Absorption of ballistic phonons by the (001) inversion layer of Si: Electron-phonon interaction in two dimensions

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We report the observation of absorption as a function of electron density of ballistic phonons in the two-dimensional electron gas in the (001) inversion layer of Si. Remarkably, the strength of the absorption is found to be an order of magnitude greater than can be explained by a conventional absorption model based on electron scattering in two dimensions and a deformation-potential electron-phonon interaction characteristic of bulk Si.

Despite two decades of experiments on the two-dimensional electron gas (2D EG) in the Si inversion layer,[1] there has been little headway in one important area, the electron— (acoustic-) phonon interaction (though intriguing hints that it is anomalously strong have occasionally surfaced). The problem is that in carrier transport, the canonical approach, the electron-phonon interaction is obscured by impurity and roughness scattering.[2] In this paper we report on a new transmission approach[3] wherein ballistic phonons are propagated through the 2D inversion layer in a Si metal-oxide-semiconductor field-effect transistor (MOSFET)—a reversal of the conventional roles of electrons and phonons which ameliorates previous problems. A negative fractional change in transmission $\Delta I/I$ is observed (hereinafter referred to as "absorption") which turns out to be an order of magnitude stronger than expected on a standard model. We suggest some possible reasons. There are also indications in this work that would recommend phonon absorption (and scattering) as a new way to probe the properties of the 2D EG proper.

It is generally believed that the mechanism for the absorption of a phonon by the 2D EG is the scattering, via the deformation potential interaction, of an electron across the 2D Fermi surface, a circle of diameter $2k_F$ for the (001) MOSFET. Since phase space for the scattering is severely restricted in 2D, the absorption is sharply peaked at phonon wave vectors of $2k_F$. In the present experiment we scan this absorption peak by varying $2k_F$ with the gate voltage of the MOSFET, in effect looking at transmission of phonons as the Fermi surface is expanded in size.

Figure 1 shows the prism-shaped Si sample with the MOSFET device fabricated on the (001) basal plane. Ballistic phonons radiate from the (pulsed) laser focal spot (area $A = 0.2$ mm^2) on one inclined face coated with an evaporated Constantan film, specularly reflect [angle $\theta = \arcsin(\frac{2}{3})^{1/2}$] from the Si-SiO$_2$ interface, and arrive at the detector, a superconducting granular Al bolometer, on the other inclined face—a path twice intersecting the inversion layer, typically 30 Å above the interface. A collimating slot in the median plane blocks direct propagation from heater to detector. The emitted phonons (wave vector $\vec{q}$, velocity v_s) have a Planckian spectrum,

$$U(q,T_h) \propto q^3/[\exp(\hbar v_s q/k_B T_h) - 1] \ ,$$

characterized by a heater temperature T_h given by[4]

$$P/A = \sigma(T_h^4 - T_0^4) \ , \tag{1}$$

where σ is a Stefan-Boltzman constant, P/A is the excitation power density, and T_0 is the lattice temperature. Isotope scattering modifies the emitted spectrum by a transmittance factor $\exp(-Alv_s^3 q^4)$, where l is the propagation distance and A is a scattering constant which for Si equals 0.16×10^{-44} sec^3, giving rise to a cutoff at $q_c \sim (3/4Alv_s^3)^{1/4} \sim 5.6 \times 10^6$ cm^{-1} (longitudinal modes).

The MOSFET is a large gate area (2.5×2.5 mm^2) device with the following conventional characterizations: (1) oxide layer 8500 Å thick; (2) p-type substrate impurity density 8×10^{12} B atoms/cm^3; (3) peak mobility 16 000 cm^2/V sec at 2.2 K; (4) threshold voltage $V_T = -1.3$ V (from conductance onset at 77 K); (5) charging rate $dn_s/dV_g = 2.53 \times 10^{10}$ (V cm^2)$^{-1}$ (from the period of Shubnikov–de Haas oscillations at 1.3 K). The electron density n_s in the inversion layer depends linearly on gate voltage V_g,

$$n_s = (dn_s/dV_g)(V_g - V_T) \ .$$

We performed the experiment by switching the gate voltage (at a 5-Hz rate) between V_T and some positive value V_g and observing the change ΔI in the intensity I of the phonon signal, time resolved by a boxcar integrator. [Figure 1 (inset) shows the ballistic spectrum of longitudinal acoustic (LA), transverse acoustic (TA), and mode conversion (MC) phonons.] Synchronous detection of ΔI followed by signal averaging gives a readout of phonon absorption $\Delta I/I$.

