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Direct generation of ultrasound by electromagnetic radiation in metals in a magnetic field. An integral-equation approach

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A study is given of the dependence of the amplitude of an electromagnetically generated acoustic wave in a metal as a function of a dc magnetic field. The incident electromagnetic wave propagates normally to a plane metal surface. The dc magnetic field is applied in this same direction. This work describes the effect on acoustic generation, under these conditions, of the manner in which the conduction electrons are scattered at the surface. The mathematical procedure consists in the solution, under anomalous skin-effect conditions, of an integral equation using the Fredholm determinantal method. The amplitude of the acoustic wave generated is obtained with the assumption that the electrons are scattered diffusely at the surface. The results obtained are in qualitative and reasonable quantitative agreement with experiments in potassium.

I. INTRODUCTION

In a recent paper[1] (hereafter referred to as I), the authors presented a theory of direct generation of ultrasound by electromagnetic radiation in metals in the presence of a uniform magnetic field $\vec{B}_0$. The analytical methods described in I provide algorithms for the calculation of the amplitudes of the acoustic and electromagnetic waves inside the metal as functions of B_0. These are, however, difficult to handle in a computer and some of the numerical results obtained in I are not sufficiently accurate.[2] The purpose of this paper is twofold. We develop an alternative algorithm to solve this problem with a precision difficult to attain with the methods of I. We also give additional results which can be compared with the experimental data. The notation of this paper is identical to that of I. In contrast to the scope of I, we limit ourselves here to the free-electron model of a metal.

We suppose an initially linearly polarized plane electromagnetic wave with an electric field of amplitude E_0 to be incident normally on the surface of a semi-infinite metal occupying the region $z \geq 0$ of space. We refer the components of all field variables to a Cartesian coordinate system (x,y,z). The unit vector $\hat{z}$ is, of course, normal to the surface and $\hat{x}$ is chosen parallel to the direction of polarization of the incident wave. The dc magnetic field $\vec{B}_0$ is taken parallel to $\hat{z}$. We designate by ω the angular frequency of the wave; the electric and magnetic fields of the wave are taken as the real parts of $\vec{E}(z)\exp(-i\omega t)$ and $\vec{B}(z)\exp(-i\omega t)$. The presence of $\vec{B}_0$ causes a rotatory power in the electron gas of the metal so that all transverse field variables inside the metal as well as the reflected electromagnetic wave possess both x and y components. For simplicity the metal is assumed electrically and elastically isotropic. We denote its density by ρ, the speed of shear acoustic waves by s and the displacement field of the transverse sound wave generated by the real part of $\vec{\xi}(z)\exp(-i\omega t)$.

If $\vec{B}_0$ is sufficiently strong, neglecting the attenuation of the acoustic wave, the equation of motion for $\vec{\xi}(z)\exp(-i\omega t)$ is

$$-\rho\omega^2\vec{\xi}(z)=\rho s^2\,\vec{\xi}\,''(z)+c^{-1}\,\vec{j}\,(z)\times\vec{B}_0\ . \qquad (1)$$

We describe later under what conditions Eq. (1) must be modified to provide a description of phenomena occurring in the region of weak $\vec{B}_0$. Here

$\vec{j}(z)\exp(-i\omega t)$ is the electric current density excited in the metal by the electromagnetic wave. Equation (1) expressed in terms of the circular components,

$$\xi_{\pm}(z)=\xi_x(z)\pm i\xi_y(z) , \tag{2}$$

is

$$\xi''_{\pm}(z)+(\omega/s)^2\xi_{\pm}(z)=\pm(iB_0/\rho s^2c)j_{\pm}(z) . \tag{3}$$

The solution of this equation subject to the condition that there be no reflected acoustic wave is of the form [see I, Eq. (3.9)]

$$\xi_{\pm}(z)=\xi_{\pm}(\infty)\exp(i\omega z/s) \tag{4}$$

for large values of z. The coefficient $\xi_{\pm}(\infty)$ is given by

$$\xi_{\pm}(\infty)=-(is/\omega)\xi'_{\pm}(+0) \pm(B_0/2\rho\omega sc)j_{\pm}(\omega/s) , \tag{5}$$

where $\xi'_{\pm}(+0)=\xi'_{\pm}(z=+0)$ and $j_{\pm}(\omega/s)$ is the cosine-Fourier transform of $j_{\pm}(z)$, namely,

$$j_{\pm}(q)=2\int_0^{\infty} j_{\pm}(z)\cos qz\, dz , \tag{6}$$

evaluated at the wave vector ω/s. The quantity $\vec{\xi}'(+0)$ is a measure of the strain at the surface and can be disregarded for sufficiently large magnetic fields B_0. Furthermore, if the scattering of the electrons at the surface $z=0$ is specular, $\vec{\xi}'(+0)$ vanishes. The relation between the electric current density $\vec{j}(z)$ and the electric field $\vec{E}(z)$ when the electron mean free path l is long compared to the skin depth δ of the metal has been discussed by many authors.[3] Under such conditions

$$\delta=(4v_Fc^2/3\pi\omega\omega_p^2)^{1/3} , \tag{7}$$

where ω_p is the plasma frequency, gives a measure of the penetration of the electromagnetic field into the material. When B_0 is sufficiently intense for the cyclotron resonance frequency $\omega_c=eB_0/mc$ to be large compared to $\omega v_F/s$ where v_F is the Fermi velocity of the conduction electrons, local transport theory is valid. It is then possible to write

$$j_{\pm}(\omega/s)=\sigma_0(1-i\omega\tau\mp i\omega_c\tau)^{-1}E_{\pm}(\omega/s) , \tag{8}$$

