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Anisotropy and anisotropy-triad dynamics in spin-glasses

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We present a physical picture for the spin triad $(\hat{n},\hat{p},\hat{q})$ and anisotropy triad $(\hat{N},\hat{P},\hat{Q})$ introduced in our earlier work. A discussion of some of the experimental implications of anisotropy relaxation is then given, and it is argued, by citing examples of less complex systems with random anisotropy, that such relaxation is independent of the processes responsible for the time-dependent remanence associated with the spin-glass state. (It is suggested that increasing the spin dimensionality will shed light on this latter question.) The form of the anisotropy torque is considered, including global triad and global single-axis anisotropy; it is also shown how global averaging over local triad anisotropy can yield global single-axis anisotropy. Expressions are derived for the macroscopic dissipative coefficients in terms of time integrals of equilibrium correlation functions. Finally, it is shown, in the limit of vanishing anisotropy, that only the macroscopic modes contribute to the total magnetization and to the new spin-space rotation angle which we introduce in this paper.

I. INTRODUCTION

We recently presented a macroscopic theory of Heisenberg spin-glasses (SG's), including the effects of movable anisotropy.[1] The macroscopic variables employed were the magnetization $\vec{m}$, the spin triad $(\hat{n},\hat{p},\hat{q})$ (this specifies the orientation of the three-dimensional SG state; $\hat{n}$ may be taken along the remanence, but $\hat{p}$ and $\hat{q}$ are not directly observable), and the anisotropy triad $(\hat{N},\hat{P},\hat{Q})$ [this specifies the anisotropy torque $\vec{\Gamma}$; if $(\hat{N},\hat{P},\hat{Q})$ coincides with $(\hat{n},\hat{p},\hat{q})$, then $\vec{\Gamma}=\vec{0}$]. Only the long-wavelength limit was considered, so only dissipation involving uniform relaxation (as opposed to diffusion) was included. Phenomenological dissipation parameters associated with each of the macroscopic variables appeared in the theory, and it was shown how some of them could be simply related to torque measurements. One of the most stressed aspects of the work was that it incorporated relaxation of the anisotropy triad [consistent with certain aspects of both torque and electron spin resonance (ESR) measurements], but relaxation of $\vec{m}$ and the spin triad was also included.

A number of aspects of the theory remains to be discussed and developed. One such development involves applying the theory of Ref. 1 to the study of ESR line shapes and linewidths; this will be done (with applications to experiment) in a separate paper.[2] This paper will be devoted to questions of the physical interpretation of the spin and anisotropy triads, the potential experimental implications of the relaxation of the anisotropy triad, and the evaluation of various parameters (both reactive and dissipative) which appear in the theory.

An outline of this paper is as follows: In Sec. II we present a qualitative physical picture for the spin and anisotropy triads. Section III provides a discussion of some recent torque experiments and their bearing on the interpretation of anisotropy in spin-glasses. In Sec. IV we discuss the relation of the relaxation of the anisotropy triad to other metastability effects in spin-glasses. (We conclude that the relation is tenuous; a complex antiferromagnet with its spins pointing in all three directions, but with random anisotropy, should also possess an anisotropy triad which can relax.) We also speculate on the nature of the ground-state degeneracy which is believed to be responsible for the metastable remanence effects associated with the spin-glass state. In Sec. V we consider the anisotropy torque, and the different forms it may take. Section VI considers the dissipative coefficients, showing how they involve time integrals over correlation functions. This permits them to be evaluated using, for example, numerical simulations. In Sec. VII we show that, in the limit where exchange dominates anisotropy, localized (nonmacroscopic) modes do not contribute either to the magnetization or to the new spin-space rotation angle $\delta\vec{\theta}$ which we introduce in this paper. In the Appendix we discuss the orthogonality of the spin-wave modes, and show how this leads to the new $\delta\vec{\theta}$.

II. PHYSICAL PICTURE

Before proceeding with the developments of this paper, it would be appropriate to discuss the physical meaning of the spin triad $(\hat{n},\hat{p},\hat{q})$ and the anisotropy triad $(\hat{N},\hat{P},\hat{Q})$. At the outset let us neglect anisotropy and note that in this case only the spin triad $(\hat{n},\hat{p},\hat{q})$ is defined. [Therefore, another name for $(\hat{n},\hat{p},\hat{q})$ might be "exchange triad," since only the exchange interaction determines the macroscopic state in this case.] For a Heisenberg SG in equilibrium, since all the spins have fixed orientations with respect to one another, one can specify the orientation of any given spin by giving its orientation with respect to a set of coordinate axes which are rigidly attached to the spin system.[3] Thus if the spin system rotates uniformly (with respect to some laboratory frame), we can specify the orientation of the spin system by indicating the orientation of the spin coordinate axes. The spin triad $(\hat{n},\hat{p},\hat{q})$ serves as the set of spin coordinate axes, where we take $\hat{n}$ along the remanence (if there is a remanence), and $\hat{p}$ and $\hat{q}$ may be chosen arbitrarily. [It is somewhat similar to the problem of attaching coordinate axes to an ellipsoid of revolution, such as an American football. One axis can point along the long direction, but the choice of the other two is arbitrary. Once painted on, the coordinate axes are marked forever. The same is true of $(\hat{n},\hat{p},\hat{q})$ for SG's.] We assume that the low-frequency modes of this system only involve two effects. First, the system can rotate as a whole; second, its spins can polarize, without changing $(\hat{n},\hat{p},\hat{q})$. The elements of this argument exist for a collinear ferrimagnet. Here, the sublattice magnetizations $\vec{M}_A$ and $\vec{M}_B$ are opposed to one another in equilibrium. The state can be changed by rotating or changing the magnitude of $\vec{M}_A$ or $\vec{M}_B$. Certainly changing either magnitude involves polarization. Furthermore, rotating $\vec{M}_A$ alone can be decomposed into part overall rotation of both $\vec{M}_A$ and $\vec{M}_B$, and part polarization of both $\vec{M}_A$ and $\vec{M}_B$ without overall rotation. In a SG, however, rotating a single spin can be decomposed into part rotation of the system as a whole, part polarization of the system, and (in addition) part localized modes[3] which are not observable with macroscopic probes (and thus are not relevant to macroscopic theories; see Sec. VII). The equations of motion for the spin-space rotation angle $\delta\vec{\theta}$ and the polarization (or magnetization) $\delta\vec{m}$ are given, in the absence of dissipation, by[4]

