpha(\beta)}(\Omega)F_{\alpha(\beta)}(\Omega)$$

[$F_{\alpha(\beta)}(\Omega)$ is the phonon density of states in α (β) film and $a^2_{\alpha(\beta)}(\Omega)$ describes the electron-phonon interaction], and $\kappa_\alpha(\omega_n)$ and $\kappa_\beta(\omega_n)$ are defined by the relation

$$\kappa_{\alpha(\beta)}(\omega_n)=[\omega_n^2+\Delta^2_{\alpha(\beta)}(\omega_n)]^{1/2} . \tag{6}$$

The quantities $\Gamma^{\alpha\beta}$ and $\Gamma^{\beta\alpha}$ are equal to (see Ref. 13)

$$\Gamma^{\alpha\beta}=\pi\tilde{T}^2\nu_\beta SL_\beta,\quad \Gamma^{\beta\alpha}=\pi\tilde{T}^2\nu_\alpha SL_\alpha$$

(ν_α,ν_β are the densities of states, $\tilde{T}$ is the tunneling matrix element, and S is the area of the contact). Hence $\Gamma^{\alpha\beta}/\Gamma^{\beta\alpha}\sim L_\beta/L_\alpha$ and, generally speaking, $\Gamma^{\alpha\beta}/\Gamma^{\beta\alpha}\ll 1$, since $L_\beta \ll L_\alpha$. This means that one can neglect the effect of the β film on the superconducting properties of the α film, that is, one can neglect the second term on the right-hand side of Eq. (4). It also is worth noting that according to[13] the quantity $\Gamma^{\beta\alpha}$ is equal to

$$\Gamma^{\beta\alpha}=V_{F\perp}\sigma/2BL_\beta , \tag{7}$$

where V_F is the Fermi velocity, σ is the barrier penetration probability, and B is a function of the ratio of the mean free path to the film thickness L_β. If β film is clean, then B is constant with value $B\simeq 2$.[13]

Consider Eq. (5) in detail. The renormalization function Z^β is equal to (see Refs. 13 and 20)

$$Z_\beta = 1 + \Gamma^{\beta\alpha}/\kappa_\alpha(\omega_n) - \widetilde{\Sigma}_{1\beta}/\omega_n \ . \tag{8}$$

Here $\widetilde{\Sigma}_{1\beta}$ is connected with the self-energy part describing the electron-phonon scattering (see, e.g., Refs. 24–26). If β film is a normal metal or a superconductor with weak coupling then

$$\widetilde{\Sigma}_{1\beta}/\omega_n = -\lambda_\beta \ , \tag{9}$$

where

$$\lambda_\beta = \int d\Omega\, g_\beta(\Omega)\Omega^{-1} \ . \tag{10}$$

Substituting (8) and (9) into (5), we arrive, after simple manipulations, at the following equation:

$$\Delta_\beta(\omega_n) = g(\omega_n)\pi T \sum_{\omega_{n'}} \int d\Omega\, g_\beta^r D(\Omega,\omega_n - \omega_{n'}) \times \kappa_\beta^{-1}(\omega_{n'})\Delta_\beta(\omega_{n'}) + S(\omega_n) \ . \tag{11}$$

Here

$$g(\omega_n) = \kappa_\alpha(\omega_n)/[\Gamma + \kappa_\alpha(\omega_n)] \ , \tag{12}$$

$$S(\omega_n) = [1 - g(\omega_n)]\Delta_\alpha(\omega_n) = \{\Gamma/[\Gamma + \kappa_\alpha(\omega_n)]\}\Delta_\alpha(\omega_n) \ , \tag{13}$$

$$\Gamma = \Gamma^{\beta\alpha}/(1+\lambda_\beta) = V_{F\perp}^r \sigma/2BL_\beta \ , \tag{14}$$

and $V_F^r = V_F(1+\lambda_\beta)$ and $g_\beta^r = g_\beta/(1+\lambda_\beta)$ are the renormalized Fermi velocity and the electron-phonon coupling. As it should be, Eq. (11) contains only renormalized quantities.

If the coupling $g_\beta = 0$, then the order parameter $\Delta_\beta(\omega_n)$ is equal to

$$\Delta_\beta(\omega_n) = S(\omega_n) \ , \tag{15}$$

where $S(\omega_n)$ is defined by Eq. (13). The function $\Delta_\alpha(\omega_n)$ is described by the usual Eliashberg equation[24]

$$\Delta_\alpha(\omega_n) = Z_\alpha^{-1}\pi T \sum_{\omega_{n'}} \int d\Omega\, g_\alpha(\Omega) D(\Omega,\omega_n - \omega_{n'}) \times \Delta_\alpha(\omega_{n'})\kappa_\alpha^{-1/2}(\omega_{n'}) \ . \tag{16}$$

We assume now that the β film is a superconductor with weak electron-phonon coupling (e.g., Al, Zn, etc.; we do not limit the strength of the electron-phonon interaction in the α film). In the weak coupling approximation, one can disregard the terms of the order of $\sim T_c^2/\widetilde{\Omega}_\beta^2$, $\widetilde{\Omega}_\beta \sim \Omega_\beta^D$; Ω_β^D is the Debye frequency of the β film. The weak coupling approximation allows one (see Ref. 27) to neglect the term ω_n in the denominator of the integrand in Eq. (11). We obtain

$$\Delta_\beta(\omega_n) = g(\omega_n)\rho_\beta \pi T \sum_{\omega_{n'}} \kappa_\beta^{-1}(\omega_{n'})\Delta_\beta(\omega_{n'})\chi(\omega_{n'}) + [1 - g(\omega_n)]\Delta_\alpha(\omega_n) \ . \tag{17}$$

Here $\rho_\beta = \lambda_\beta/(1+\lambda_\beta)$, λ_β and the function $\kappa_\beta(\omega_n)$ are defined by Eqs. (6–10), and

$$\chi(\omega_{n'}) = \rho_\beta^{-1} \int d\Omega\, g_\beta^r(\Omega) D(\Omega,\omega_{n'}) \ . \tag{18}$$