where $\tau=l/v_F$ is the average time between two successive collisions of an electron and $\sigma_0=(\omega_p^2\tau/4\pi)$ is the dc conductivity of the metal. Use of the Maxwell equations in the frequency region in which ω is much less than τ^{-1} and $4\pi\sigma_0(s/c)^2$, taking due account of the electromagnetic boundary conditions at $z=0$, gives

$$\xi_{\pm}(\infty)=\pm E_0B_0/2\pi\rho\omega s . \tag{9}$$

Thus for large magnetic fields $\xi_x(\infty)$ is equal to zero and $\xi_y(\infty)$ becomes linear in B_0, a result in agreement with the experimental work of Wallace *et al.*[4] and of Chimenti *et al.*[5] We remark that the data is obtained measuring the amplitude of the acoustic wave far from $z=0$ but is displayed corrected to its value at the plane of incidence using the measured coefficient of attenuation.[5] Thus $\vec{\xi}(\infty)$ corresponds to the quantity actually measured.

At zero and low magnetic field intensities two observed features, inconsistent with Eq. (9), appear: (1) $\xi_x(\infty)=\xi(\infty)$ does not vanish at $B_0=0$, and (2) $\xi_y(\infty)$, starting from zero at $B_0=0$, increases with increasing B_0, going first through a maximum, then a minimum, before becoming linear in B_0.

The first phenomenon can be understood under anomalous-skin-effect conditions ($l\gg\delta$).[6] In such circumstances, in addition to the elastic and Lorentz forces displayed in Eq. (1) there is an alternating torque as we now explain. The positive ions of the metal, which we take to have charge γe, are accelerated by the electric field of the electromagnetic wave. The electrons also experience an acceleration giving rise, due to collisions, to a steady-state current density $\vec{j}(z)\exp(-i\omega t)$. In this process the electrons transfer continuously their excess momentum to the positive ions producing an effective force per ion $\vec{F}_c$, called the collision drag force. These two forces are equal and opposite because the metal is electrically neutral. However, if $l\gg\delta$, their resultants are displaced relative to each other, $\gamma e\vec{E}$, the direct force, being effective within the penetration depth δ and $\vec{F}_c$ over the longer distance l from the surface. This is the origin of the shear stress effective even when $B_0=0$.

The explanation given above does not depend on the manner in which the electrons are scattered at the surface of the metal. The simplest assumption is that the scattering is specular. The theory of ultrasonic generation with this boundary condition was discussed by Quinn,[6] by Alig,[7] and used by Chimenti *et al.*[5]; in this form, it is unable to account for the nonmonotonic behavior of $\xi_y(\infty)$ as a function of B_0. However, if we suppose that the electrons are scattered diffusely at $z=0$ (i.e., upon collision with the surface, an electron transfers to it, on the average, the tangential component of its momentum) then the effect can be understood.

In this case, in addition to the forces $\gamma e\vec{E}$ and $\vec{F}_c$, there is a surface force $\vec{F}_s$ acting only on the atoms on the surface $z=0$ and lying parallel to this plane. This force, together with $\gamma e\vec{E}$ and $\vec{F}_c$, produces the torque responsible for acoustic generation even when $B_0=0$. As B_0 increases, all these forces acquire a y component. The nonmonotonic behavior of $\xi_y(\infty)$ can be attributed to a change in the relative phases of the torques resulting from γeE_y and

F_{cy} on the one hand and $\gamma e E_y$ and F_{sy} on the other. In I we estimated that, under anomalous skin-effect conditions, this occurs when $\omega_c \tau \sim \sqrt{3}$. However, it should be emphasized that this marks only the beginning of the change and does not coincide with the value of B_0 for which $\xi_y(\infty)$ is a minimum. Even though the resultant of $\vec{F}_c$ and $\vec{F}_s$ must be equal and opposite to $\gamma e \vec{E}$, it is not legitimate, as was done by Banik and Overhauser,[8] to assume that the torque arising from these forces is smaller in the case of diffuse scattering than it is for specular scattering. In fact, there is not only the change in relative phase of the forces as the magnetic field increases but, as we shall see later, the electric field at the surface is larger for diffuse scattering than it is for specular scattering.[9] As will be shown in Sec. III, for $B_0 \neq 0$, the component E_y is enhanced by a factor of approximately 2.

In I, the calculations of $\vec{E}(z)$ and $\vec{\xi}(z)$ in the case of diffuse scattering were carried out using the Wiener-Hopf method.[10] In the present work we use the Fredholm determinantal method to solve an appropriate integral equation for the Fourier transform of the electric field. In addition we have performed similar calculations based on an iteration procedure to compare our results with other work in the field.

The analytical part of this program is carried out in Sec. II. In Sec. III we display the results of the numerical calculations for parameters appropriate to potassium and discuss their relevance to the experimental results. The numerical studies have been made for a frequency $(\omega/2\pi) = 8.97$ MHz and for various electron mean free paths of the order of those estimated for their samples by Chimenti *et al.*[5]

II. ANALYSIS OF THE FIELDS WITHIN A METAL

A. Integral equation for the Fourier components of the electric field

The components of the electric field $\vec{E}(z)$ for $z \geq 0$ satisfy the equations[1]

$$E''_{\pm}(z) = -(4\pi i \omega \sigma_0 / c^2) \times \int_0^\infty d\zeta E_{\pm}(\zeta)[G_{\pm}(z-\zeta) + pG_{\pm}(z+\zeta)] , \quad (10)$$

where p is the probability that an electron is scattered specularly at $z=0$, while $1-p$ is the probability of diffuse scattering. The functions $G_{\pm}(z)$ and their Fourier transforms are

$$G_{\pm}(z) = (3/4l) \int_0^{\pi/2} \sin^2\theta \tan\theta \exp[-(1-i\alpha_{\pm})\,|z|\,/l \cos\theta] d\theta , \quad (11)$$

and

$$G_{\pm}(q) = \int_{-\infty}^{\infty} G_{\pm}(z) \exp(-iqz) dz = \tfrac{3}{4} \int_0^{\pi} \sin^3\theta (1 - i\alpha_{\pm} + i\beta \cos\theta)^{-1} d\theta . \quad (12)$$