$$\frac{\partial m_\alpha}{\partial t} = -\gamma \frac{\delta f}{\delta \theta_\alpha}, \quad \frac{\partial \theta_\alpha}{\partial t} = \gamma \frac{\delta f}{\delta m_\alpha}, \tag{2.1}$$

where γ is the gyromagnetic ratio and f is the free-energy density. Once f is known, one can compute the equations of motion. The free-energy density is generally taken to be of the form

$$f = \frac{m^2}{2\chi} - \vec{m}\cdot\left[\vec{H} + \frac{m_r}{\chi}\hat{n}\right] + f_{an}, \tag{2.2}$$

where now we introduce anisotropy: f_{an} yields $\vec{\Gamma}$ via $\Gamma_\alpha = -\gamma\,\partial f/\partial\theta_\alpha$. The form (2.2) yields $\vec{m} = m_r\vec{n} + \chi\vec{H}$ when $\vec{m}$ is in equilibrium ($\partial f/\partial\vec{m} = \vec{0}$). The form of f_{an} is not generally agreed upon. In Sec. V we present an argument that, locally, f_{an} should depend upon the orientation of the spin triad $(\hat{n},\hat{p},\hat{q})$ with respect to the anisotropy triad $(\hat{N},\hat{P},\hat{Q})$. Let us now consider the meaning of $(\hat{N},\hat{P},\hat{Q})$.

When anisotropy is turned on, it is possible for the spin system to "see" the lattice orientation. However, we must keep the anisotropy weak compared to the exchange, otherwise the spins reorient so much in the presence of anisotropy that we must revise our view of the spin triad. In terms of the football analogy, we may think of exchange (rather than leather and air) as producing the shape of the football, and anisotropy as providing springs attached to it. If the spring tension is too large, the football will become very distorted. We wish to consider weak anisotropy (or weak springs), which does not much distort the strongly exchange-coupled SG state (or the sturdily built football), but which give the SG state (or the football) an overall orientation. The preferred overall orientation for $(\hat{n},\hat{p},\hat{q})$ is given by $(\hat{N},\hat{P},\hat{Q})$, if the anisotropy is of a triad nature. If the anisotropy is of a single-axis nature, only $\hat{n}$ will tend to align with $\hat{N}$, $\hat{p}$ and $\hat{q}$ being oblivious to their orientation about $\hat{n}$.

III. ON THE RELAXATION OF SPIN-GLASS ANISOTROPY

In this section, we will discuss a number of experiments designed to determine the symmetry of the anisotropy (triad or single-axis), as well as experiments designed to study the relaxation of spin-glass anisotropy.

Triad and single-axis anisotropy are distinguishable by the fact that, for triad anisotropy, there is a restoring torque for rotations about any axis, whereas for single-axis anisotropy there is a restoring torque only for the two rotation axes which change that preferred axis. In the context of spin-glasses, the simplest way to see the restoring torque associated only with the third axis would be to observe a longitudinal resonance.[5-7] That is, for a system prepared in a cooling field $\vec{H}_c$, and in an applied field $\vec{H}$ along $\vec{H}_c$, one applies an rf field $\vec{H}_{ac}$ along $\vec{H}_c$, and looks for a resonance. Such longitudinal resonance experiments have been very useful in studying the anisotropy energy in superfluid ^{3}He.[8,9] Such a measurement would give unambiguous evidence that SG anisotropy is of a triad nature. Unfortunately, such measurements are more difficult for SG's than for ^{3}He, since ESR in SG's requires microwave spectrometers of nearly fixed frequency, whereas nuclear magnetic resonance (NMR) in ^{3}He has a more flexible frequency range.

