Static shielding of an impurity near a surface in the presence of a magnetic field

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The static shielding of a nonmagnetic impurity embedded in a quantum plasma with a surface is examined. Linear-response theory is assumed for the interaction between the impurity and conduction electrons in a uniform positive jellium background and the response function is calculated in the non-self-consistent Hartree single-particle approximation. A uniform magnetic field is applied in a direction perpendicular to the surface. The unperturbed electron eigenstates which are used are those appropriate to a jellium model with planar boundaries simulated by infinite potential barriers. Two separate cases are considered which correspond to a contact interacton and a screened Coulomb potential between the impurity and conduction electrons. The Friedel-Kohn oscillations which are exhibited in the induced electron-number density are due to the logarithmic singularity in the response function.

I. INTRODUCTION

Since Friedel's work dealing with the static shielding of a fixed impurity embedded in a plasma was published in 1958,[1] there have been several important contributions to this problem. For example, in bulk shielding, Langer and Vosko[2] used linear-response theory and calculated the required response function in the random-phase approximation (RPA). The result of Langer and Vosko[2] for the induced electron-number density and shielded potential at temperature $T=0$ exhibits a Friedel-Kohn oscillatory behavior which varies as $\cos(2k_F r)/(k_F r)^3$, where k_F is the Fermi wave number and r is the distance from the impurity. It is the logarithmic singularity at $k=2k_F$ of the single-particle response function in the RPA bulk dielectric function which gives rise to Friedel-Kohn oscillations. However, the non-self-consistent Hartree single-particle approximation neglects the contribution to the induced electron-number density which is due to collective plasma modes. The mean-field Hartree result can be obtained by expanding the RPA dielectric function to linear order in the polarizability. Therefore, Friedel oscillations are common to both the RPA and non-self-consistent Hartree single-particle descriptions. Of course, an analytical relationship between the two shielding phenomena in bulk could be developed using the contour integral analysis of RPA shielding as described by Fetter and Walecka.[3] However, the Debye-Thomas-Fermi (DTF) shielding law is obtained in the RPA and not in the non-self-consistent Hartree single-particle approximation since the latter does not have the necessary pole structure to describe DTF type shielding.[4] Another reason why the Hartree single-particle theory is not satisfactory is that the induced electron-number density and shielded potential are not given in terms of the density response properties for excitations within a plasma. This means that in the case of dynamical shielding, where the frequency variable is not set equal to zero as it is for static shielding, the model response function in Hartree single-particle theory does not have poles which correspond to the active plasma resonances.

For an electron in a static, homogeneous magnetic field, the total kinetic energy is the sum of translational energy in the direction parallel to the magnetic field and the energy for cyclotron motion in the plane perpendicular to the field (Landau quantization). With the use of the non-self-consistent Hartree single-particle theory, Rensink[5] and Horing[6,7] have shown that, as a result of Landau quantization due to the presence of an external magnetic field, the induced electron-number density at large distances from the impurity is modified qualitatively compared with the result in the absence of a magnetic field. The calculations in Refs. 5—7 were restricted to bulk shielding. Beck, Celli, Lo Vecchio and Magnaterra[8] have calculated the static induced electron-number density and shielded potential in a semi-infinite plasma due to an impurity embedded near the surface, in the absence of a magnetic field. Linear-response theory was assumed for the interaction between the impurity and conduction electrons in a uniform positive jellium background, and the response function was calculated in the RPA. We note that Gadzuk[9] has also calculated some effects due to static shielding of an impurity by a quantum plasma with a surface in the absence of a magnetic field. However, Gadzuk's results take no account of the shielding due to a surface charge distribution. This means that the electron-gas dispersion is not adequately included in the calculation.

In this work we use the non-self-consistent Hartree single-particle approximation to study the effect of a surface and Landau quantization in determining the static shielding law of an impurity embedded in a plasma. This study should be qualitatively reasonable at large distances from the impurity in view of the approximate treatment of the model response function. In Sec. II we formulate the problem and in Sec. III we conclude with a discussion.

II. THE NON-SELF-CONSISTENT HARTREE SINGLE-PARTICLE THEORY

We calculate the electron-number density which is induced by an impurity with charge Ze embedded in a bounded plasma. As a model for the surface, an infinite-barrier model (IBM) is used. Linear-response theory is assumed for the interaction between the impurity and conduction electrons in a uniform positive jellium background, and the response function is calculated in the non-self-consistent Hartree single-particle theory. The point charge is inserted at $\vec{r}_0=(\vec{r}_{\|}^{(1)}, z_0)$ *within* the otherwise uniform plasma. The film is bounded by infinite potential barriers at $z=0$ and $z=L$, but is unbounded in the $r_{\|}$ plane (parallel to the surfaces). A magnetic field of strength H_0 is applied in the z direction.