In Fig. 2 the LA-phonon absorption plotted versus $(V_g - V_T)^{1/2}$, which is directly proportional to $2k_F = 2\sqrt{\pi n_s}$ (upper scale), is seen to be rising steeply with increasing

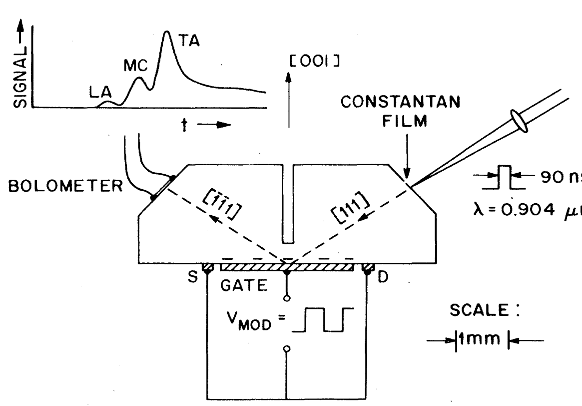

FIG. 1. Physical layout of the prism sample showing square-wave voltage modulation applied to the gate of the MOSFET. Inset: a typical time-resolved ballistic phonon spectrum.

$2k_F$, assuming a rather broad maximum at a magnitude of 1 to 2%, and then falling off. With increasing T_h the maxima of the profiles shift both upwards and toward large $2k_F$. These features can be understood intuitively by realizing that $\Delta I/I$ is a convolution of the absorption function $G(q)$ with $U(q,T_h)$. $G(q)$ is relatively sharp, so the convolution tends to replicate the latter when $2k_F$ is scanned. We see that this is qualitatively so. The $2k_F$ shift with T_h is a Wien-type displacement, but it is very nonlinear due to truncation of $U(q,T_h)$ by isotope scattering.

Turning next to a theory we follow a conventional approach to calculate phonon attenuation in a 2D EG. The absorption of a phonon (energy $\hbar\omega_q$) at 0 K scatters an electron from an occupied state $|\vec{k}| \leq k_F$ to an unoccupied state $|\vec{k}'| > k_F$, subject to conservation of crystal momentum (in the plane), $\vec{k}' = \vec{k} + \vec{q}_\parallel$, and energy, $E(\vec{k}') = E(\vec{k}) + \hbar\omega_q$, where $q_\parallel = q\sin\theta$. The transition rate for this process is given by Fermi's Golden Rule,

$$\begin{aligned}\Gamma(q) &= \frac{2\pi}{\hbar}|M_{e-\mathrm{ph}}|^2 g_s g_v \frac{A}{(2\pi)^2}\int_{|\vec{k}|\leq k_F,\ |\vec{k}+\vec{q}_\parallel|>k_F} d^2k\, \delta(E(\vec{k}+\vec{q}_\parallel)-E(\vec{k})-\hbar\omega_q) \\ &= \frac{2\pi}{\hbar}|M_{e-\mathrm{ph}}|^2 g_s g_v \frac{A}{(2\pi)^2}\mathcal{J} ,\end{aligned} \tag{2}$$

where $M_{e-\mathrm{ph}}$ denotes the matrix element of the electron-phonon interaction, A is a normalization area, and $g_s g_v$ represents the spin degeneracy $g_s = 2$ and valley degeneracy $g_v = 2$. Evaluation of the phase-space integral $\mathcal{J}$ gives

$$\frac{g_s g_v}{(2\pi)^2}\mathcal{J} = \frac{1}{q_\parallel}G(q_\parallel) \equiv \frac{g_s g_v m}{2\pi^2\hbar^2 q_\parallel}\begin{cases} 0, & k_F + ms/\hbar \leq q_\parallel/2 \\ \beta_-(q_\parallel), & k_F - ms/\hbar \leq q_\parallel/2 < k_F + ms/\hbar \\ \beta_-(q_\parallel) - \beta_+(q_\parallel), & 0 \leq q_\parallel/2 < k_F - ms/\hbar \end{cases} \tag{3}$$

for the case $ms/\hbar < k_F$, where

$$\beta_\pm(q_\parallel) = [k_F^2 - (q_\parallel/2 \pm ms/\hbar)^2]^{1/2}$$

and $s = v_s/\sin\theta$. The function $G(q_\parallel)$ is shown in the inset in Fig. 3.