Approximately,

$$\chi(\omega_{n'}) \simeq \widetilde{\Omega}_\beta^2/(\widetilde{\Omega}_\beta^2 + \omega_{n'}^2), \quad \widetilde{\Omega}_\beta \simeq \Omega_\beta^D \ . \tag{19}$$

In order to make the transformation to the BCS description, which is valid in the weak coupling approximation, one should put $\chi(\omega_{n'}) = 1$ in Eq. (17). Then we obtain the well-known logarithmic divergence which has been avoided by the cutoff at the frequency $\Omega \sim \Omega_\beta^D$. The presence of the function $\chi(\omega_{n'})$ in Eq. (17) results in the vanishing of the divergence, and one can obtain the same solution of Eq. (17) as in the BCS approximation.[26,27]

Our goal is the find the solution of the nonlinear equation (17) and then to evaluate the current I_M according to Eq. (3). One can seek the solution of Eq. (17) in the form

$$\Delta_\beta(\omega_n) = g(\omega_n)\beta + S(\omega_n) \ , \tag{20}$$

where $g(\omega_n)$ and $S(\omega_n)$ are defined by Eqs. (12) and (13) and the quantity β does not depend on ω_n. Substituting (20) into (17) we arrive at the following equation for the parameter β:

$$\beta = \rho_\beta \pi T \sum_{\omega_n} \frac{g(\omega_n)\beta + S(\omega_n)}{\{\omega_n^2 + [S(\omega_n) + g(\omega_n)\beta]^2\}^{1/2}} \chi(\omega_n) \ . \tag{21}$$

Let us introduce the dimensionless quantity

$$\delta = \beta/\epsilon_\alpha \ , \tag{22}$$

where $\epsilon_\alpha \equiv \epsilon_\alpha(T)$ is the energy gap of the superconductor α. With the use of Eqs. (6), (12), and (22), after some manipulations one can reduce Eq. (21) to the form

$$\delta = \rho_\beta \frac{\pi T}{\epsilon_\alpha} \sum_n \frac{f_\alpha + \delta t\widetilde{\kappa}}{[x_n^2(1+t\widetilde{\kappa})^2 + (f_\alpha + \delta t\widetilde{\kappa})^2]^{1/2}} \chi(x_n\epsilon_\alpha) \ . \tag{23}$$

Here

$$x_n = \omega_n/\epsilon_\alpha = (2n+1)\pi T/\epsilon_\alpha, \quad f_\alpha = \Delta_\alpha(x_n\epsilon_\alpha)/\epsilon_\alpha \ , \tag{24}$$

$$\widetilde{\kappa} = (x_n^2 + f_\alpha^2)^{1/2}\alpha \ , \tag{25}$$

$$\alpha = \epsilon_\alpha/\pi T_c \ . \tag{26}$$

The function f_α can be obtained from the theory of strong coupling (see Sec. III C). In the weak coupling approximation $f_\alpha = 1$.

We introduce a parameter t which is defined by the relation

$$t = l/S_0 \ . \tag{27}$$

Here $l = L_\beta/L_0$ is a dimensionless quantity, where L_0 is some fixed thickness (we have chosen $L_0 = 10^2$ Å). The parameter S_0 is defined by the relation

$$S_0 = \Gamma_0/\pi T_c \ , \tag{28}$$

where $\Gamma_0 = V_{F\perp}^r \sigma/2BL_0$ [cf. Eq. (14)].

Solving Eq. (23), one can find the quantity δ and then, according to (6), (12), (13), (20), and (22), the order parameter $\Delta^\beta(\omega_n)$. The solution of Eq. (23) will be obtained below (see Sec. V).

Now let us turn to the evaluation of the maximum current I_M. According to (3), (20), and (22), we obtain, after simple manipulations,

$$I_M eR = r2\pi T \sum_{n\geq 0} \frac{(f_\alpha+\delta t\tilde{\kappa})f_\gamma}{[x_n^2(1+t\tilde{\kappa})^2+(f_\alpha+\delta t\tilde{\kappa})^2]^{1/2}(x_n^2+r^2f_\gamma^2)^{1/2}} . \tag{29}$$

Here $r=\epsilon_\gamma/\epsilon_\alpha$; the quantities x_n, t, $\tilde{\kappa}$, and f_α are defined by Eqs. (24)—(27), $f_\alpha=\Delta_\gamma(x_n\epsilon_\alpha)/\epsilon_\gamma$, and the parameter δ is the solution of Eq. (23).

If $T\to 0$ one can pass from summation to integration, according to the rule (see, e.g., Ref. 19)

$$(2\pi T/\epsilon_\alpha)\sum_n \to \int dx ,$$

and we obtain

$$I_M eR = \epsilon_\gamma(0)\int_0^\infty dx \frac{f_\alpha(x)+\delta t\tilde{\kappa}(x)}{\{x^2[1+t\tilde{\kappa}(x)]^2+[f_\alpha(x)+\delta t\tilde{\kappa}(x)]^2\}^{1/2}} \frac{f_\gamma}{(x^2+f_\gamma^2r^2)^{1/2}} . \tag{30}$$

With the use of the Poisson formula, it is easy to prove that the expression (30) is valid if $T \ll \epsilon_\alpha(T)$, that is, up to $T/T_c \sim 0.5$. A more detailed discussion of Eqs. (23) and (29) will be given below (see Sec. V).