In non-self-consistent Hartree single-particle theory, the nonlocal equation relating the static induced electron-number density $\delta\rho(\vec{r})$ with the effective potential V due to a single Coulomb center at $\vec{r}_0$ is

$$\delta\rho(\vec{r})=Ze\int d\vec{r}\,'\chi^0(\vec{r},\vec{r}\,';\omega=i0^+)V(\vec{r}\,'-\vec{r}_0) \ , \tag{1}$$

where the static response function $\chi^0(\vec{r},\vec{r}\,';\omega=i0^+)$ is given by

$$\chi^0(\vec{r},\vec{r}\,')=\sum_{i,i} \frac{f_0(E(k_z,n))-f_0(E(k_z',n'))}{E(k_z,n)-E(k_z',n')+i0^+} \times\phi_i(\vec{r})\phi_{i'}^*(\vec{r})\phi_i^*(\vec{r}\,')\phi_{i'}(\vec{r}\,') \ . \tag{2}$$

Here $\phi_i(\vec{r})$ is an electron eigenfunction of energy $E(k_z,n)$ where the label i stands for the quantum numbers (k_y,k_z,n), with the Landau levels labeled by $n=0,1,2,\ldots,$ and f_0 is the Fermi distribution function. For the IBM, where the single-particle potential within the plasma is assumed uniform in the absence of the inserted charge, a complete set of unperturbed eigenstates is given in Ref. 10.

For the purpose of identifying the contributions to $\delta\rho(\vec{r})$, which arise from classical specular reflection and quantum interference between incident and reflected electrons scattered off the surface, we rewrite Eq. (1) in terms of the Fourier transform of χ^0. The result is

$$\delta\rho(\vec{r})=Ze\frac{1}{L^2}\sum_{q_z,q_z'}\frac{1}{(2\pi)^2}\int d\vec{q}_{\|}\chi^0(q_z,q_z';q_{\|})\cos(q_z z)e^{i\vec{q}_{\|}\cdot\vec{r}_{\|}} \times\int_0^L dz'\int d\vec{r}_{\|}'\cos(q_z'z')e^{-i\vec{q}_{\|}'\cdot\vec{r}_{\|}'}V(\vec{r}\,'-\vec{r}_0) \ , \tag{3a}$$

where q_z and q_z' have the values 0, $\pm\pi/L$, $\pm 2\pi/L,\ldots,$ and

$$\chi^0(q_z,q_z';q_{\|})=\frac{1}{2L}\sum_{k_y,k_z}\sum_{n,n'}\frac{f_0(E(k_z,n))-f_0(E(k_z+q_z,n'))}{E(k_z,n)-E(k_z+q_z,n')} \times C_{nn'}(q_{\|})\frac{L}{2}(\delta_{q_z,\pm q_z'}-\delta_{q_z+2k_z,\pm q_z'}) \ . \tag{3b}$$

Here the matrix element $C_{nn'}(q_{\|})$ is given by

$$C_{nn'}(q_{\|})\equiv\frac{n!}{n'!}e^{-s}s^{n'-n}[L_n^{n'-n}(s)]^2 \ , \tag{3c}$$

where $s\equiv\hbar q_{\|}^2/2m^*\omega_c$, with the scalar effective mass of an electron equal to m^*. ω_c is the cyclotron frequency and $L_n^{n'-n}$ is a Laguerre polynomial. In Eq. (3b) the k_z sum extends over all multiples of π/L as well as zero and the first (diagonal) and second (nondiagonal) terms in square brackets are associated with classical specular scattering and quantum interference effects, respectively.

In this work, two cases are considered which correspond to a contact interaction and a screened Coulomb potential between the charged impurity and conduction electrons.

Case (1): $V(\vec{r}-\vec{r}_0)=e^2\delta(\vec{r}-\vec{r}_0)$. Substituting Eq. (3b) for $\chi^0(q_z,q_z';q_{||})$ into Eq. (3a), we obtain the induced electron-number density which is written as $\delta\rho(\vec{r})=\delta\rho_{\rm cl}(\vec{r})+\delta\rho_{q-i}(\vec{r})$, with $\delta\rho_{\rm cl}$ due to classical specular scattering and $\delta\rho_{q-i}$ due to quantum interference between the incident and reflected electrons scattered off the surface. Taking the limit $L\to\infty$ for the thickness of the film and making use of the symmetry of $C_{nn'}$ under interchange of n and n', it is a simple matter to show that at $T=0$,