The evaluation of $M_{e-\mathrm{ph}}$ proceeds as in 3D except that the electronic wave function is localized in the z direction perpendicular to the plane,

$$\psi = \zeta_0(z) A^{-1/2}\exp(i\vec{k}\cdot\vec{x}) ,$$

where

$$\zeta_0(z) = (2a_0^3)^{-1/2} z \exp(-z/2a_0)$$

is a variational wave function with thickness parameter a_0 for the lowest subband 0 ($\vec{x}$ is the coordinate vector in the plane). We thus obtain, for intrasubband scattering $0 \rightarrow 0$,

$$|M_{e-\mathrm{ph}}|^2 = \left(\frac{\hbar}{2\rho\omega_q V}\right) N q^2 D_l^2(\theta) F(q_z) , \tag{4}$$

where $q_z = q\cos\theta$, ρ is the crystal density, V is a normalization volume containing N phonons. The quantity $F(q_z)$ is a form factor, $F(q_z) = (1 + q_z^2 a_0^2)^{-3}$, cutting off the absorption at $q_z \approx 1/a_0$; $D_l(\theta)$ is a deformation potential (mode l) which in Herring's notation is $D_{\mathrm{LA}} = \Xi_d + \Xi_u\cos^2\theta$ for LA modes.

The absorption is $\Delta I(q)/I = -\Gamma(q)V/ANv_g\cos\theta$, where v_g is the phonon's group velocity, so finally we have

$$\frac{\Delta I(q)}{I} = -\frac{D_l^2(\theta)}{\rho v_s v_g \cos\theta\sin\theta}\pi G(q_\parallel) F(q_z) . \tag{5}$$

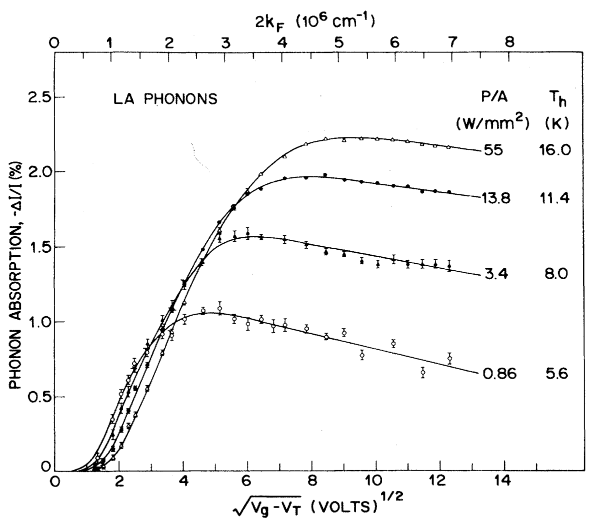

FIG. 2. Profiles of LA-phonon absorption in a 2D EG measured at $T_0 = 2.15$ K and angle $\theta = 54.7°$ for different heater excitation power densities P/A. Corresponding heater temperatures T_h are based on Eq. (1) (see text). Solid curves are drawn as an aid to the eye.