Equations (29), (30), and (23) are the basic equations of the theory. They allow one to evaluate the Josephson current I_M in the S_α-M_β-I-S_γ junction. It is easy to get an expression describing the system S_α-M_β-I-M_γ-S_δ containing two proximity sandwiches (see Sec. III B). The expressions describing the junction S_α-N_β-I-S_γ, where N_β is a normal metal and $\rho_\beta=0$, can be obtained from (29) and (30) if we put $\delta=0$, that is,

$$[I_M eR]_{S_\alpha\text{-}N_\beta\text{-}I\text{-}S_\gamma} = r2\pi T \sum_{n\geq 0} \frac{f_\alpha f_\gamma}{[x_n^2(1+t\tilde{\kappa})^2+f_\alpha^2]^{1/2}(x_n^2+r^2f_\gamma^2)^{1/2}} \tag{31}$$

and

$$[I_M eR]_{S_\alpha\text{-}N_\beta\text{-}I\text{-}S_\gamma, T\to 0} = [\epsilon_\alpha(0)]\int_0^\infty dx \frac{f_\alpha(x)f_\gamma(x)}{\{x^2[1+t\tilde{\kappa}(x)]^2+f_\alpha^2(x)\}^{1/2}[x^2+r^2f_\alpha^2(x)]^{1/2}} . \tag{32}$$

Equations (23) and (29)—(32) describe the thickness and temperature dependence of I_M. As one can see, the behavior of functions $I_M(L_\beta)$ and $I_M(T)$ depends on the strength of the electron-phonon interaction [see Eqs. (16), (24), and (29)] on the ratio $r=\epsilon_\gamma/\epsilon_\alpha$, and on the properties of the S_α-S_β contact [see Eq. (23)]. In the weak coupling approximation, one should put $f_\alpha=f_\gamma=1$. If $L_\beta\to 0$, we obtain [see Eqs. (29) and (30)] the expression describing the usual Josephson contact S_α-I-S_γ. Note that if $L_\beta\to 0$ and $f_\alpha=f_\gamma=1$ (weak coupling), we get the well-known expression that was obtained by Anderson[28] and by Ambegaokar and Baratoff.[29]

Hence the maximum Josephson current in the presence of a proximity system is described by Eqs. (29) and (31), and in the low-temperature region by Eqs. (30) and (32). The parameter δ, appearing in Eqs. (29) and (30), is the solution of Eq. (23). Now we turn to evaluation of the dependences $I_M(L_\beta)$ and $I_M(T)$ for different systems.

III. THICKNESS DEPENDENCE OF THE MAXIMUM dc JOSEPHSON CURRENT

A. S_α-N_β-I-S_γ system

Consider a Josephson junction containing the proximity sandwich S_α-N_β, where N_β is a normal metal and the electron-phonon coupling $\rho_\beta=0$. Then the parameter $\delta=0$ [see Eq. (23)] and, according to Eq. (32), the current I_M in the low-temperature region ($T \ll \epsilon_\alpha$, that is, $T<0.5T_c$) is described by the following expression:

$$\eta = G_r(t)/G_r(0) . \tag{33}$$

Here $\eta=I_MR/(I_MR)_0$, $(I_MR)_0 \equiv I_MR$ ($L_\beta=0$), and

$$G_r(t)=\int_0^\infty dx\{x^2[1+\alpha t(x^2+1)^{1/2}]^2+1\}^{-1/2} \times(x^2+r^2)^{-1/2} . \tag{34}$$

The quantities α and t are defined by Eqs. (26) and (27) and

$$r=\epsilon_\gamma/\epsilon_\alpha \tag{35}$$

is the ratio of the energy gaps. Equations (33) and (34) are written in the weak coupling approximation (the effect of strong coupling will be considered in Sec. III C) so that $f_\alpha=f_\gamma=1$ and $\alpha=\alpha_{\rm BCS}=0.56$.

The thickness dependence of I_M, as obtained from Eqs. (33) and (34), is shown in Fig. 2(a). We see that I_M decreases with increasing L_β and that the sharpness of this decrease depends on the ratio r. Curve 2 in Fig. 2(a) corresponds to the simplest structure S_α-N_β-I-S_α ($r=1$). Hence this dependence is described by a universal function of the parameter $t=l/S_0$. The value of the parameter S_0 depends on the quality of the proximity contact and on the critical temperature T_c [see Eq. (28)], and this value determines how rapidly I_M drops with increasing L_β. A decrease in S_0 results in a slower decrease of $I_M(L_\beta)$. The numerical value of S_0 can be obtained experimentally (see Refs. 14—16). As is known, if the β film is clean, the parameter S_0 does not depend on L. The simplest method is to measure I_MR at definite L_β. For example, if we investigate the properties of the junction S_α-N_β-I-S_α and we know the value of $\eta_1\equiv\eta$ ($L_\beta=10^2$ Å) (then $l=1$ and $t=S_0^{-1}$), we can use curve 2 in Fig. 2(a) to determine S_0. Indeed, in this case this curve describes the dependence $\eta(S_0^{-1})$ and this allows us to find S_0 directly by measuring η. This value can be used subsequently in order to obtain the dependences $I_M(L_\beta)$ and $I_M(T)$. Moreover, the Josephson tunneling appears here as a method of investigating a proximity system.

Thus I_M decreases with increasing L_β and the sharpness of this decrease depends on the ratio of the energy gaps r and on the parameter S_0. The asymptotic dependence

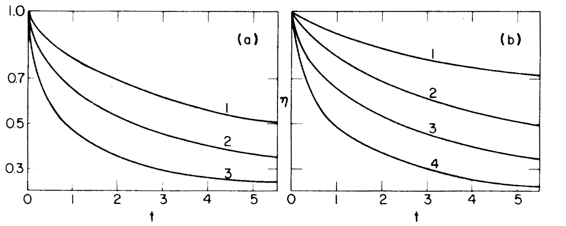

FIG. 2. Thickness dependence of $I_M R$ for (a) S_α-N_β-I-S_γ system: (1) $r=\frac{1}{6}$, (2) $r=1$, (3) $r=6$; $r=\epsilon_\gamma/\epsilon_\alpha$ is the ratio of the energy gaps. (b) S_α-N_β-I-N_γ-S_δ system: (1) $\tilde{t}'=10$, (2) $\tilde{t}'=1$, (3) $t'=0$, (4) $t'=t$, $\tilde{t}'=0.56t'$: $t=l_a/S_0^{\alpha\beta}$; $t'=l_\gamma/S_0^{\alpha\gamma}$; $\eta=I_M R/(I_M R)_0$.