Here

$$\beta = ql \quad (13)$$

and

$$\alpha_{\pm} = (\omega \pm \omega_c)\tau . \quad (14)$$

We note that, since the functions $G_{\pm}(z)$ are even in z, $G_{\pm}(-q) = G_{\pm}(q)$. The equations (10) are transformed into equivalent integral equations for the Fourier transforms of $E_{\pm}(z)$ in a straightforward procedure. Since the field $\vec{E}(z)$ is defined for $z \geq 0$ we use the cosine-Fourier transform

$$E_{\pm}(q) = 2 \int_0^\infty E_{\pm}(z) \cos qz \, dz , \quad (15)$$

yielding even functions of q. The inverse of (15) is

$$E_{\pm}(z) = \frac{1}{\pi} \int_0^\infty E_{\pm}(q) \cos qz \, dq \quad (16)$$

and gives the components of $\vec{E}(z)$ for $z \geq 0$. We obtain

$$-2E'_{\pm}(+0) - q^2 E_{\pm}(q) = -(4i\omega E_0 / c) - q^2 E_{\pm}(q) = -\frac{4\pi i \omega \sigma_0}{c^2} \left[G_{\pm}(q) E_{\pm}(q) + \frac{1-p}{\pi^2} \int_0^\infty dq' E_{\pm}(q') \int_{-\infty}^{\infty} dk \frac{G_{\pm}(k) k^2}{(k^2 - q^2)(k^2 - q'^2)} \right] \quad (17)$$

where in the last integral the Cauchy principal value is intended. Here[11]

$$E'_{\pm}(+0)=E'_{\pm}(z=+0)=(2i\omega E_0/c)\ . \qquad (18)$$

Substitution of the values of $G_{\pm}(k)$ using Eq. (12) leads to the more tractable equations,

$$F_{\pm}(\beta)-(1-p)S_{\pm}(\beta)\int_0^{\infty}U_{\pm}(\beta,\beta')F_{\pm}(\beta')d\beta'=S_{\pm}(\beta)\ . \qquad (19)$$

Here we have used the following notation:

$$S_{\pm}(\beta)=\left[i\beta^2+\frac{4}{3\pi}\left[\frac{l}{\delta}\right]^3\widetilde{G}_{\pm}(\beta)\right]^{-1}, \qquad (20)$$

where we have denoted $G_{\pm}(\beta/l)$ by $\widetilde{G}_{\pm}(\beta)$,

$$U_{\pm}(\beta,\beta')=\frac{2}{\pi^2}\left[\frac{l}{\delta}\right]^3(\beta^2-\beta'^2)^{-1}\times\left[f\left[\frac{\beta'}{1-i\alpha_{\pm}}\right]-f\left[\frac{\beta}{1-i\alpha_{\pm}}\right]\right] \qquad (21)$$

with

$$f(z)=\tfrac{1}{2}(1+z^{-2})\ln(1+z^2)\ . \qquad (22)$$

The functions $F_{\pm}(\beta)$ are a measure, in dimensionless form, of the components of the Fourier transform of $\vec{E}(z)$ and are defined by

$$F_{\pm}(\beta)=(c/4\omega l^2E_0)E_{\pm}(\beta/l)\ . \qquad (23)$$

In the definition of $f(z)$ for z complex we select the Riemann sheet for which $\ln(1+z^2)$ has the value zero at $z=0$.

When $p=1$ (specular scattering), Eq. (19) simply gives

$$F_{\pm}^{(S)}(\beta)=F_{\pm}(\beta)=S_{\pm}(\beta)\ .$$

In the other extreme of purely diffuse scattering, $p=0$, the components of the electric field were calculated numerically. The procedure is described in Sec. III. It is possible, however, to solve Eq. (19) by iteration, the zero-order solution being $F_{\pm}^{(S)}(\beta)$. Since the surface impedances $Z^{(D)}$ and $Z^{(S)}$ for diffuse and specular scattering differ by a factor of $\frac{9}{8}$ for $B_0=0$ we expect this procedure to be rapidly convergent.[3] Replacing p by zero and $F_{\pm}(\beta')$ by $S_{\pm}(\beta')$ in the integral in Eq. (19) we find a first approximation to $F_{\pm}^{(D)}(\beta)$, the reduced Fourier transforms of the components of the electric field. It is more convenient to display $E_{\pm}^{(D)}(z)$ using

$$E_{\pm}(z)=(4\omega lE_0/\pi c)\times\int_0^{\infty}F_{\pm}(\beta)\cos(\beta z/l)d\beta\ . \qquad (24)$$

We obtain

$$E_{\pm}^{(D)}(z)\cong E_{\pm}^{(S)}(z)+(4\omega lE_0/\pi c)\times\int_0^{\infty}d\beta\cos(\beta z/l)S_{\pm}(\beta)\times\int_0^{\infty}d\beta' U_{\pm}(\beta,\beta')S_{\pm}(\beta')\ . \qquad (25)$$