$$\delta\rho_{\rm cl}(\vec{r})=-\frac{Ze^3}{(2\pi)^3}\left[\frac{m^*\omega_c}{2\pi^2\hbar}\right]\times\int_{-\infty}^{\infty}dq_z\int d\vec{q}_{||}\cos(q_z z)\frac{e^{i\vec{q}_{||}\cdot\vec{R}_{||}}}{\hbar^2 q_z/2m^*}\cos(q_z z_0)\sum_{n,n'}\eta_+[\mu-(n+\tfrac{1}{2})\hbar\omega_c]C_{nn'}(q_{||})\times\ln\left|\frac{\hbar(\frac{1}{2}q_z^2+k_F^{(n)}q_z)/m^*+(n'-n)\omega_c}{\hbar(\frac{1}{2}q_z^2-k_F^{(n)}q_z)/m^*+(n'-n)\omega_c}\right| , \tag{4a}$$

$$\delta\rho_{q-i}(\vec{r})=\frac{Ze^3}{(2\pi)^3}\left[\frac{m^*\omega_c}{2\pi^2\hbar}\right]\times\int_{-\infty}^{\infty}dq_z\int d\vec{q}_{||}\cos(q_z z)\frac{e^{i\vec{q}_{||}\cdot\vec{R}_{||}}}{\hbar^2 q_z/m^*}\times\sum_{n,n'}\eta_+[\mu-(n+\tfrac{1}{2})\hbar\omega_c]C_{nn'}(q_{||})\times\left[\cos\left[\frac{z_0(n'-n)\omega_c}{\hbar q_z/2m^*}\right][\mathrm{ci}(u_{n,n'}^{(+)})-\mathrm{ci}(u_{n,n'}^{(-)})-\mathrm{ci}(v_{n,n'}^{(+)})+\mathrm{ci}(v_{n,n'}^{(-)})]+\sin\left[\frac{z_0(n'-n)\omega_c}{\hbar q_z/2m^*}\right][\mathrm{si}(u_{n,n'}^{(+)})-\mathrm{si}(u_{n,n'}^{(-)})+\mathrm{si}(v_{n,n'}^{(+)})-\mathrm{si}(v_{n,n'}^{(-)})]\right] . \tag{4b}$$

Here $\vec{R}_{||}\equiv\vec{r}_{||}-\vec{r}_{||}^{(1)}$ and

$$u_{n,n'}^{(\pm)}(q_z)\equiv z_0\left[(n'-n)\omega_c+\frac{\hbar q_z^2}{2m^*}\pm\frac{\hbar q_z}{m^*}k_F^{(n)}\right]\left[\frac{2m^*}{\hbar q_z}\right] , \tag{5a}$$

$$v_{n,n'}^{(\pm)}(q_z)\equiv z_0\left[-(n'-n)\omega_c-\frac{\hbar q_z^2}{2m^*}\pm\frac{\hbar q_z}{m^*}k_F^{(n)}\right]\left[\frac{2m^*}{\hbar q_z}\right] , \tag{5b}$$

$$k_F^{(n)}\equiv\left[\frac{2m^*}{\hbar^2}[\mu-(n+\tfrac{1}{2})\hbar\omega_c]\right]^{1/2} , \tag{5c}$$

where μ is the chemical potential. si and ci are the sine and cosine integrals, respectively,[11] and the step function η_+ in Eq. (4) sets an upper limit on the sum over occupied states.

In the quantum strong field limit where all electrons are in the lowest Landau level ($\frac{1}{2}\hbar\omega_c<\mu<\frac{3}{2}\hbar\omega_c$), only the $n=0$ term contributes to the sums in Eq. (4). Equation (3c) gives $C_{n=0,n'}(q_{||})=e^{-s}s^{n'}/n'!$. For this case, the integration over $\vec{q}_{||}$ is easily done to yield

$$\int d\vec{q}_{\|} e^{i\vec{q}_{\|}\cdot\vec{R}_{\|}} \exp\left[-\frac{\hbar q_{\|}^2}{2m^*\omega_c}\right]\left[\frac{\hbar q_{\|}^2}{2m^*\omega_c}\right]^{n'} = \pi(n'!)\left[\frac{2m^*\omega_c}{\hbar}\right] {}_1F_1\left[n'+1;1;-\frac{m^*\omega_c}{2\hbar}R_{\|}^2\right] , \tag{6}$$

where ${}_1F_1$ is a degenerate hypergeometric function. Also, in the limit of very high magnetic field, the arguments of the logarithmic function in Eq. (4a) and the sine and cosine integrals in Eq. (4b) become

$$u_{n'}^{(\pm)}(q_z) \equiv z_0 q_z^{-1}[q_z \pm k_F(1+i\alpha_{n'})][q_z \pm k_F(1-i\alpha_{n'})] = -v_{n'}^{(\mp)}(q_z) , \tag{7}$$

where $i\alpha_{n'} \equiv (1-n'\hbar\omega_c/\mu)^{1/2}$. $\alpha_{n'}$ is real for $n' \geq 1$ but purely imaginary for $n'=0$. k_F is defined by $k_F \equiv (2m^*\mu/\hbar^2)^{1/2}$.