$I_M(L_\beta)$ turns out to be nontrivial. This asymptotic dependence can be evaluated analytically in the general form (see Appendix). It appears that in the region of small thicknesses ($l\to 0$)

$$\Delta(I_M R)\sim l\ln l \;, \tag{36}$$

where

$$\Delta(I_M R)=I_M R(L_\beta)-(I_M R)_0 \;.$$

One can also determine the dependence of I_M for $t\to\infty$. According to Eq. (27), large values of t correspond, for example, to small values of S_0 (e.g., small values of the probability σ). Analytical evaluation of the integral (34) (see Appendix) results in the dependence

$$[I_M R]_{t\to\infty}\sim \ln t/t \;. \tag{37}$$

B. Josephson junction with two proximity systems

We have described (see above) the thickness dependence of I_M in the S_α-N_β-I-S_γ system. Let us consider a more complicated structure S_α-N_β-I-N_γ-S_δ containing two proximity systems. The approach developed above (see Sec. II) can be generalized for the case when both electrodes are proximity sandwiches. I_M is described by Eq. (3) and $\Delta_\beta(\omega_n)$ can be found from Eqs. (11)—(13) ($\rho_\beta=0$). An analogous equation can be used to determine $\Delta_\gamma(\omega_n)$ and finally we obtain [cf. Eq. (33)]

$$\eta=f(t,t')/f(0,t') \;, \tag{38}$$

where $\eta=I_M R/(I_M R)_0$, $(I_M R)_0\equiv I_M R$ ($L_\beta=0$), and

$$f(t,t')=\int_0^\infty dx\{x^2[1+\alpha t(x^2+1)^{1/2}]^2+1\}^{-1/2} \times\{x^2[1+\alpha t'(x^2+r^2)^{1/2}]+r^2\}^{-1/2} \;. \tag{39}$$

Here $\alpha=0.56$ [see Eq. (26)], $r=\epsilon_\delta/\epsilon_\alpha$, the parameter t is defined by Eq. (27), and t' is the same parameter for the system S_δ-N_γ. One can see that the behavior of η has become more complicated than in the picture considered in Sec. III A. The decrease of η with increasing t depends parametrically not only on r, but also on the properties of the S_δ-N_γ sandwich (parameter t'). The dependence $\eta(t)$ for different values t' (if $r=1$) is shown in Fig. 2(b).

A specific dependence $\eta(t)$ appears in the case when $t=t'$, that is, in the case when the Josephson tunneling between two identical proximity systems is studied. The equality $L_\beta=L_\gamma$ should be kept during the experiment. In this case the function $\eta(t)$ drops more rapidly with increasing L_β. One can obtain the asymptotic dependence of $I_M R$ [cf. Sec. III A, Eqs. (36) and (37)]. Evaluation of the integrals (see Appendix) results in the dependence $\Delta(I_M R)\sim t$ for $t\to 0$ and $I_M R\sim t^{-1}$ for $t\to\infty$ [cf. Eq. (37)]. For systems with $t=t'$ (see above), the asymptotic behavior is different. Namely, $\Delta(I_M R)\sim t\ln t$ [$t\to 0$ or $l\to 0$, see Eq. (27)] and $I_M R\sim \ln t/t$ ($t\to\infty$).

C. Strong coupling effect

Let us consider again the system S_α-N_β-I-S_γ and take into account the strong coupling factor. The correction due to strong electron-phonon interaction is noticeable (see below) if we consider junctions containing such superconductors as Pb, NbN, Nb, etc. In the low-temperature region $T\ll\epsilon_\alpha,\epsilon_\gamma$, one can use the general expression (32), and we obtain

$$I_M eR=\epsilon_\alpha(0)G_r^f(t) \;. \tag{40}$$

Here

$$G_r^f(t)=\int_0^\infty dx\, f_\alpha(x)f_\gamma(x)\{x^2[1+t\tilde{\kappa}(x)]^2+f_\alpha^2(x)\}^{-1/2}[x^2+r^2 f_\alpha^2(x)]^{-1/2} \;, \tag{41}$$

where $f_{\alpha(\gamma)}(x)$, $\tilde{\kappa}(x)$, t, and r are defined by Eqs. (24)—(27) and (35). In the weak coupling approximation $f_\alpha=f_\gamma=1$ and (if $r=1$) we obtain the well-known expression $(I_M eR)_0=(\pi/2)\epsilon_\alpha(0)$ (see Ref. 29). According to Eq. (40), the ratio $\eta=I_M R/(I_M R)_0$ is described by the relation [cf. Eqs. (33) and (34)]

$$\eta=G_r^f(t)/G_r^f(0) \;,$$

where G_r^f is defined by Eq. (41).

The frequency-dependent order parameter $\Delta_\alpha(\omega_n)$ is the solution of the Eliashberg equation (16) [$\omega_n=(2n+1)\pi T$; $\Delta_\gamma(\omega_n)$ satisfies an analogous equation]. In the low-temperature region, in accordance with the Poisson equation, ω_n becomes a continuous variable but the functions $\Delta_{\alpha(\gamma)}(\omega)$ have to be evaluated on the basis of the method of the thermodynamic Green's function. As is known, the problem of evaluation of the usual Green's function is connected with the analytical continuation of the thermodynamic Green's function (see, e.g., Ref. 19). The function $\Delta(\omega)$ can be calculated on the basis of the theory of strong coupling developed by Scalapino *et al.*[25,30] For our purpose it is convenient to use the method of the theory of strong coupling which has been developed by Geilkman, Masharov, and the present author.[26] This method is based on the theory of the thermodynamic Green's function and allows one to evaluate directly the functions of interest $\Delta_\alpha(\omega_n)$ and $\Delta_\gamma(\omega_n)$.