This expression can be reduced in the limit of large l/δ to a more convenient one by noticing that the largest contributions to integrals containing $S(\beta)$ [$S_{\pm}(\beta)$ for $B_0\neq 0$] come from a region in β centered at about l/δ with a width of the same order. Since, for $\beta\gg 1$ and $B_0=0$, $\widetilde{G}(\beta)\approx 3\pi/4\beta$, the contributions just mentioned are of order δ/l while those for $\beta\ll l/\delta$ and $\beta\gg l/\delta$ are of higher order in δ/l. Thus, denoting l/δ by β_0 we find

$$E^{(D)}(z)\approx E^{(S)}(z)-\frac{8\omega lE_0\beta_0^3}{\pi^3c}\int_0^{\infty}d\beta\frac{\beta\cos(\beta z/l)}{\beta^3-i\beta_0^3}\int_0^{\infty}d\beta'\frac{\beta'\ln(\beta'/\beta)}{(\beta'^2-\beta^2)(\beta'^3-i\beta_0^3)}\ . \qquad (26)$$

Setting $\beta'=\beta e^{t'}$ in the last integral and subsequently $\beta=\beta_0e^{t}$ in the other and using the further transformation $t+t'=v, t'=u$ we find

$$E^{(D)}(z)\approx E^{(S)}(z)-\frac{1}{\pi^2}\int_0^{\infty}du\frac{u}{\sinh u+\sinh 2u}\left[\frac{e^{-u}E^{(S)}(z)-E^{(S)}(ze^{-u})}{1-e^{-u}}\right]\ . \qquad (27)$$

This result is similar to, but differs from, that given by Chambers.[12] In fact, Chambers's result would predict $E^{(D)}(0)=E^{(S)}(0)$ in agreement with his plot of these fields in Fig 4.5, p. 205 of Ref. 12. Setting $z=0$ in Eq. (27) gives

$$E^{(D)}(0)=E^{(S)}(0)\left[1+\frac{1}{\pi^2}\int_{-\infty}^{\infty}du\frac{u}{\sinh u+\sinh 2u}\right]\approx 1.093E^{(S)}(0)\ , \qquad (28)$$

in agreement with Eq. (4.68) of Chambers' paper[12] but in disagreement with his own Eq. (4.69). Thus we conclude that Eq. (4.69) and Fig. 4.5 of Ref. 12 are incorrect.

B. Amplitude of the acoustic wave generated

To obtain the equation of motion for $\vec{\xi}(z)$ we add to the right-hand side of Eq. (1) the direct force per unit volume $(\rho\gamma e/M)\vec{E}(z)$ and the collision drag force per unit volume[13]

$$(\rho/M)\vec{F}_c = -(\rho\gamma e/M\sigma_0)\,\vec{j}\ . \tag{29}$$

Here M is the mass of the positive ions in the metal. The differential equations for $\xi_\pm(z)$ can be solved in a manner identical to that used to obtain the solutions of Eq. (1). The surface force is taken into account by obtaining $\xi'_\pm(+0)$ from the stress tensor of the electron gas. From the result of I we have

$$\xi'_\pm(+0) = \frac{3ne}{4\rho s^2}(1-p)\int_0^{\pi/2} d\theta \sin^3\theta \int_0^\infty dz\, E_\pm(z)\exp\left[-\frac{1-i\alpha_\pm}{l\cos\theta}z\right]. \tag{30}$$

Substituting for $E_\pm(z)$ its Fourier transform [Eq. (16)] we find, after some transformations,

$$\xi'_\pm(+0) = -(1-p)(1-i\alpha_\pm)(2\gamma e\omega l^2 E_0/cMs^2)K_\pm(0)\ , \tag{31}$$

where

$$K_\pm(\beta) = \frac{3}{2\pi}\int_0^\infty d\beta' F_\pm(\beta')(\beta'^2-\beta^2)^{-1}\left[f\left[\frac{\beta}{1-i\alpha_\pm}\right] - f\left[\frac{\beta'}{1-i\alpha_\pm}\right]\right]. \tag{32}$$

The final expression for $\xi_\pm(\infty)$ is

$$\xi_\pm(\infty) = (2i\gamma e l^2 E_0/cMs)\{[1-\tilde{G}_\pm(\beta_a)]F_\pm(\beta_a) + (1-p)[(1-i\alpha_\pm)K_\pm(0) - K_\pm(\beta_a)]\}\ , \tag{33}$$

where

$$\beta_a = \omega l/s\ . \tag{34}$$

For convenience we express $\xi_\pm(\infty)$ in units of $E_0H_0/2\pi\rho\omega s$ where H_0 is the intensity of a magnetic field selected at will. This allows us to give graphs of $\xi_\pm(\infty)$ as functions of B_0 in a more convenient fashion. The dimensionless expression for $\xi_\pm(\infty)$ is

$$X_\pm(\infty) = (2\pi\rho\omega s/E_0H_0)\xi_\pm(\infty)\ . \tag{35}$$

Thus for large values of the magnetic field B_0,

$$X_\pm(\infty) = \pm(B_0/H_0)\ , \tag{36}$$

a result which can be verified by taking the asymptotic limit of Eq. (33) when $\omega_c\tau \gg 1$.