One may verify for the quantum strong-field limit that the contributions to $\delta\rho_{\text{cl}}(\vec{r})$ due to the higher-order $(n' \geq 1)$ terms are exponentially smaller than the $n'=0$ term by factors $\exp(-\alpha_{n'}k_F|z-z_0|)$ and $\exp[-\alpha_{n'}(z+z_0)]$. Therefore, neglecting these terms when $k_F|z-z_0| \gg 1$, $k_F(z+z_0) \gg 1$, and $(m^*\omega_c/2\hbar)^{1/2}R_{\|} \gg 1$, we obtain

$$\delta\rho_{\text{cl}}(\vec{r}) = \frac{Ze^3}{2\pi}\frac{\bar{n}p_H^2}{\hbar}\left[\frac{m^*}{2\mu}\right]^{1/2}\left[\frac{\cos(2k_F|z-z_0|)}{2k_F|z-z_0|} + \frac{\cos[2k_F(z+z_0)]}{2k_F(z+z_0)}\right]\exp(-\tfrac{1}{4}p_H^2R_{\|}^2) \tag{8}$$

since the degenerate hypergeometric function in Eq. (6) is equal to an exponential of the third argument when the first two arguments are equal. $p_H \equiv k_F(\hbar\omega_c/\mu)^{1/2}$ and, in the quantum strong-field limit, the electron density is given by $\bar{n} \equiv (m^{*3}\omega_c^2\mu/2)^{1/2}/\pi^2\hbar^2$. Equation (8) thus gives the analog of the Friedel-Kohn oscillatory behavior for static shielding of an impurity embedded in a semi-infinite quantum plasma. The result in Eq. (8) is due to classical specular scattering of electrons off the surface with the $|z-z_0|$ term and $(z+z_0)$ term due to the impurity and its image in the surface, respectively. The Gaussian decay in a direction perpendicular to the magnetic field is the same as that for a bulk plasma.

We calculate the contribution due to quantum interference in the non-self-consistent Hartree single-particle approximation by calculating $\delta\rho(\vec{r})$ in Eq. (4) numerically. For this calculation, we note that the cosine integral ci(x) has a logarithmic singularity at $x=0$. Therefore, ci(x) must be evaluated in the Cauchy principal value sense for negative values of x. The sine integral si(x) is an *entire* function of x. Figure 1 shows plots of the induced electron-number density of Eq. (4) in the presence of an extremely large magnetic field. The magnetic field H_0 is chosen as 200 kG, the scalar effective mass of an electron is $m^*=0.01m_e=0.911\times10^{-29}$ g, the electon-number density $\bar{n}=10^{17}$ cm^{-3}, and the chemical potential $\mu=0.125$ eV. Thus $k_F=(6\bar{n}\pi^2)^{1/3}=0.018$ Å^{-1}, $\hbar\omega_c=0.231$ eV, and $\hbar\omega_p=0.117$ eV. The quantum strong field limit $(\frac{1}{2}\hbar\omega_c<\mu<\frac{3}{2}\hbar\omega_c)$ is thus satisfied, i.e., all electrons with the same spin are in the lowest Landau level. The scalar effective mass of an electron and the electron-number density are those appropriate for a semiconductor such as InSb. In a metal such as Cu with an electron density $\bar{n}=10^{21}$ cm^{-3}, the value of the magnetic field which should be applied so that all electrons with the same spin are in the lowest Landau level is beyond presently available steady magnetic fields. It is known that an accumulation layer could be present at the surface of a semiconductor in an external electric field.[12] Aspects due to an accumulation layer are not considered in this work.