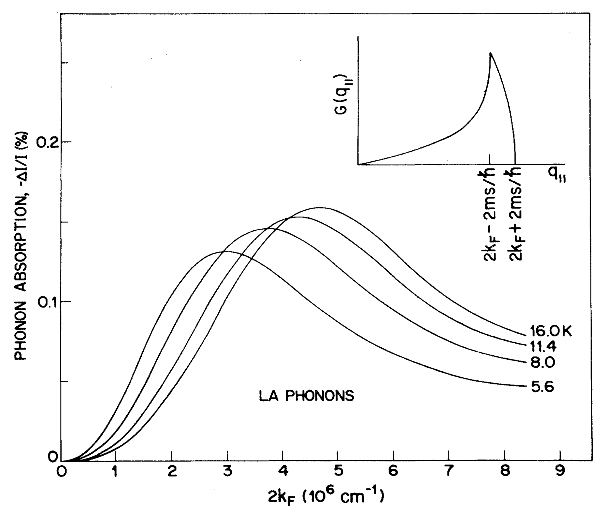

FIG. 3. LA-phonon absorption profiles for a 2D EG calculated by the convolution of the phonon absorption function $G(q_\parallel)$ (see inset) with phonon distribution functions for the given heater temperatures. Inset: $G(q_\parallel)$ for the case $ms/\hbar < k_F$.

Ignoring $F(q_z)$ [of order unity below cutoff, $q_z a_0 \sim 1$, which invariably falls outside the $2k_F$ cutoff of $G(q_{\parallel})$] we see that $\Delta I(q)/I$ separates into two parts: a prefactor governed by the strength of the electron-phonon interaction and a structural factor $G(q_{\parallel})$ which expresses the entire $\vec{q}$ dependence. We convolve Eq. (5) (doubled for 2 passes) with the phonon distribution function to obtain integrated LA absorption profiles as a function of $2k_F$ shown in Fig. 3. The material parameters (representative of "bulk" Si) are $\Xi_u = +9.0$ eV, $\Xi_d = -6.0$ eV, $m = 0.19 m_0$, $v_g = v_s = 9.4 \times 10^5$ cm/sec, and $\rho = 2.33$ g/cm^3.

Comparison of the theory, Fig. 3, with the data in Fig. 2 shows some features in common but on the whole the agreement is disappointing. Observed absorption is an order of magnitude stronger than theory, and at large $2k_F$ the shapes diverge, the observed absorption being relatively "flat" as compared to the theory. Inasmuch as the calculation involves no free parameters, it is impossible within the context of the model to adjust the fit. We conclude that one must seek explanations beyond the model. Of the possibilities considered, the two most likely candidates are the following:

(i) Dynamical properties of the 2D EG. Our simple model is a "static" model in that it regards the inversion layer as a passive absorber and ignores its dynamical response to the phonons. In other words, we have focused exclusively on the imaginary part of $\chi(\vec{q}, \omega)$, the complex linear response function of the 2D EG, as an attenuation mechanism. However, it may not be permissible to ignore phonon radiation from the driven 2D EG for which both $\mathrm{Re}\chi$ and $\mathrm{Im}\chi$ come into play.

(ii) Interfacial effects. The model, strictly speaking, is for the ideal case of transmission through a 2D EG isolated in homogeneous medium. In actuality, however—and it may be highly significant—absorption takes place within a phonon wavelength of the Si-SiO_2 interface.

The determination of T_h raises an interesting point. Our use of the simple, blackbody radiation law [Eq. (1)] for estimating T_h in Fig. 3 is dictated by the fact that it is reasonably consistent with the data. So-called acoustic-mismatch models,[5] with emissivity < 1, would raise T_h considerably and shift the calculated profiles away from the data towards larger $2k_F$. (On the other hand, inclusion of radiation loss into the He bath would act oppositely.) Direct measurements of the phonon distribution by the present procedure, made possible by the easy tunability of the absorption function $G(q_{\parallel})$ with voltage, could in principle sort out such matters and ultimately find wide application in phonon spectroscopy.

In passing we mention that we see no structural evidence in our data of a valley-occupancy transition in the 2D EG, a first-order, one- to two-valley transition predicted[6] to occur somewhere in the vicinity of $2k_F = 2 \times 10^6$ cm^{-1}.

We have gathered extensive absorption data for TA phonons. The TA phonon profiles are rather similar to the LA (and show the same discrepancies with theory) except that they are dispaced to larger $2k_F$ owing to a smaller v_s.

In summary, we have observed a surprisingly strong absorption of ballistic phonons in the 2D EG of the inversion layer of a Si MOSFET, an order of magnitude in fact stronger than can be accounted for on a simple absorption model. Moreover, there are structrual discrepancies manifested in the shapes of the absorption profiles. Some ideas have been advanced, but for the moment a viable explanation is still wanting. For the future the direction is toward monochromatic phonons; such experiments are underway.

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