III. NUMERICAL CALCULATIONS

The analysis of the preceding section provides us with equations allowing the calculation of the electric and acoustic displacement fields. In the detailed calculations performed here we have used parameters appropriate to potassium to facilitate a comparison with the work of Chimenti *et al.*[5] We have selected the experimental frequency of 8.97 MHz and a mean free path such that $(\omega l/s) = 4.5$ or multiples thereof. The other quantities used are $v_F = 8.633\times10^7$ cm s^{-1}, $\rho = 0.91$ g cm^{-3}, and $s = 1.780\times10^5$ cm s^{-1}. For these parameters, $\delta = 2.357\times10^{-4}$ cm. We expect the electric field $\vec{E}(z)$ to decrease rapidly for $z > \delta$ and hence, $\vec{F}(\beta)$ is appreciable different from zero only for $\beta \lesssim l/\delta$. Thus, when $\omega l/s = 4.5$, the range of β should extend sufficiently beyond $\beta_0 = (l/\delta) \simeq 60$ to insure convergence.

The integration in Eq. (19) was performed by an extended Simpson rule[14] in which the range of β' was subdivided into three uneven intervals and cut off at different values of β' depending on l. For example, for $l = 1.42\times10^{-2}$ cm the integral was extended to $\beta' = 2000$. The equations (19) were thus reduced to the problem of inverting complex matrices of dimensions up to 120. This procedure was verified by replacing the integral in Eq. (19) by Gaussian multiplicative quadratures over six uneven intervals ranging from $\beta' = 0$ to $\beta' = 500$ (for $l = 1.42\times10^{-2}$ cm). A twenty point Gauss-Legendre quadrature was chosen in each interval with appropriate weight factors. The results obtained by both methods were in numerical agreement. The inverse Fourier transforms, necessary to obtain $\vec{E}(z)$ were carried out by the same methods.

Figure 1 gives the x component of the electric

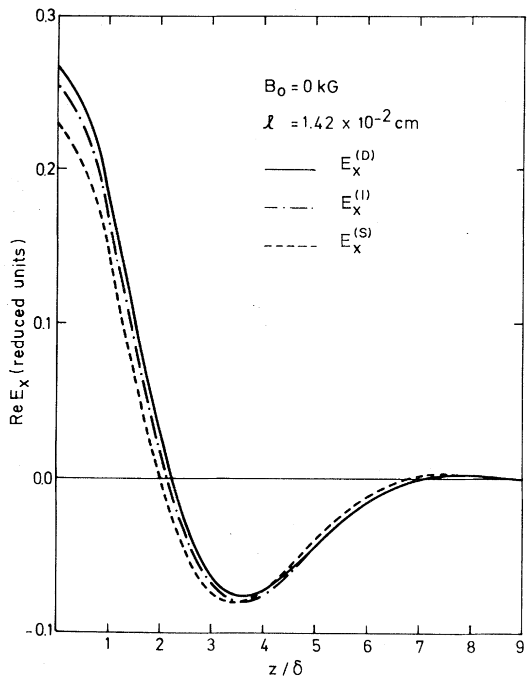

FIG. 1. $\mathrm{Re}E_x(z)$ for $B_0=0$ as a function of z/δ where z is the depth in the metal and δ the skin depth defined in Eq. (7). The indices D and S indicate results for diffuse and specular scattering, respectively. The index I is for diffuse scattering using the first approximation in an iteration procedure. The numerical values were obtained for potassium at a frequency of 8.97 MHz and assuming a mean free path $l=1.42\times10^{-2}$ cm. The field is amplified by a factor $(c/4\omega l^2E_0)$ described in the text.

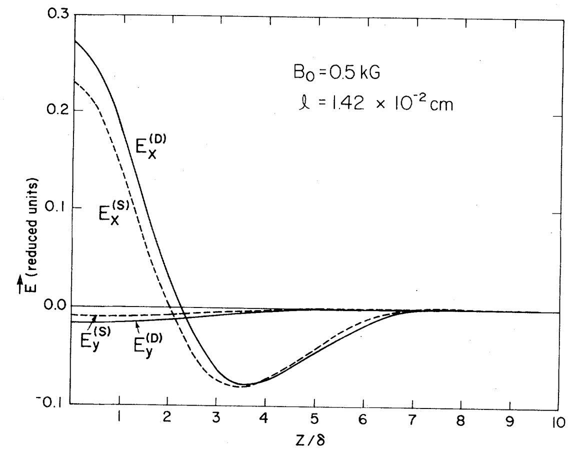

FIG. 2. Components of the electric field $\mathrm{Re}\vec{E}(z)$ as functions of z/δ for $B_0=0.5$ kG. Other conditions as in Fig. 1.

field at zero magnetic field for $(\omega l/s)=4.5$ corresponding to a mean free path of $l=1.42\times10^{-2}$ cm. The display gives E_x as a function of z for specular (S) and diffuse (D) scattering. In addition, a third curve, labeled (I), gives the numerical results for $E^{(D)}(z)$ using the first approximation in the solution of Eq. (19) by iteration. We note that, when $z/\delta\lesssim4$, $E^{(D)}(z)>E^{(S)}(z)$, contrary to the results obtained by Chambers[12] and used by Banik and Overhauser.[8] The electric field is expressed in the form given by Eqs. (23) and (24). What is actually shown in the figure is $c\vec{E}(z)/4\omega l^2E_0$ having the dimensions of a reciprocal length. Figures 2 and 3 show the results of calculations of $\vec{E}(z)$ as a function of z for magnetic fields of 0.5 and 1.0 kG. We note the difference of a factor of almost 2 in the y components of $\vec{E}$ between the specular and diffuse forms of electron scattering. We note also that $E_x(z)$ is not very dependent on B_0 for small values of this field. Figure 4 shows the results of a similar calculation for $E_y(z)$ at $B_0=0.75$ kG displayed in an enlarged scale. It includes the graph obtained using the first approximation in the iteration method.