Another approach to the study of anisotropy employs torque measurements. These typically involve rotation of the magnetization about a perpendicular axis.[10–13] This method is not capable of distinguishing between triad and single-axis anisotropy because only one of the three axes is probed. Nevertheless, such studies have proved very useful, for they have revealed the fact that the anisotropy triad (or axis) moves, even for T/T_g values as low as 0.05 (where T_g is the SG transition temperature).[12] Reference 1 considered such motion, according to a single relaxation time. However, recent work at $T/T_g \approx 0.3$ indicates that relaxation of the anisotropy torque takes place on multiple time scales.[13] This complicates the data interpretation, so to avoid such effects anisotropy torque relaxation must be suppressed as much as possible. The experiments[12,13] indicate two ways to achieve this: to allow T/T_g to remain very small, or to keep the remanence close to $\vec{H}_c$ (corresponding to only small rotations of the magnetization). It is not unlikely that many of the experiments which have been performed, involving large rotations of the remanence, probably involved a partially relaxed anisotropy, rendering their interpretation difficult. In addition, studies where the remanence is reversed by an $\vec{H}$ applied opposite to $\vec{H}_c$ may also be difficult to interpret.[12]

One study, by Fert and Hippert, has been done at a very low value of T/T_g (~ 0.015), and the results indicate very little relaxation of the anisotropy triad.[14] Furthermore, by performing torque measurements about two mutually perpendicular axes, Fert and Hippert were able to obtain results consistent with triad anisotropy, and an anisotropy free-energy density $f_{\rm an}$ consistent with the form,

$$f_{\rm an} = -K_1 \cos\psi - \tfrac{1}{2} K_2 \cos^2\psi \qquad (3.1)$$

with $K_1 \gg K_2$. Here $\hat{n}\cdot\hat{N} + \hat{p}\cdot\hat{P} + \hat{q}\cdot\hat{Q} = 1 + 2\cos\psi$. (The work of Ref. 12 is also consistent with $K_1 \gg K_2$.) To help confirm these results, it would be of value to have ESR measurements on the same type of material, CuMn ($c = 20\%$), for the same range of T/T_g. In particular, with anisotropy relaxation suppressed, one would expect to find ESR resonances in agreement with the predictions of Henley *et al.*,[7] based on triad anisotropy. [*Note Added in Proof.* This has recently been done by E. M. Gullikson, D. R. Fredkin, and S. Schultz, Phys. Rev. Lett. 50, 537 (1983).] Note that, in the past, ESR measurements have been performed at relatively high T/T_g, where anisotropy relaxation may have taken place.[15,16]

IV. RELATION OF ANISOTROPY RELAXATION TO OTHER SPIN-GLASS METASTABILITY EFFECTS

Because SG's possess the unusual property of anisotropy relaxation, as well as unusual time-dependent remanence effects,[17] it is natural to inquire whether or not these properties are linked. The answer, we believe, is that they are not. Specifically, we will suggest other systems which should possess anisotropy relaxation.

In accordance with the picture presented in Sec. II, we assume that the exchange interaction is much larger than any microscopic anisotropy interactions. Consider now what is perhaps the classic SG, *Cu*Mn, whose Mn go onto Cu sites in the host fcc lattice. The Mn sites are nearly random, so the Ruderman-Kittel-Kasuya-Yosida (RKKY) exchange constants between Mn are nearly random, thus leading to a series of nearly degenerate ground states, or equilibrium configurations (EC's).[3] Assume that the system is in one of these EC's, where the spins point in all three spin directions, and that we have not yet turned on the anisotropy. (Of course, we are treating the spins as classical.) The state may be specified by indicating: (1) which EC is involved [this means giving a table of numbers for the spin directions, which identifies an EC for its spin triad in some standard orientation like $(\hat{i},\hat{j},\hat{k})$], (2) the actual orientation $(\hat{n},\hat{p},\hat{q})$ of the spin triad, and (3) the orientation $(\hat{A},\hat{B},\hat{C})$ of the lattice. Of course, without anisotropy, $(\hat{A},\hat{B},\hat{C})$ is irrelevant. If we now turn on anisotropy, which is random, each spin reorients slightly in the anisotropy field and the adjusted exchange field of its neighbors. In the language of quantum mechanics, the wave function changes to first order and the energy changes to second order. The reason the anisotropy in SG's is metastable (i.e., it can change direction) is that the ground-state energy is the same for all lattice orientations, if one averages over a large sample. Thus if one rapidly rotates the zeroth-order part of the wave function $(\hat{n},\hat{p},\hat{q})$, by rotating the remanence, and then waits, the system slowly adjusts (i.e., metastability) the first-order part of its wave function to minimize the total energy for its new $(\hat{n},\hat{p},\hat{q})$. This is quite unlike what one has for a ferromagnet in a crystal with a preferred axis, where the anisotropy energy comes in to first order, and does not produce any internal adjustments of the spins. In this case there are only two orientations of the magnetization $\vec{m}$ which minimize the anisotropy energy (i.e., $\vec{m}$ along or opposite to the preferred axis). The anisotropy axis is fixed to the crystal in this case.