The main feature of the theory of strong coupling is that physical quantities are not described, as in the weak coupling approximation, by universal functions but depend on the phonon spectrum of the superconductor. The corresponding corrections usually are of the order $\sim T_c^2/\tilde{\Omega}^2$, where $\tilde{\Omega}$ is the characteristic phonon frequency (see below).

According to the theory[26] the function $\Delta(\omega)$ is described by the expression

$$V(\omega)=N_0^{-1}\int d\Omega\, g(\Omega)D(\Omega,\omega)[1+\epsilon\omega^2/(\omega^2+\delta\Omega^2)]\ , \tag{42}$$

where

$$V(\omega)=\Delta(\omega)/\Delta(0),\ \ N_0=\int d\Omega\, g(\Omega),\ \ g(\Omega)=\alpha^2(\Omega)F(\Omega)$$

[see Eq. (5)], $\epsilon=1.4\rho$, $\delta=1.5$,

$$D=\Omega^2/(\Omega^2+\omega^2),\ \ \rho=\lambda/(1+\lambda),\ \ \lambda=\int d\Omega\, g(\Omega)\Omega^{-1}\ .$$

The function $g(\Omega)$ is known from tunneling measurements (see Ref. 31). The energy gap $\epsilon_{\alpha(\gamma)}(0)$ can be obtained from the relation[26,32]

$$\epsilon(0)=1.76T_c\left[1+5.3\left[\frac{T_c}{\tilde{\Omega}}\right]^2\ln\left[\frac{\tilde{\Omega}}{T_c}\right]\right]\ , \tag{43}$$

where $\tilde{\Omega}=\Omega_t$ (Ω_t corresponds to the transverse branch) is the frequency of the lowest peak (the presence of the highest peak near Ω_h results in a correction of the order $\sim T_c^2/\Omega_h^2$ which is usually small). Equation (43) is in good agreement with experimental data (see Refs. 33 and 34).

One can prove (see Ref. 26) that the function $f_\alpha(x,\epsilon_\alpha)$ [see Eq. (24)] for superconductors such as Pb (T_c^2/Ω_h^2 is negligibly small and $\rho_h \ll \rho_t$) can be written with high accuracy in the form

$$f_\alpha(x,\epsilon_\alpha)=A^2/(x^2+A^2)\ , \tag{44}$$

where $A=\tilde{\Omega}/\epsilon_\alpha$; ϵ_α is defined by Eq. (43). For example, for Pb, $\tilde{\Omega}=4.5$ meV, $\epsilon(0)=2.1T_c$, and $A=3.45$. Nb is characterized (see Ref. 35) by the presence of two closely situated peaks. In this case one can use the general expression (42) which also can be rewritten as a sum of two parts corresponding to different peaks [see Ref. 25, Eq. (6.24)]. Equation (44) also can be taken as good approximation with the value of the frequency $\tilde{\Omega}$ intermediate between Ω_t and Ω_h. Orlando *et al.*[35] used measurements of the heat capacity in order to find the value of the frequency $\tilde{\Omega}$. Their method is based on the relation obtained by Parchomenko and the present author[36]:

$$(C_e^S-C_e^n)/\gamma T_c=1.4\left\{1+1.8\left[\ln\left[\frac{\tilde{\Omega}}{T_c}\right]+0.5\right]\left[\frac{\pi T_c}{\tilde{\Omega}}\right]^2\right\}\ . \tag{45}$$

Based on this method, the authors[35] found the value $\tilde{\Omega}=11.4$ meV for Nb_3Sn and $\tilde{\Omega}=24$ meV for V_3Si. This method is very efficient in the absence of tunneling data.

Substituting (44) into Eqs. (40) and (41) one can evaluate the maximum current I_M. Based on Eqs. (40) and (41), we have calculated the thickness dependence I_M for the system Pb-Cu-I-Pb (see Fig. 3). For comparison we also show the thickness dependence of I_MR obtained in the weak coupling approximation [see Eqs. (33) and (34) and Fig. 2(a)]. One can see that the correction caused by strong electron-phonon interaction also depends on the thickness L_β (if $L_\beta=0$, this correction is $\sim 20\%$ in accordance with Ref. 17).

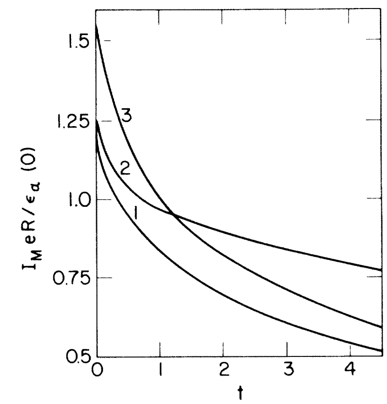

FIG. 3. Thickness dependence of I_MR for (1) Pb-N_β-I-Pb, $\lambda_\beta=0$, (2) Pb-Zn-I-Pb, (3) S_α-N_β-I-S_α (weak coupling). The difference between (1) and (3) is caused by strong coupling in Pb; the difference between (1) and (2) is due to the coupling λ_β in Zn.

D. Thickness dependence in the region of $T\sim T_c$

Consider the Josephson junction S_α-N_β-I-N_γ-S_α, where N_β and N_γ are normal films and $\lambda_\beta=\lambda_\gamma=0$, and S_α is a superconductor. If $T\to T_c$, the order parameters $\Delta_\beta,\Delta_\gamma\to 0$ and one can neglect their values in the denominator of Eq. (3). Then we obtain

$$I_MeR=\pi T\sum_{\omega_n}\Delta_\beta(\omega_n)\Delta_\gamma(\omega_n)/\omega_n^2\ . \tag{46}$$

The quantities Δ_β and Δ_γ can be evaluated according to Eqs. (11)–(13). In the weak coupling approximation (the generalization for strong coupling is straightforward and will be given elsewhere), we obtain (in our case $\Delta_\alpha=\Delta_\delta=\epsilon$; ϵ is the energy gap in the superconducting films)

$$I_MeR=2\Gamma\Gamma'\epsilon^2\pi T\sum_{\omega_n>0}\omega_n^{-2}(\Gamma+\omega_n)^{-1}(\Gamma'+\omega_n)^{-1}\ , \tag{47}$$

where $\Gamma\equiv\Gamma^{\beta\alpha}$ and $\Gamma'\equiv\Gamma^{\gamma\alpha}$ [see Eq. (7)], and they correspond to the systems S_α-N_β and S_α-N_γ, respectively.