In Fig. 1 we have plotted the induced electron-number density $\delta\rho$ in the IBM and the classical infinite-barrier model (CIBM), where quantum interference between incident and reflected electrons scattered off the surface is ignored. The induced number density vanishes at the boundary $z=0$ for the IBM. This result is ensured since the unperturbed electron wave functions in Eq. (1) vanish at the boundary.[10] However, in the CIBM there could be an induced electron-number density at the boundary since the single-particle response function is approximated by its bulk value. This means that $\chi^0(q_z,q_z')$ in Eq. (3b) is approximated by the *diagonal* elements $\delta q_{z,\pm q_z'}$ only. The results of Fig. 1 show that the long-range oscillatory behavior of the induced electron-number density in the quantum strong-field limit has a Friedel-Kohn-type "wiggle" in a direction parallel to the magnetic field. With the calculations of Fig. 1, we could examine the effect due an impurity at varying depths below the surface of a degenerate plasma in the presence of an external magnetic field which satisfies the quantum strong-field limit. For an impurity at $z_0=2k_F^{-1}$, $5k_F^{-1}$, and $15k_F^{-1}$ [Figs. 1(a)–1(c)] within the plasma, the effect due to the off-diagonal quantum interference terms is to produce significant quantita-

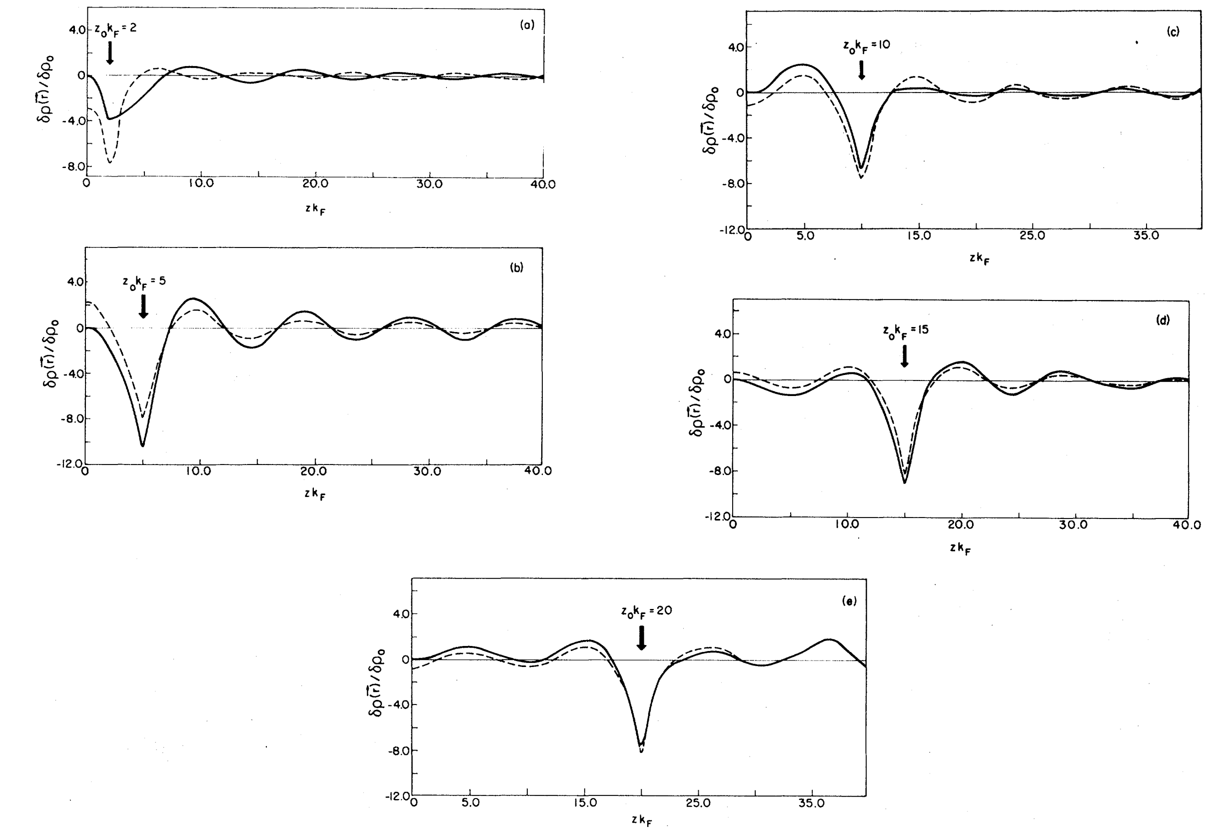

FIG. 1. Comparison of the induced electron-number density $\delta\rho(\vec r)$ for the infinite barrier model (IBM) and the classical infinite barrier model (CIBM) of a semi-infinite plasma. $\delta\rho$ is plotted as a function of distance into the plasma, along the polar axis perpendicular to the surface. An impurity is on the polar axis at z_0 within the plasma. The solid line shows the Friedel-Kohn oscillatory behavior for the IBM and the dashed line is for the CIBM. The magnetic field $H_0 = 200$ kG and the interaction between the impurity and conduction electrons is a contact potential. The calculations are based on the non-self-consistent Hartree single-particle theory. $\delta\rho_0$ is defined by $\delta\rho_0 \equiv Ze^3(k_F^2 p_H)^2/2\mu(2\pi)^4$.