It is possible to imagine two other situations, one like a SG and one like a ferromagnet, both of which have metastable anisotropy. The first case is a complicated antiferromagnet, with the spins pointing in all three directions ("3D antiferromagnet") making it very similar to the particular EC we considered for the SG. If the anisotropy is random for this 3D antiferromagnet, there will be no preferred anisotropy triad fixed in the lattice; the system will be very similar to the case of the SG we have already considered. The second case is a disordered ferromagnet, where the spins are located randomly in some matrix (which need not even be crystalline), and the anisotropy is likely to be random. Thus the anisotropy energy does not have any preferred direction to first order, and the anisotropy must enter to second order to show a preferred axis. Because the random anisotropy makes the spins noncellinear, the state is three dimensional thus requiring a spin triad and an anisotropy triad.[18]

We thus are of the opinion that the metastability displayed by the anisotropy in SG's is not unique to SG's, but rather is due to the random anisotropy interactions in SG's. One can probably study anisotropy relaxation in SG's independently of remanence relaxation. This is consistent with the fact that these two types of relaxation have very different types of characteristic time dependences, the anisotropy relaxing exponentially, and the remanence relaxing as a power law with a small exponent (nearly logarithmic).[19]

It is currently believed that the peculiar remanence effects of SG's are associated with nonergodic behavior: The SG has many nearly degenerate EC's, and it undergoes transitions between these EC's, but there are large barriers between different EC's, so they are not populated according to a thermal equilibrium distribution.[3,20] Presently, relatively little is known about these different EC's, nor about the rate at which transitions take place from one EC to another.[21,22]

One of the difficulties in dealing with the SG state is that it is difficult to see how it develops in a continuous (or nearly continuous) fashion. One is given N spins with more or less random exchange interactions, and one finds, as stated earlier, that it has many nearly degenerate ground states, or EC's.[3] Typically one studies spin dimensionality $n=1$ (Ising spins) and $n=3$ (Heisenberg spins). Consider now the question of how the number of EC's N_{EC} depends on n. This is done in the work of Bray and Moore,[22] using the long-range model of Sherrington and Kirkpatrick. For $N=200$ they find the sequence $N_{\mathrm{eq}} \approx 2\times 10^{17}$, 105, 5 when $n=1,2,3$. Thus it is not unlikely that N_{EC} decreases as n increases, until a spin dimension is reached ($n=n_c$) where $N_{\mathrm{EC}}=1$. For larger values of n the energy may decrease, but no additional EC's are found. It is also quite possible that for $n>n_c$ the energy cannot decrease any further, so that (in the sense of energy minimization) the "frustration" in the system has been totally released. It should be noted that for $n\rightarrow\infty$ (the spherical model), the Sherrington-Kirkpatrick model is believed to be ergodic,[20,24] consistent with the above hypothesis, since a system with $N_{\mathrm{EC}}=1$ should have ergodic behavior. Such a system may only be a very complex type of antiferromagnet.

We close this section by remarking that it may be of interest to determine the nature of the different EC's that develop as n is decreased, or equivalently, how the different EC's coalesce as n is increased. I would suggest that, as n increases, different EC's may relax topological defects, in a sense similar to the way that an $n=2$ ferromagnet with line defects can have those defects disappear when n is increased to 3 ("escape into the third dimension"). This does not, however, answer the question of how the defects would arise, because the topology of the SG state for any n is likely to be extremely complex. This is to be contrasted to the case of, e.g., an $n=3$ ferromagnet, where decreasing n to 2 need not introduce topological defects.

V. THE ANISOTROPY TORQUE

From very general considerations, one can determine the form expected for the metastable anisotropy torque (and energy) in the spin-glass state. Let us write down the Hamiltonian for the system:

$$H=H_0+H_1\ , \tag{5.1}$$

$$H_0=-\tfrac{1}{2}\sum_{i,j} J_{ij}\vec{S}_i\cdot\vec{S}_j,\quad J_{ij}=J_{ji} \tag{5.2}$$

$$H_1=-\tfrac{1}{2}\sum_{\substack{i,j\\ \alpha,\beta}} D_{ij\alpha\beta}S_{i\alpha}S_{j\beta},\quad D_{ij\alpha\beta}=D_{ji\beta\alpha}\ . \tag{5.3}$$

The local field at site i is given by

$$H_{i\alpha}\equiv -\frac{\partial H}{\partial S_{i\alpha}}=\sum_j J_{ij}S_{j\alpha}+\sum_{j,\beta} D_{ij\alpha\beta}S_{j\beta}\ . \tag{5.4}$$

The total torque on the spin system is given by

$$\Gamma_\mu=\gamma\sum_{\alpha,\beta}\epsilon_{\alpha\beta\mu}\sum_i S_{i\alpha}H_{i\beta}$$

$$=\gamma\sum_{\alpha,\beta}\epsilon_{\alpha\beta\mu}\sum_{i,j}S_{i\alpha}\left[J_{ij}S_{j\beta}+\sum_\delta D_{ij\beta\delta}S_{j\delta}\right]$$

$$=\gamma\sum_{\alpha,\beta}\epsilon_{\alpha\beta\mu}\sum_{i,j,\delta}D_{ij\beta\delta}S_{i\alpha}S_{j\delta}\ . \tag{5.5}$$

(Note that the torque due to exchange sums to zero.) Now assume that there is an equilibrium orientation $\vec{S}_i^{(0)}$ for the ith spin and that the spins have all been rotated via

$$S_{i\alpha}=\sum_{\alpha'}R_{\alpha\alpha'}S_{i\alpha'}^{(0)}\ .$$

Then

$$\Gamma_\mu=\gamma\sum_{\substack{\alpha,\alpha'\\ \beta,\delta,\delta'}}\epsilon_{\alpha\beta\mu}R_{\alpha\alpha'}R_{\delta\delta'}\left[\sum_{i,j}D_{ij\beta\delta}S_{i\alpha'}^{(0)}S_{j\delta'}^{(0)}\right]\ . \tag{5.6}$$