Summing over ω_n, we arrive, after cumbersome calculations, at the following result:

$$\frac{I_MR}{(I_MR)_0}=\phi(t,t')\ , \tag{48}$$

where

$$\phi(t,t')=1-(4/\pi^2)\left[\frac{t^2}{t-t'}\Psi\left[\frac{1}{2}+\frac{1}{2t}\right]+\frac{t'^2}{t'-t}\Psi\left[\frac{1}{2}+\frac{1}{2t'}\right]-(t+t')\Psi(\tfrac{1}{2})\right]\ . \tag{49}$$

Here $\Psi(x)$ is the digamma function and t,t' are defined according to (27) [cf. Eqs. (38) and (39)]. In the special case of $t=t'$ we obtain

$$\frac{I_M R}{(I_M R)_0}=\phi(t,t)=1-(8t/\pi^2)\left[\Psi\left[\frac{1}{2}+\frac{1}{2t}\right]-\Psi(\tfrac{1}{2})\right]$$
$$-\frac{2}{\pi^2}\Psi'\left[\frac{1}{2}+\frac{1}{2t}\right]. \quad (50)$$

If we put $L_\gamma=0$ (then $t'=0$) in Eq. (48), we obtain an expression describing the thickness dependence of I_M for the S_α-N_β-I-S_γ system:

$$\frac{I_M(R)}{(I_M R)_0}=\phi(t,0)$$

or

$$\frac{I_M R}{(I_M R)_0}=1-(4t/\pi^2)\left[\Psi\left[\frac{1}{2}+\frac{1}{2t}\right]-\Psi(\tfrac{1}{2})\right]. \quad (51)$$

It is worth noting that according to Eq. (51), $\Delta(I_M R)_{l\to 0}\sim l\ln l$, as for the low-temperature region [see Eq. (36)]. If $t\to\infty$, one can obtain, after simple calculations,

$$I_M R\sim\epsilon^2 t^{-1} \quad (52)$$

[cf. Eq. (37)].

Equations (48)—(52) are valid if $T\sim T_c$. As is known, the de Gennes—Werthamer (GW) theory (see, e.g., Ref. 37) also describes this region. It is worth noting that the GW theory is valid in the "dirty" limit, whereas our method, based on the McMillan tunneling model, describes the case of a clean β film.

IV. TEMPERATURE DEPENDENCE OF I_M

The general expression (29), which has been obtained on the basis of the theory of the thermodynamic Green's function, is valid for any temperature. Based on this equation, one can evaluate the temperature dependence $I_M(T)$ for fixed thickness L_β. For the system S_α-N_β-I-S_γ (the contact S_α-S_β will be considered in Sec. V), we obtain

$$\frac{I_M(T)}{I_M(0)}=\frac{2S}{\alpha}\frac{T}{T_c^\alpha}\sum_{n\geq 0}\nu(x_n)\Big/\int_0^\infty \nu(x)dx . \quad (53)$$

Here

$$\nu(x_n)=f_\alpha f_\gamma\{x_n^2[1+\alpha t(x_n^2+f_\alpha^2)^{1/2}]^2+f_\alpha^2\}^{-1/2}$$
$$\times(x_n^2+r^2f_\gamma^2)^{-1/2}, \quad (54)$$

where x_n, f_α, f_γ, α, and r are defined by Eqs. (24), (26), and (35); $S=\epsilon_\gamma(T)/\epsilon_\gamma(0)$.

In the weak coupling approximation one should put $f_\alpha=f_\gamma=1$. Then the function $I_M(T)$ parametrically depends on t, that is [see Eq. (27)], on the thickness L_β. The behavior of $I_M(T)$ for different values of t ($r=1$) is shown in Fig. 4. One can see that deviation from the well-known Ambegaokar-Baratoff formula[29] [this formula can be obtained from (53) if we put $t=0$] increases with increasing t. This increase in t can be achieved [see Eq. (27)] by increasing the thickness L_β or by decreasing the value of S_0. The small value of S_0 can be connected with smallness of the probability of penetration δ. Moreover, if the β film is a degenerate semiconductor (or a semimetal), S_0 can be decreased by decreasing the electron concentration and, correspondingly, the Fermi velocity V_F [see Eq. (14)].

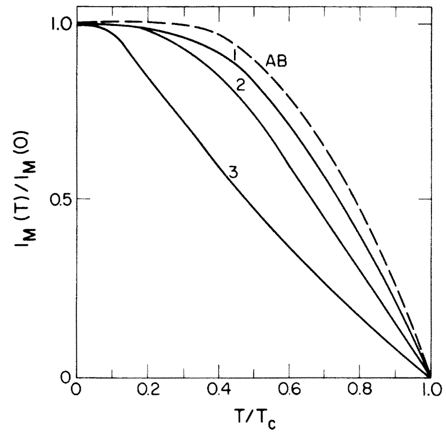

FIG. 4. Temperature dependence of I_M (S_α-N_β-I-S_α junction) for (1) $\alpha t=0.3$, (2) $\alpha t=1$, (3) $\alpha t=5$; the curve AB corresponds to S-I-S contact (Ref. 29).

If one or both of the films α and γ are superconductors with strong coupling, one should take into account the corresponding correction. Based on Eqs. (44) and (53), we have calculated the dependence $I_M(T)$ for the Pb-Cu-PbO-Pb junction (see Figs. 5 and 6). One can see that the dependence described by Eqs. (53) and (54) is in good agreement with experimental data obtained by Greenspoon and Smith[1] for the junction with a thin Cu film (Fig. 6) in the low-temperature region.