Figures 5–10 display different components of $\vec{\xi}(\infty)$ and $|\vec{\xi}(\infty)|$ as functions of B_0 for a few choices of the electron mean free path. The units used are reduced in the sense described by Eqs. (35) and (36) taking $H_0=10$ kG for convenience. Some of the graphs are labeled "arbitrary units" but these are the same as the reduced units multiplied by a factor of 100 for ease of display. Finally, Fig. 11 shows graphs of $|\vec{\xi}(\infty)|$ at $B_0=0$ as a function of τ^{-1} for both specular ($p=1$) and diffuse scattering. This result is to be compared with Fig. 5 in the work of Banik and Overhauser.[8] We note that there is no qualitative difference between the behavior of $|\vec{\xi}(\infty)|$ for $B_0=0$ as a function of τ^{-1} for the two extreme choices of boundary condition.

Even though the results of the calculations give

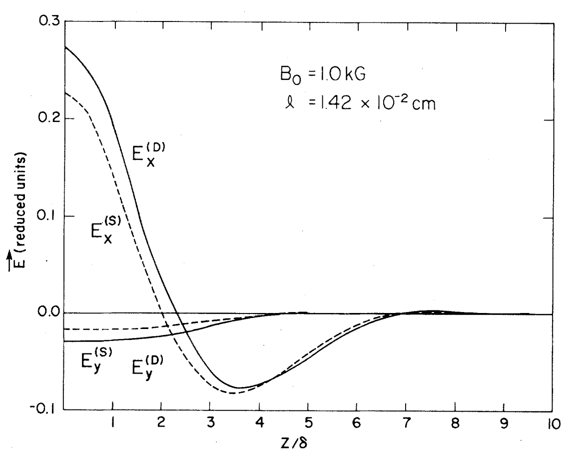

FIG. 3. Same as Fig. 2 for $B_0=1.0$ kG.

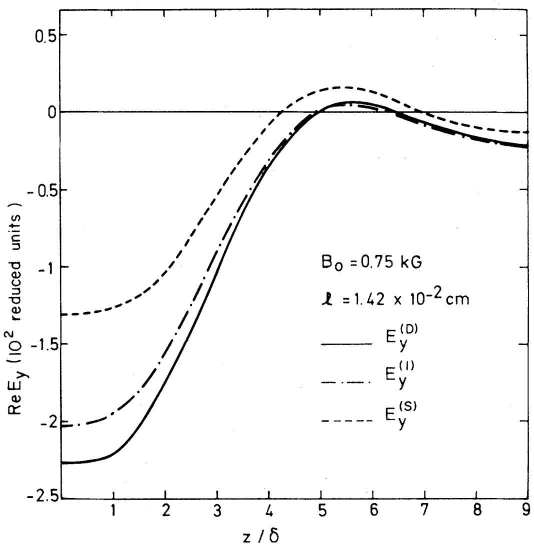

FIG. 4. $\mathrm{Re}E_y(z)$ as a function of z/δ at $B=0.75$ kG under the conditions described in Fig. 1. Note that the vertical scale has been amplified by 100.

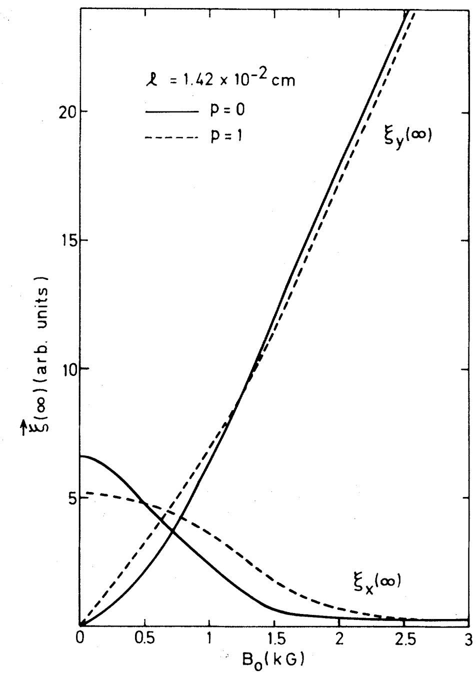

FIG. 6. Components of $\vec{\xi}(\infty)$ as functions of B_0 for specular ($p=1$) and diffuse scattering ($p=0$). The units are the reduced units amplified by 100. Other parameters are as in Fig. 1.

qualitative agreement with the experimental work of Chimenti *et al.*[5] it is difficult to make a complete comparison. We remember that in carrying out the correction of their measured acoustic amplitudes to obtain $\vec{\xi}(\infty)$ Chimenti *et al.* noticed that the measured value of the coefficient of acoustic attenuation was twice as large as that predicted by free-electron theory. This correction is most important when $B_0=0$ since the coefficient of attenuation decreases rapidly with increasing magnetic field. Thus, if their values were corrected using the theoretical decay constant, $|\vec{\xi}(\infty)|$ at $B_0=0$ would be smaller

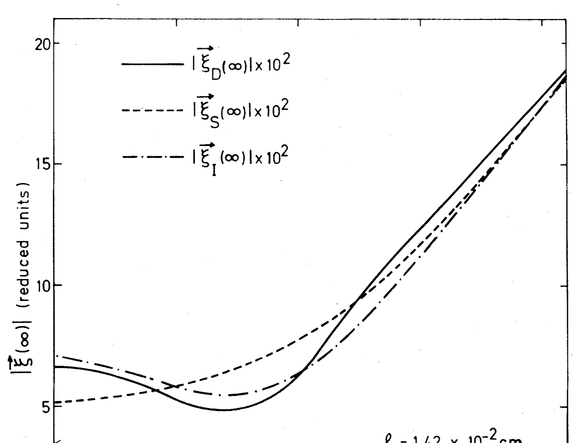

FIG. 5. Total amplitude $|\vec{\xi}(\infty)|$ of the acoustic wave as a function of B_0 for specular (S) and diffuse (D) scattering. Also the result of the first approximation using the iteration procedure (I) is shown. The units are selected so that for $B_0=10$ kG, $|\vec{\xi}(\infty)|=1$ (see text). The electron mean free path is indicated. Other parameters are as in Fig. 1.