Explicitly, the rotation matrix $\underline{R}$ is given by

$$R_{\alpha\alpha'}=\delta_{\alpha\alpha'}\cos\psi+\hat{\psi}_\alpha\hat{\psi}_{\alpha'}(1-\cos\psi)-\epsilon_{\alpha\alpha'\nu}\hat{\psi}_\nu\sin\psi\ , \tag{5.7}$$

where $\hat{\psi}$ is the axis of rotation, and ψ is the angle of rotation. Now consider

$$A_{\beta\delta,\alpha'\delta'}\equiv\gamma\sum_{i,j}D_{ij\beta\delta}S_{i\alpha'}^{(0)}S_{j\delta'}^{(0)}$$

$$=\gamma\sum_{i,j}D_{ji\delta\beta}S_{j\delta'}^{(0)}S_{i\alpha'}^{(0)}$$

$$=\gamma\sum_{i,j}D_{ij\delta\beta}S_{i\delta'}^{(0)}S_{j\alpha'}^{(0)}$$

$$=A_{\delta\beta,\delta'\alpha'}\ . \tag{5.8}$$

If the spins are uncorrelated to the lattice, then only β and δ, and α' and δ', contract. This occurs to zeroth order in perturbation theory. If the spins are correlated to the lattice (e.g., from first-order perturbation theory), then more contractions occur:

$$A_{\beta\delta,\alpha'\delta'}=A\delta_{\beta\delta}\delta_{\alpha'\delta'}+B\delta_{\beta\alpha'}\delta_{\delta\delta'}+C\delta_{\beta\delta'}\delta_{\delta\alpha'}\ . \tag{5.9}$$

Note that (5.9) is consistent with (5.8). Placing (5.9) in (5.6) and employing (5.7), one finds that

$$\Gamma_\mu=-2\hat{\psi}_\mu[B\sin\psi+(B+C)\sin(2\psi)]\ . \tag{5.10}$$

This implies an anisotropy energy density of the form

$$f_{\rm an}=-2B\cos\psi-(B+C)\cos(2\psi)\ . \tag{5.11}$$

Note that this symmetry is independent of the detailed form of $D_{ij\alpha\beta}$; only the bilinear nature (with respect to the spins) of the spin-lattice interaction H_1 is required to obtain this form. Also note that for interactions satisfying $D_{ij\alpha\beta}=\pm D_{ij\beta\alpha}$ we have

$$C=\pm B\quad (D_{ij\alpha\beta}=\pm D_{ij\beta\alpha})\ . \tag{5.12}$$

The minus sign is appropriate to the Dzyaloshinsky-Moriya interaction, and yields only a $\cos\psi$ dependence in (5.11), a result obtained in Refs. 25 and 7. It corresponds to $K_2=0$ in Eq. (3.1). The plus sign is appropriate to interactions like to dipolar interaction, and yields a $\cos(2\psi)$ amplitude as large as the $\cos\psi$ amplitude in (5.11). It corresponds to $K_2=4K_1$ in Eq. (3.1), a result also obtained in Ref. 7.

We should also observe that if the SG has some remanence, Eq. (5.9) will become more complex. $A_{\beta\delta,\alpha'\delta'}$ will now contain the terms

$$A_1\delta_{\beta\delta}m_{\alpha'}m_{\delta'}+B_1\delta_{\beta\alpha'}m_\delta m_{\delta'}+C_1\delta_{\beta\delta'}m_\delta m_{\alpha'}+A_2m_\beta m_\delta\delta_{\alpha'\delta'}+B_2m_\beta m_{\alpha'}\delta_{\delta\delta'}+C_2m_\beta m_{\delta'}\delta_{\delta\alpha'}$$

which causes the torque, Eq. (5.6), to contain the terms

$$-2B_1\hat{\psi}_\mu\sin\psi(\vec{m}\cdot\vec{m}')-B_2(\vec{m}\times\vec{m}')_\mu R_{\alpha\alpha}+(C_1+C_2)(\vec{m}'\times\vec{m}'')_\mu\ ,$$

where

$$m'_\alpha=\sum_\beta R_{\alpha\beta}m_\beta,\quad m''_\alpha=\sum_\beta R^{-1}_{\alpha\beta}m_\beta\ .$$

Thus the B_1 and B_2 terms involve both SG and axial symmetry, and the C_1+C_2 term involves only axial symmetry. All three terms should appear for a mixed phase (SG and ferromagnet) if there is random anisotropy. Note that for a ferromagnet with a preferred anisotropy axis $\hat{a}$, $A_{\beta\delta,\alpha'\delta'}$ will contain the term

$$A_3\hat{a}_\beta\hat{a}_\delta m_{\alpha'}m_{\delta'}\ ,$$

so the torque will contain the term

$$A_3(\vec{m}'\cdot\hat{a})(\vec{m}'\times\hat{a})\ ,$$

as expected for a uniaxial ferromagnet.

Let us now consider the possibility that the $(\hat{N},\hat{P},\hat{Q})$ is not uniform over a sample of SG. Because of the strong exchange coupling throughout the sample, we expect $(\hat{n},\hat{p},\hat{q})$ to be uniform, but no such energy drives $(\hat{N},\hat{P},\hat{Q})$ to uniformity. According to Ref. 1 $(\hat{N},\hat{P},\hat{Q})$ relaxes to $(\hat{n},\hat{p},\hat{q})$, driven dominantly by the anisotropy torque. If a sample of SG is quenched below T_g very rapidly, it is possible that the $(\hat{N},\hat{P},\hat{Q})$ relaxation is not total, so one may not be in a local equilibrium state. We wish to show that, for an appropriate distribution of $(\hat{N},\hat{P},\hat{Q})$, the local triad anisotropy can simulate global single-axis anisotropy.