tive changes in the static shielding of the source. When the impurity is moved to points farther away from the surface as shown in Figs. 1(d) and 1(e), the effect of quantum interference near the source is not as large. This is consistent with our discussion above that the effect of the off-diagonal matrix elements for the single-particle response function is to ensure that $\delta\rho$ vanishes at the boundary for the full IBM in contrast to the CIBM where the electronic properties are described by the bulk response function. Thus the closer to the surface the impurity is inserted within the plasma the larger would be the difference in $\delta\rho$ near the source since the response for the CIBM and the IBM is treated differently at the boundary. The non-self-consistent Hartree single-particle approximation gives only a qualitative representation of the shielding of a source. This method of calculation, the Hartree single-particle approximation, neglects plasmon dispersion. This means that in the case of dynamical shielding the model response function has no poles which correspond to the active plasmon resonances (bulk and surface). The RPA method of calculation treats plasmon dispersion satisfactorily. It would therefore be useful to examine the shielding of a source with the RPA description of the response properties of a plasma with a surface, in the presence of an external magnetic field.

Case (2): $V(\vec r - \vec r_0) = e^2(e^{-\kappa|\vec r - \vec r_0|}/|\vec r - \vec r_0|)$. For this shielded Coulomb potential with inverse screening length κ, we have

$$\int_0^L dz \int d\vec r_\parallel \cos(q_z' z) e^{-i\vec q_\parallel \cdot \vec r} V(\vec r - \vec r_0) = \frac{4\pi e^2}{Q^2 + (q_z')^2} e^{-i\vec q_\parallel \cdot \vec r_\parallel^{(1)}} \{\cos(q_z' z_0) - \tfrac{1}{2}[e^{-Qz_0} + (-1)^{m'} e^{-Q(L - z_0)}]\} , \qquad (9)$$

where $q_z' = m'\pi/L$ and $Q \equiv (q_{||}^2 + \kappa^2)^{1/2}$. Substituting Eqs. (3b) and (9) into Eq. (3a) and taking the limit $L \to \infty$, it is a simple matter to show that the induced electron-number density for a semi-infinite degenerate plasma for arbitrary magnetic field strength is given by $\delta\rho(\vec{r}) = \delta\rho_{cl}(\vec{r}) + \delta\rho_{q-i}(\vec{r})$, where the contribution due to classical specular scattering is

$$\delta\rho_{cl}(\vec{r}) = -\frac{Ze^3}{(2\pi)^3}\left[\frac{m^*\omega_c}{2\pi^2\hbar}\right]\int_{-\infty}^{\infty} dq_z \int d\vec{q}_{||}\cos(q_z z)\frac{e^{i\vec{q}_{||}\cdot\vec{R}_{||}}}{\hbar^2 q_z/2m^*}[\cos(q_z z_0) - \tfrac{1}{2}e^{-Qz_0}]\frac{4\pi}{Q^2+q_z^2} \times \sum_{n,n'} \eta_+[\mu - (n+\tfrac{1}{2})\hbar\omega_c]C_{nn'}(q_{||}) \times \ln\left|\frac{\hbar(\frac{1}{2}q_z^2 + k_F^{(n)}q_z)/m^* + (n'-n)\omega_c}{\hbar(\frac{1}{2}q_z^2 - k_F^{(n)}q_z)/m^* + (n'-n)\omega_c}\right| \tag{10a}$$

and the contribution due to quantum interference is

$$\delta\rho_{q-i}(\vec{r}) = -\frac{Ze^3}{(2\pi)^3}\left[\frac{m^*\omega_c}{2\pi^2\hbar}\right]\int_{-\infty}^{\infty} dq_z \int d\vec{q}_{||}\cos(q_z z)e^{i\vec{q}_{||}\cdot\vec{R}_{||}} \times \sum_{n,n'}\eta_+[\mu - (n+\tfrac{1}{2})\hbar\omega_c]C_{nn'}(q_{||}) \times [R_{nn'}(q_z,q_{||}) - \tfrac{1}{2}e^{-Qz_0}R_{nn'}^0(q_z,q_{||})] \tag{10b}$$

where

$$R_{nn'}(q_z,q_{||}) \equiv \int_{q_z-2k_F^{(n)}}^{q_z+2k_F^{(n)}} dx \frac{\cos(z_0 x)}{(n-n')\hbar\omega_c - (\hbar^2 q_z/2m^*)x}\frac{4\pi}{Q^2+x^2} . \tag{10c}$$

The function R depends on the value of z_0 and R^0 is the value of R for $z_0 = 0$.