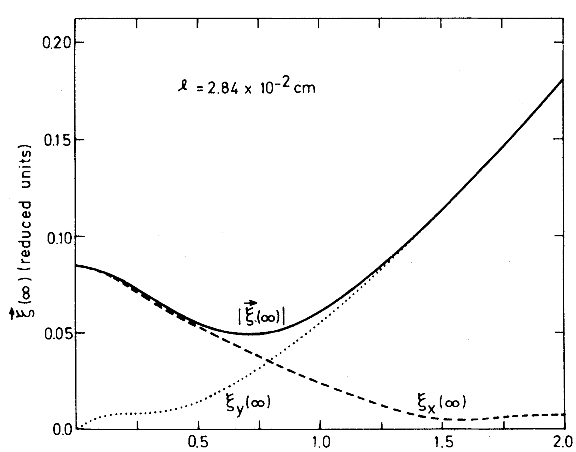

FIG. 7. Components and magnitude of $\vec{\xi}(\infty)$ as functions of B_0 for diffuse scattering taking $l=2.84\times10^{-2}$ cm. All other parameters are as in the previous figures.

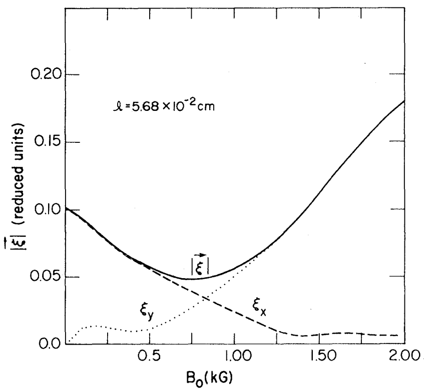

FIG. 8. Same as Fig. 7 for $l=5.68\times10^{-2}$ cm.

than displayed in their graphs and the position of the minimum in $|\vec{\xi}(\infty)|$ as a function of B_0 would be shifted to lower values of this quantity. The values shown in Fig. 9 corresponding to a mean free path $l=5.68\times10^{-2}$ cm would appear to be, therefore, in reasonable agreement with the experimental work. We note that the position of the maximum of $|\vec{\xi}(\infty)|$ as a function of B_0 is shifted to larger magnetic fields as the mean free path l is increased. However, this shift does not continue indefinitely but saturates rapidly beyond this value of l.

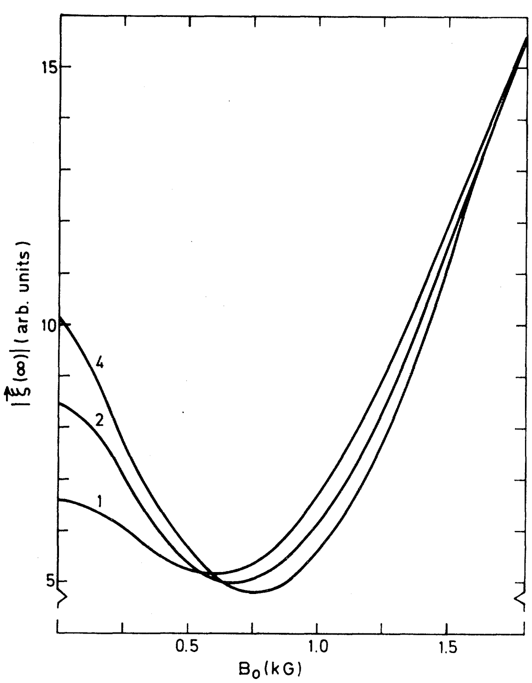

FIG. 9. $|\vec{\xi}(\infty)|$ as a function of B_0 for diffuse scattering and $l=l_0=1.42\times10^{-2}$ cm (1), $l=2l_0=2.84\times10^{-2}$ cm (2) and $l=4l_0=5.68\times10^{-2}$ cm (4). The vertical scale is in reduced units multiplied by 100. Other parameters are as in Fig. 1.

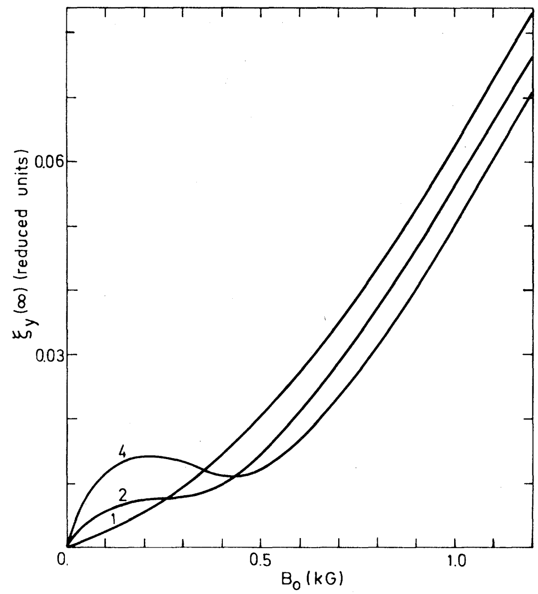

FIG. 10. Same as Fig. 9 but displaying only $\xi_y(\infty)$ as a function of B_0.