Consider (3.1) for the anisotropy free-energy density $f_{\rm an}$. If we express $\cos\psi$ in terms of the Euler angles (α,β,γ) we obtain

$$1+2\cos\psi=\cos\beta[1+\cos(\alpha+\gamma)]+\cos(\alpha+\gamma)\ . \tag{5.13}$$

A random average over $\alpha+\gamma$, with $\cos\beta=\hat{n}\cdot\hat{N}$ held fixed, yields

$$\langle f_{\rm an}\rangle=-(\tfrac{1}{2}K_1-\tfrac{1}{8}K_2)(\hat{n}\cdot\hat{N})-\tfrac{3}{16}K_2(\hat{n}\cdot\hat{N})^2\ , \tag{5.14}$$

up to a constant, for a given value of $\hat{n}\cdot\hat{N}$. If, further, one has a distribution of values $\hat{n}\cdot\hat{N}$ about some average value, the fully averaged anisotropy will have even smaller coefficients than given by (5.14). The extreme limit of this is complete randomness, with no dependence on $\hat{n}\cdot\hat{N}$, which has been averaged out.

VI. THE DISSIPATIVE COEFFICIENTS

In this section we derive expressions for the dissipative coefficients, in a form such that they may be determined by numerical simulations. We therefore consider a SG consisting of classical spins. Our method is a generalization of that given in the textbook by Reif.[26] There, one considers the dissipative coefficient of a single particle in a fluid, by considering the time dependence of its velocity. In this case it is most convenient to study the time dependence of the internal field $\vec{h}$ and the anisotropy torque $\vec{\Gamma}$, both of which can be monitored for a given state of the system. ($\vec{h}$ will be discussed in Sec. VII; $\vec{\Gamma}$ was discussed in Sec. V.) Near equilibrium in $\vec{H}=\vec{0}$, both $\vec{\Gamma}$ and $\vec{h}$ will be small. In that case the differential $d\epsilon$ of the energy density ϵ takes the form[1]

$$d\epsilon=T\,dS+\vec{h}\cdot d\vec{m}-(m_r/\chi)\vec{m}\cdot d\hat{n}-\vec{\Gamma}\cdot(d\vec{\theta}-d\vec{\Theta})\ . \tag{6.1}$$

Here $d\vec{\theta}$ rotates $(\hat{n},\hat{p},\hat{q})$ and $d\vec{\Theta}$ rotates $(\hat{N},\hat{P},\hat{Q})$. Neglecting the change in entropy density dS, and with

$$\vec{h}\equiv\chi^{-1}(\vec{m}-m_r\hat{n})\ , \tag{6.2}$$

$$\vec{\Gamma}\approx-K(\vec{\theta}-\vec{\Theta}),(K\equiv K_1+K_2)\ , \tag{6.3}$$

for small rotations $\vec{\theta}$ of $(\hat{n},\hat{p},\hat{q})$ and $\vec{\Theta}$ of $(\hat{N},\hat{P},\hat{Q})$, Eq. (6.1) becomes

$$d\epsilon\approx\chi\vec{h}\cdot d\vec{h}+K^{-1}\vec{\Gamma}\cdot d\vec{\Gamma}\ . \tag{6.4}$$

This establishes that the fluctuations possess the thermal averages

$$\langle\chi h^2\rangle=\langle K^{-1}\Gamma^2\rangle=(3k_BT)V^{-1} \tag{6.5}$$

(where V is the volume of the system), on application of the equipartition theorem.

The dissipative part $\vec{h}^{(0)}$ of $\vec{h}$ is given by[1]

$$\dot{\vec{h}}^{(D)}=-(\gamma/\chi)[(D+m_r^2E)\vec{h}-m_r^2E(\vec{h}\cdot\hat{n})\hat{n}+m_r(E+E')\vec{\Gamma}\times\hat{n}]\ . \tag{6.6}$$

Changes in $\vec{h}$ due to this source, on going from time t to $t+\tau$, where τ is large compared to a "collision time" but small enough that $|\Delta\vec{h}|\ll|\vec{h}|,|\Delta\vec{\Gamma}|\ll|\vec{\Gamma}|$ over this interval, are given, for a macroscopic state by

$$\vec{h}(t+\tau)-\vec{h}(t)\approx-(\gamma\tau/\chi)[(D+m_r^2E)\vec{h}-m_r^2E(\vec{h}\cdot\hat{n})\hat{n}+m_r(E+E')\vec{\Gamma}\times\hat{n}] \tag{6.7}$$

and, for a microscopic state, by

$$\vec{h}(t+\tau)-\vec{h}(t)=\int_t^{t+\tau}\langle\dot{\vec{h}}(t)\rangle dt'=-\beta\int_t^{t+\tau}dt'\langle\Delta E(t')\dot{\vec{h}}(t)\rangle_0 dt'\ . \tag{6.8}$$