For magnetic fields so strong that all electrons are in the lowest Landau level, only the $n=0$ term contributes to the sum over n in Eq. (10). For the sum over n', the contributions to $\delta\rho$ due to the higher-order ($n' \geq 1$) terms are exponentially smaller than the $n'=0$ term at large distances from the source and its image in the surface along the direction parallel to the magnetic field. That is, when $k_F|z - z_0| \gg 1$ and $k_F(z+z_0) \gg 1$, it could be shown with the use of Eq. (10a) that for a degenerate plasma in the quantum strong-field limit

$$\delta\rho_{cl}(\vec{r}) = \frac{Ze^3}{(2\pi)^2}\frac{2\bar{n}}{\hbar}\left[\frac{m^*}{2\mu}\right]^{1/2}\int d\vec{q}_{||}e^{i\vec{q}_{||}\cdot\vec{R}_{||}}\exp\left[\frac{-q_{||}^2}{p_H^2}\right] \times \left[\frac{\cos(2k_F|z-z_0|)}{2k_F|z-z_0|} + \frac{\cos[2k_F(z+z_0)]}{2k_F(z+z_0)} - e^{-Qz_0}\frac{\cos(2k_F z)}{2k_F z}\right]\frac{4\pi}{Q^2+(2k_F)^2} . \tag{11}$$

The dependence of $\delta\rho_{cl}(\vec{r})$ on the variable $\vec{r}_{||}$ is obtained by evaluating the integrals

$$\mathcal{S}_1 = \frac{1}{(2\pi)^2}\int d\vec{q}_{||}e^{i\vec{q}_{||}\cdot\vec{R}_{||}}\frac{4\pi}{Q^2+(2k_F)^2} \times \exp\left[\frac{-q_{||}^2}{p_H^2}\right] , \tag{12}$$

$$\mathcal{S}_2 = \frac{1}{(2\pi)^2}\int d\vec{q}_{||}e^{i\vec{q}_{||}\cdot\vec{R}_{||}}\frac{4\pi}{Q^2+(2k_F)^2} \times \exp\left[-z_0 Q - \frac{q_{||}^2}{p_H^2}\right] . \tag{13}$$

The calculation of $\mathcal{S}_1$ for large values of $R_{||}$ has been discussed in detail by Horing[6] for the unshielded ($\kappa=0$) Coulomb potential. With a straightforward generalization of the method of Ref. 6 to the case when the value of κ is finite, we obtain

$$\mathscr{S}_1 \approx (2\pi)^{1/2} \exp\left[\frac{\kappa^2+(2k_F)^2}{p_H^2}\right] \times \frac{\exp\{-R_{||}[\kappa^2+(2k_F)^2]^{1/2}\}}{R_{||}^{1/2}[\kappa^2+(2k_F)^2]^{1/4}} \quad (14)$$

when $R_{||}p_H \gg 1$ and $R_{||}[\kappa^2+(2k_F)^2] \gg 1$. It is straightforward to show that $\mathscr{S}_2$ in Eq. (13) could be rewritten in terms of the Bessel function J_0 as

$$\mathscr{S}_2 = 2\int_0^\infty dx \exp[-x^2 - z_0(\kappa^2+x^2p_H^2)^{1/2}] \times \frac{x}{x^2+[\kappa^2+(2k_F)^2]/p_H^2} J_0(R_{||}p_H x) \ . \quad (15)$$

In general, the approximate evaluation of integrals with oscillating kernels could be done using the comparatively simple method of integration by parts. Thus we obtain for $R_{||}p_H \gg 1$

$$\mathscr{S}_2 \approx \frac{2}{R_{||}^2} \mathscr{I}_2(0) \frac{e^{-\kappa z_0}}{\kappa^2+(2k_F)^2} \ , \quad (16)$$

where the function $\mathscr{I}_2$ is defined by

$$\mathscr{I}_1(t) = \int_t^\infty dt' J_0(t') \ , \quad (17a)$$

$$\mathscr{I}_n(t) = \int_t^\infty dt' \mathscr{I}_{n-1}(t'), \quad n \geq 2 \ . \quad (17b)$$