The condition $L_\beta \ll \xi_\beta$ (see Sec. IV) can be satisfied in the low-temperature region only, because a decrease in temperature results in an increase of ξ_β.[38] Hence the good agreement of the theory with experimental data (see Fig. 6) can also be considered as evidence of the increase of ξ_β with decreasing T.[39]

V. JUNCTION CONTAINING S_α-S_β PROXIMITY SYSTEM

In this section we consider the properties of the Josephson junction S_α-S_β-I-S_γ, where the proximity system S_α-S_β contains two superconductors. Then one should use

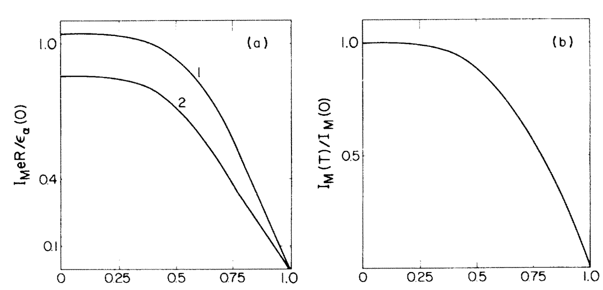

FIG. 5. Effect of strong coupling and the inequality $\lambda_\beta\neq 0$ on the dependence $I_M(T)$: (a) Temperature dependence of $I_M eR/\epsilon_\alpha(0)$ for (1) Pb-Al-PbO-Pb, (2) Pb-Cu-PbO-Pb, $\alpha t=0.6$ (e.g., $L_\beta=100$ Å, $S_0=1.1$; $\alpha_{\rm Pb}=0.67$). One can see the difference between S_α-N_β-I-S_α and S_α-S_β-I-S_α junctions. (b) Temperature dependence $I_M(T)/I_M(0)$ for Pb-Al-PbO-Pb ($\alpha t=0.6$).

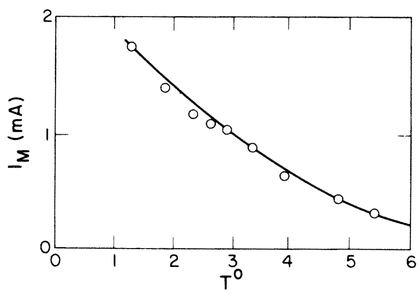

FIG. 6. Dependence $I_M(T)$ for Pb-Cu-PbO-Pb junction; solid line is the theoretical curve; ○, experimental data (Ref. 1) ($L_{\mathrm{Cu}}=10^3$ Å, $S_0=0.12$.)

Eqs. (23) and (29) [if $T \ll \epsilon_\alpha$, one can use Eqs. (23) and (30)]. We consider the case $\rho_\beta \ll \rho_\alpha$ (correspondingly, $T_{c\beta} \ll T_{c\alpha}$; $T_{c\alpha\,(\beta)}$ are the critical temperatures of isolated films). We assume that β film is a superconductor with weak coupling [see the discussion following Eq. (16)].

In the first place we consider Eq. (23), which allows one to evaluate the parameter δ. The value of δ depends on temperature, on the value of t and, hence, on the thickness of the β film and on the function $g_\beta(\Omega)$ [see Eq. (10)] describing the electron-phonon interaction in the β film. It is worth noting that, generally speaking, the function $g_\beta(\Omega)$ also depends on the thickness L_β. For Al films, for example, it has been observed that this function changes noticeably as the thickness of the film decreases and this results in an increase of T_c (see, e.g., Ref. 40). Measurements of T_c allow one to determine the dependences of $g_\beta(\Omega)$ and ρ_β on the thickness L_β.

Equation (23) is nonlinear. If $L_\beta \to 0$ [$t \to 0$, see Eq. (27)] we obtain the expression describing the usual S_α-I-S_γ Josephson junction. Let us consider the opposite case $t \to \infty$ [e.g., the coefficient $\sigma \to 0$, see Eq. (28)]. Then $\Gamma \to 0$, $S(\omega_n) \to 0$, and $g(\omega_n) \to 1$ [see Eqs. (12) and (13)]. Then we obtain [see Eq. (21)]

$$\beta = \rho_\beta \pi T \sum_{\omega_n} \frac{\beta}{(\omega_n^2+\beta^2)^{1/2}}\,, \qquad (55)$$

that is, we obtain the equation describing the energy gap of isolated β film. We put $X=1$, as in the BCS approximation (see Sec. II). Hence if $t \to \infty$, the quantity $\delta \to \epsilon_\beta/\epsilon_\alpha$, and $\delta \to 0$ if $T \to T_c$. If $t \to \infty$, Eq. (29) becomes

$$I_M eR = (\epsilon_\gamma/\epsilon_\alpha) 2\pi T \sum_{n>0} \frac{\delta f_\gamma}{(x_n^2+\delta^2)^{1/2}[x_n^2+(\epsilon_\gamma/\epsilon_\alpha)^2 f_\gamma^2]^{1/2}}\,. \qquad (56)$$

Based on Eq. (24) and using the relation $\delta_{t\to\infty} = \epsilon_\beta/\epsilon_\alpha$, we obtain

$$I_M eR = 2\pi T \sum_{\omega_n>0} \frac{\epsilon_\beta \Delta_\gamma(\omega_n)}{(\omega_n^2+\epsilon_\beta^2)^{1/2}[\omega_n^2+\Delta_\gamma^2(\omega_n)]^{1/2}}\,, \qquad (57)$$

and hence we obtain the expression [cf. Eq. (31)] describing the S_β-I-S_γ contact (we have not made any assumption about the electron-phonon strength in the γ film).