(Here the angular brackets with subscript zero denote the *equilibrium* statistical average.) Use of (6.4) gives

$$\begin{aligned}\Delta E(t')&=V\int_t^{t'}[\chi\vec{h}(t'')\cdot\dot{\vec{h}}(t'')+K^{-1}\vec{\Gamma}(t'')\cdot\dot{\vec{\Gamma}}(t'')]dt''\\&\approx V\chi\vec{h}(t)\cdot\int_t^{t'}\dot{\vec{h}}(t'')dt''+VK^{-1}\vec{\Gamma}(t)\cdot\int_t^{t'}\dot{\vec{\Gamma}}(t'')dt''\ .\end{aligned} \tag{6.9}$$

Placing (6.9) into (6.8) and rearranging the domain of integration[26] yields

$$\langle \vec{h}(t+\tau)-\vec{h}(t)\rangle = -V\beta\tau\chi\vec{h}(t)\cdot\int_0^\infty \langle \dot{\vec{h}}(0)\dot{\vec{h}}(s)\rangle_0 ds - V(\beta\tau/K)\vec{\Gamma}(t)\cdot\int_0^\infty \langle \dot{\vec{\Gamma}}(0)\dot{\vec{h}}(s)\rangle_0 ds \ . \tag{6.10}$$

Comparison of (6.7) for a macroscopic state with the statistically averaged value over microscopic states, (6.10), yields

$$(D+m_r^2E)\delta_{\alpha\beta}-m_r^2E\hat{n}_\alpha\hat{n}_\beta=(\beta\chi^2V/\gamma)\int_0^\infty \langle \dot{\vec{h}}_\alpha(0)\dot{\vec{h}}_\beta(t)\rangle_0 dt \tag{6.11}$$

and

$$m_r(E+E')=-(\beta\chi V/2\gamma K)\int_0^\infty \langle \dot{\vec{\Gamma}}(0)\cdot\dot{\vec{h}}(t)\times\hat{n}\rangle_0 dt \ . \tag{6.12}$$

Similarly, study of $\vec{\Gamma}$ yields

$$(E+2E'+F)\delta_{\alpha\beta}=\frac{\beta V}{\gamma K^2}\int_0^\infty \langle \dot{\Gamma}_\alpha(0)\dot{\Gamma}_\beta(t)\rangle_0 dt \tag{6.13}$$

and a repetition of (6.12). These equations yield the four dissipative coefficients. Note that, in (6.12), it is assumed that $\hat{n}$ does not fluctuate significantly.

It is not clear that Eqs. (6.11)–(6.13) provide a practical means to determine the dissipative coefficients (via, e.g., numerical simulations). However, they serve the purpose of showing the correlations on which the dissipative coefficients depend. Only D and E have a ready interpretation. From (6.11) one has

$$D=\frac{\beta\chi^2V}{\gamma}\int_0^\infty \langle \hat{n}\cdot\dot{\vec{h}}(0)\hat{n}\cdot\dot{\vec{h}}(\tau)\rangle_0 dt \ , \tag{6.14}$$

$$m_r^2E=\frac{\beta\chi^2V}{2\gamma}\int_0^\infty \langle \dot{\vec{h}}(0)\cdot\dot{\vec{h}}(t)-3\hat{n}\cdot\dot{\vec{h}}(0)\hat{n}\cdot\dot{\vec{h}}(t)\rangle_0 dt \ . \tag{6.15}$$

Thus D depends upon the autocorrelations of $\hat{n}\cdot\vec{h}$, and E depends upon the nonisotropic nature of the autocorrelation of $\vec{h}$. No such obvious interpretation arises for E' and F.

VII. ON THE SPIN TRIAD

The calculations of the previous sections depend upon our ability to define various quantities. Of these quantities, $\vec{h}$ still is incompletely defined, since it requires $\vec{m}$, m_r, χ, and $\hat{n}$:

$$\vec{h}=\chi^{-1}(\vec{m}-m_r\hat{n}) \ . \tag{7.1}$$

For χ, one may use standard expressions in terms of equilibrium correlation functions, as has been done by Walker and Walstedt[3]; for $\vec{m}$ one employs

$$\vec{m}=\frac{\gamma}{V}\sum_{i\in\mathscr{R}}\vec{S}_i \ , \tag{7.2}$$

where one sums over the spins $\vec{S}_i$ in the region $\mathscr{R}$, of volume V; for m_r one employs the value of $\vec{m}$ when the system is in equilibrium ($\vec{h}=\vec{H}$) for $\vec{H}=\vec{0}$. The only quantity which has not yet been determined is $\hat{n}$.

Reference 4 provides a definition of the rotation angle $\delta\vec{\theta}$ by which the SG has been rotated slightly from some standard orientation $(\hat{n}_0,\hat{p}_0,\hat{q}_0)$. From $\delta\vec{\theta}$ one has $\hat{n}=\hat{n}_0+\delta\vec{\theta}\times\hat{n}_0$ (to lowest order in $\delta\vec{\theta}$), so that $\hat{n}$ is determined from $\delta\vec{\theta}$. Specifically, Ref. 4 gives

$$\delta\theta_\gamma=(2q)^{-1}\sum_{\alpha,\beta}\epsilon_{\alpha\beta\gamma}\delta t_{\alpha\beta} \ , \tag{7.3}$$

$$\delta t_{\alpha\beta}=n^{-1}\sum_{i\in\mathscr{R}}\langle S_{i\alpha}\rangle\delta S_{i\beta} \ , \tag{7.4}$$

$$q\equiv(3n)^{-1}\sum_{i\in\mathscr{R}}\langle S_{i\alpha}\rangle^2 \ . \tag{7.5}$$

Here $\langle S_{i\alpha}\rangle$ is the restricted ensemble average (about an EC) at the αth spin component of the ith spin, $\delta S_{i\beta}$ is the small change in $S_{i\beta}$, and n is the number of spins in region $\mathscr{R}$.