In addition to explaining the nonmonotonic variation of $\xi_y(\infty)$ as a function of B_0, diffuse surface scattering leads to an enhancement of the x component of the zero-field acoustic amplitude compared to that predicted by Quinn.[6] In the present exact solution of the equations governing the behavior of the electromagnetic and acoustic-wave amplitudes within the metal this enhancement results from the increased penetration of the electromagnetic fields in the sample under the assumption of diffuse scattering as compared to specular reflection. In the work of Banik and Overhauser[8] it was assumed, as we mentioned above, that the electric field inside the metal is approximately the same for these two boundary conditions. This assumption was based on the work of Chambers[12] which, as we proved in Sec. II A, is inaccurate. Our calculations give conversion efficiencies for potassium samples with a mean free path of 1.4×10^{-2} cm in a field of frequency 9 MHz of 4.4×10^{-12} and 6.3×10^{-12} for specular and diffuse scattering, respectively. These values correct those quoted in Ref. 10 (see footnote 11 of Ref. 1). Babkin and Kravchenko,[15] even though they recognize the increased penetration of the ac field into the metal under diffuse scattering conditions, do not take this change into account quantitatively.

The enhancement of the x component of the acoustic-wave amplitude depends, of course, on the choice of mean free path. This is demonstrated in

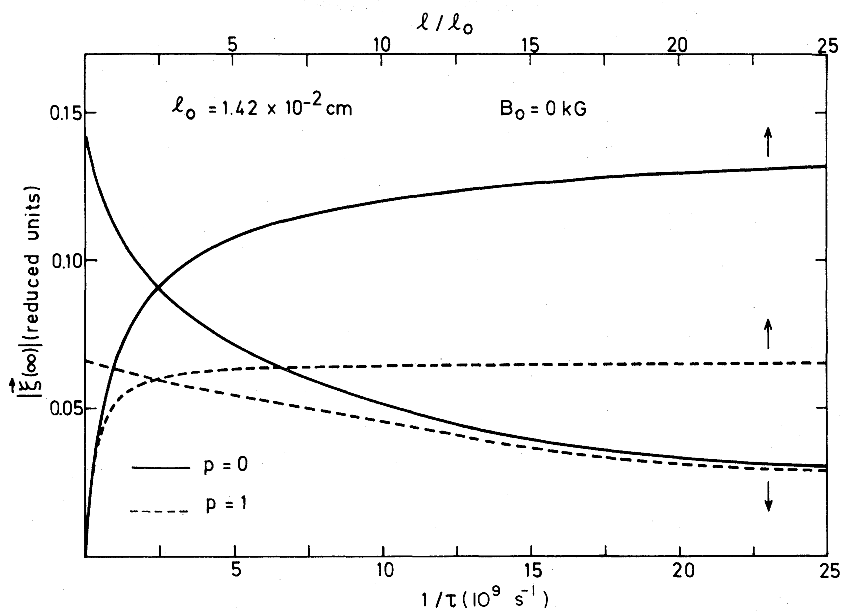

FIG. 11. $|\vec{\xi}(\infty)|$ at $B_0=0$ as a function of either τ^{-1} (lower scale) and l/l_0 (upper scale) for diffuse ($p=0$) and specular scattering ($p=1$) for the parameters appropriate to potassium at a frequency of 8.97 MHz; $l_0=1.42\times10^{-2}$ cm.

Fig. 9. This figure shows $|\vec{\xi}(\infty)|$ as a function of B_0 for several choices of the mean free path. At $B_0=0$, of course, $|\vec{\xi}(\infty)|=|\xi_x(\infty)|$.

The comparison between theory and the experiment of Chimenti *et al.*[5] is complicated by the fact that $|\vec{\xi}(\infty)|$ was obtained by a measurement at the far end of the sample but corrected to its value at the surface of incidence using the measured coefficient of ultrasonic attenuation. But their measured attenuation is twice the value obtained using Pippard's theory.[16] In this work (and in that of previous authors) it was assumed that the effect under investigation is due to the interaction of the motion of the electrons with that of positive ions. If we accept the possibility that in the experiments of Ref. 5 an additional nonelectronic source of acoustic attenuation was present, $\vec{\xi}(\infty)$ should be obtained by making this correction with the theoretical coefficient of ultrasonic attenuation. When that is done, owing to the rapid decrease of this coefficient with increasing B_0, the corrected values of $\vec{\xi}(\infty)$ near $B_0=0$ will be lower than indicated by Chimenti *et al.*[5] and closer to those predicted in this work.

In the work of Wallace *et al.*[4] $\xi_x(\infty)$ at $B_0=0$ is approximately 7 times larger than that expected using Quinn's theory.[6] In Ref. 5 this factor is only about 3. Reference 4 does not contain a statement regarding the correction discussed above. An additional complication is the fact that for propagation along the [110] direction (the direction of incidence in the experiments of Wallace *et al.*) the sample has a large elastic anisotropy. This requires additional investigation. However, Wallace *et al.*[4] suggest that the discrepancy may be due to the effect of the surface force caused by electron diffuse scattering but do not pursue the subject quantitatively.

In conclusion, the important features of the low-magnetic-field behavior exhibited by the acoustic amplitude in the works of Chimenti *et al.*[5] and Wallace *et al.*[4] can be explained within the framework of the free-electron model assuming that the electrons are scattered diffusively at the surface of the metal.

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[10]See, for example, L. R. Ram Mohan, E. Kartheuser, and S. Rodriguez, Phys. Rev. B 20, 3233 (1979). This paper contains additional references for the mathematical methods used.

[11]The relation of the fields at $z=0$ and E_0 is discussed in some detail in I.

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[13]See Eq. (3.2) in Ref. 1.

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