Equations (23), (29), and (30) can be solved for any t, that is, for different thicknesses L_β. For example, the curve 1 in Fig. 5(a) and the curve in Fig. 5(b) correspond to $t=0.9$. We would like to emphasize that the parameter $\delta \neq 0$ (if $t \neq 0$) even if $T > T_{c\beta}$, and the corresponding contribution to I_M has to be taken into account (see Fig. 5). This result has a clear physical meaning. The electron-phonon interaction ρ_β is not equal to zero. If $T > T_{c\beta}$, the superconducting state of the isolated β film is destroyed by thermal motion. In our case the superconducting state is caused by the proximity effect and the electron-phonon interaction ρ_β also contributes to this state. The parameter δ describes this contribution. Thus Eqs. (23), (29), and (30) allow us to evaluate the current I_M for S_α-S_β-I-S_γ systems. The temperature dependence of I_M for the Pb-Al-I-Pb system is shown in Fig. 5.

VI. CONCLUSION AND SUMMARY

Based on the thermodynamic Green's-function method, we have carried out an analysis of the properties of the Josephson junction with a proximity system. The junction can contain one (S_α-M_β-I-S_γ) or two (S_α-M_β-I-M_δ-S_γ) proximity systems. Moveover, the film $M_{\beta\,(\delta)}$ can represent a normal metal or a superconductor (e.g., S_α-S_β-I-S_γ system; $T_{c\beta} < T_{c\alpha}$). The probelm is to evaluate the thickness and temperature dependences of the maximum Josephson current I_M in the presence of a proximity system.

The main results can be summarized as follows.

(1) The value of I_M decreases with increasing thickness L_β. The sharpness of this decrease depends strongly on the ratio $r = T_c^\gamma/T_c^\alpha$ and increases as r increases (see Fig. 2).

(2) Junctions containing two proximity systems (S_α-M_β-I-M_δ-S_γ) have also been considered. The function $I_M(L_\beta)$ depends on the properties of the added proximity system M_δ-S_γ. The sharpest dependence appears to be in the case when both proximity systems are identical, or more exactly, if $t=t'$ [see Fig. 2(b)].

(3) The asymptotic behavior of $I_M(L_\beta)$ depends on the structure of the system [see Sec. III A, Eqs. (36) and (37), and Sec. III B] and on the temperature [cf. Eqs. (36), (37), and (52)].

(4) The temperature dependence $I_M(T)$ differs noticeably from the usual dependence, and this deviation increases with increasing L_β (see Fig. 4). The obtained dependence is in good agreement with experimental data (see Fig. 6).

(5) The effect of strong electron-phonon coupling on the properties of the junction has been investigated. Increase of coupling results in a decrease of the function $I_M R/\epsilon_\alpha(0)$ and the correction caused by strong coupling also depends on the thickness L_β.

(6) The behavior of the junction containing a proximity system with two superconductors [e.g., S_α-S_β-I-S_α, $T_c^\beta < T_c^\alpha$; $T_c^{\alpha\,(\beta)}$ is the critical temperature of the isolated α (β) film] turns out to be very interesting. Equations (23), (29), and (30) describe this system. The presence of the attractive electron-phonon coupling results in an increase in $I_M(L_\beta)$ [see Fig. 5(a)]. The inequality $\lambda_\beta \neq 0$ affects the value of $I_M(L_\beta)$ even if $T > T_c^\beta$.

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APPENDIX

Let us note the two following points.

(1) The integral (34) can be written as a sum

$$J_1=J_1^0+J_1'+J_1'' \ . \tag{A1}$$

Here

$$J_1'=-2t\int_0^\infty u_1(x)dx,\ \ J_1''=-t^2\int_0^\infty u_2(x)dx \ , \tag{A2}$$

$$u_1(x)=x^2/R,\ \ u_2(x)=x^2\kappa_0(x)/R \ , \tag{A3}$$

$$R=\{x^2[1+t\kappa_0(x)]^2+1\}\kappa_0(x) \times(\kappa_0(x)+\{x^2[1+t\kappa_0(x)]^2+1\}^{1/2}) \ , \tag{A4}$$

$$\kappa_0(x)=(x^2+1)^{1/2} \ , \tag{A5}$$

$$J_1^0\equiv G_1(0)=\pi/2,\ \ J_1=G_1(t)$$

[see Eq. (34)], and we consider the S_α-N_β-I-S_α system.

In order to find the behavior of J_1 in the limit of $t\to 0$, we divide each of the integrals (A2) into two parts, e.g.,

$$J_1'=2t\int_0^A u_1(x)dx+2t\int_A^\infty u_1(x)dx \ , \tag{A6}$$

where A has been chosen so that $1\ll A\ll t^{-1}$. Then we can neglect the quantity $t\kappa_0(x)$ in the integrand of the first term in the right-hand side of Eq. (A6) and write

$$x^2[1+t\kappa_0(x)]^2+1\simeq x^2(1+tx)$$

in the integrand in the second term. After a long but simple calculation, we find that the main term in the limit $t\to 0$ is $J_1\simeq t\ln t$.

(2) The integral (34) in the limit $t\to\infty$ can be written in the form

$$J_1\simeq\int_0^\infty \phi(x)dx \ , \tag{A7}$$

where

$$\phi(x)=[t^2x^2\kappa_0^2(x)+1]^{-1/2}\kappa_0^{-1}(x)$$

[$\kappa_0(x)$ is defined by Eq. (A5)] or

$$J_1=t^{-1}\int_0^\infty dz[z^2(1+z^2/t^2)+1]^{-1/2}(1+z^2/t^2)^{-1/2} \ .$$

This integral can be divided into two terms, J_1' and J_2'. The first term contains integration from zero to B, and the second from B to infinity, where $1\ll B\ll t$. One can neglect the term Z^2/t^2 in the first integrand; the second term is equal to

$$I_2\simeq(t^{-1})\int_B^\infty dz\,z^{-1}(1+z^2/t^2)^{-1} \ .$$

After simple calculations we obtain in the limit $t\to\infty$

$$J\sim t^{-1}\ln t \ . \tag{A8}$$

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