Comparing the results in Eqs. (14) and (16), we find that for $\kappa z_0 \sim 1$, $\mathscr{S}_2$ makes a more significant contribution to $\delta\rho_{cl}$ compared with $\mathscr{S}_1$ in the limit $R_{||}p_H \gg 1$ for a screened Coulomb potential. That is, we have for $\kappa z_0 \sim 1$, at large distances from the source and its image in the surface in a direction parallel to the magnetic field and with $R_{||}p_H \gg 1$,

$$\delta\rho_{cl}(\vec{r}) \approx Ze^3 \frac{4\bar{n}}{\hbar}\left[\frac{m^*}{2\mu}\right]^{1/2} \frac{e^{-\kappa z_0}}{\kappa^2+(2k_F)^2} \times \frac{1}{R_{||}^2} \frac{\cos(2k_F z)}{2k_F z} \ . \quad (18)$$

For $\kappa z_0 \gg 1$, the contibution due to $\mathscr{S}_1$ dominates when the conditions $k_F|z-z_0| >> 1$, $k_F(z+z_0) \gg 1$, and $R_{||}p_H \gg 1$ are satisfied, and

$$\delta\rho_{cl}(\vec{r}) \approx Ze^3 \frac{2\bar{n}}{\hbar}\left[\frac{m^*}{2\mu}\right]^{1/2} \exp\left[\frac{\kappa^2+(2k_F)^2}{p_H^2}\right]\left[\frac{2\pi}{[\kappa^2+(2k_F)^2]^{1/2}R_{||}}\right]^{1/2} \times \exp\{-[\kappa^2+(2k_F)^2]^{1/2}R_{||}\}\left[\frac{\cos(2k_F|z-z_0|)}{2k_F|z-z_0|} + \frac{\cos[2k_F(z+z_0)]}{2k_F(z+z_0)}\right] \ . \quad (19)$$

This means that for $\kappa z_0 \sim 1$, the damping is exponential in a direction perpendicular to the magnetic field compared with a power-law damping when $\kappa z_0 \gg 1$. The condition $\kappa z_0 \gg 1$ thus deprives $\delta\rho_{cl}$ of long-range oscillations in a direction perpendicular to the magnetic field. However, for both Eqs. (18) and (19), the Friedel-Kohn oscillatory behavior persists in the direction parallel to the magnetic field. We note that the term arising from e^{-Qz_0} in Eq. (11) is due to the Fourier transform of the screened Coulomb potential in Eq. (9) for the bounded plasma. That is, $\mathscr{S}_2$ is due to a surface effect on the finite range of the interaction. Numerical results have been obtained for this case involving a finite-range potential with the impurity at varying distances from the surface. The results are in qualitative agreement with the results of Fig. 1 for the contact potential in the non-self-consistent Hartree single-particle approximation.

III. CONCLUDING REMARKS

With the use of a high-resolution, electrostatic electron spectrometer, and a Mössbauer spectrometer, depth-selective Mössbauer spectra from the surface of appropriate materials could be obtained.[13] In this experiment, emitted conversion electrons are detected at selected electron energies. Depth-selective conversion-electron Mössbauer spectroscopy (DCEMS) would thus appear to be a plausible technique to examine the depth dependence of the static shielding of impurites in a Mössbauer absorber. A difficulty, however, with such a surface experiment is in extracting from the data the information which one could attribute to static shielding. An external magnetic field which is so strong that the data in a DCEMS experiment is significantly different from the data in the absence of an external magnetic field would therefore be a useful tool in examining depth-dependent shielding.

[1]J. Friedel, Nuovo Cimento Suppl. 2, 287 (1958).
[2]J. S. Langer and S. H. Vosko, J. Phys. Chem. Solids 12, 196 (1959).
[3]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971), pp. 175–180.
[4]N. J. Horing, Ann. Phys. (N.Y.) 54, 405 (1969).
[5]M. E. Rensink, Phys. Rev. 174, 744 (1968).
[6]N. J. Horing, Phys. Rev. 186, 434 (1969).
[7]N. J. Horing, J. Phys. Soc. Jpn. 21, Suppl. 704 (1966).
[8]D. E. Beck, V. Celli, G. Lo Vecchio, and A. Magnaterra, Nuovo Cimento B 68, 230 (1970).
[9]J. W. Gadzuk, Solid State Commun. 5, 743 (1967); J. Phys. Chem. Solids 30, 2307 (1969).
[10]G. Gumbs and A. Griffin, J. Phys. F 12, 1185 (1982).
[11]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic, New York, 1980), p. 928.
[12]J. A. Appelbaum and G. A. Baraff, Phys. Rev. B 4, 1235 (1971).
[13]Z. Bonchev, A. Jordanov, and A. Minkova, Nucl. Instrum. Methods 70, 36 (1969).