This definition is intuitively reasonable, but one can fault it if one finds that a set of spin displacements $\{\delta S_{i\beta}^{(\nu)}\}$ corresponding to a localized mode ν can contribute to $\delta\vec{\theta}$. In the Appendix, we obtain an orthogonality relation for the normal modes (neglecting anisotropy), finding that this leads to another definition for the spin-space rotation angle $\delta\vec{\theta}$. The new $\delta\vec{\theta}$ has the same commutation relations as the old one and, further, no localized mode can contribute to the new $\delta\vec{\theta}$. Specifically, we have

$$\delta\theta_\alpha=\chi^{-1}\sum_{i,j,\beta}\chi_{ij,\alpha\beta}\delta\theta_{j\beta} \ , \tag{7.6}$$

where $\delta\theta_{j\beta}=(\vec{S}_j^{(0)}\times\delta\vec{S}_j)_\beta\,|\langle\vec{S}_j^{(0)}\rangle|^{-2}$, and $\chi_{ij,\alpha\beta}$ is the local susceptibility of the ith site due to a field at the jth site [see Eqs. (A16) and (A12) of the Appendix]. Thus in calculating

$$\dot{\vec{\mathrm{h}}}=\chi^{-1}(\dot{\vec{\mathrm{m}}}-m_r\hat{n})=\chi^{-1}(\dot{\vec{\mathrm{m}}}-m_r\dot{\vec{\theta}}\times\hat{n}) , \quad (7.7)$$

needed for the correlation functions in Sec. VI, one should employ the $\delta\vec{\theta}$ of (7.6) rather than that of (7.3).

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APPENDIX

In this appendix we extend some of Ginzburg's results,[27] discuss orthogonality of the normal modes, and introduce a new definition for the macroscopic spin-space rotation angle $\delta\vec{\theta}$.

Setting $\gamma=\hbar=1$, the equation of motion for the ith spin is given (when only exchange is considered) by

$$\dot{S}_{i\alpha}=\sum_{\beta,\gamma}\epsilon_{\alpha\beta\gamma}S_{i\beta}\sum_{j}J_{ij}S_{j\beta} . \quad (A1)$$

Thus in equilibrium one has

$$0=\sum_{\beta,\gamma}\epsilon_{\alpha\beta\gamma}S_{i\beta}^{(0)}\sum_{j}J_{ij}S_{j\gamma}^{(0)} . \quad (A2)$$

As a consequence, $\sum_j J_{ij}S_{j\gamma}^{(0)}$ points along $S_{i\gamma}^{(0)}$:

$$\sum_{i,j}J_{ij}S_{j\gamma}^{(0)}=\sum_{i}\lambda_i S_{i\gamma}^{(0)} . \quad (A3)$$

Ordinarily, to determine the spin waves one works with $\delta\vec{\mathrm{S}}_i=\vec{\mathrm{S}}_i-\vec{\mathrm{S}}_i^{(0)}$, and linearizes (A1) to obtain

$$\begin{aligned}\delta\dot{S}_{i\alpha}&=\sum_{\beta,\gamma}\epsilon_{\alpha\beta\gamma}\left[\delta S_{i\beta}\lambda_i S_{i\gamma}^{(0)}+S_{i\beta}^{(0)}\sum_j J_{ij}\delta S_{j\gamma}\right]\\ &=\sum_{\beta,\gamma}\epsilon_{\alpha\beta\gamma}S_{i\beta}^{(0)}\left[\sum_j (J_{ij}-\lambda_i\delta_{ij})\delta S_{j\gamma}\right]\\ &=-\sum_{\beta,\gamma}\sum_j(\epsilon_{\alpha\beta\gamma}S_{i\beta}^{(0)}\lambda_{ij})\delta S_{j\gamma} , \end{aligned} \quad (A4)$$

$$\lambda_{ij}\equiv\lambda_i\delta_{ij}-J_{ij}=\lambda_{ji} . \quad (A5)$$

One then differentiates (A4) with respect to time, and employs (A4) to eliminate $\delta\dot{S}_{j\gamma}$:

$$\begin{aligned}\delta\ddot{S}_{i\alpha}&=\sum_{\beta,\gamma}\sum_j\epsilon_{\alpha\beta\gamma}S_{i\beta}^{(0)}\lambda_{ij}\sum_{\mu,\nu}\sum_k(-\epsilon_{\gamma\mu\nu}S_{j\mu}^{(0)}\lambda_{jk})\delta S_{k\nu}\\ &=-M_{ik,\alpha\nu}\delta S_{k\nu} ,\end{aligned} \quad (A6)$$

$$M_{ik,\alpha\nu}\equiv-\sum_{\beta,\mu}\sum_j(\delta_{\alpha\mu}\delta_{\beta\nu}-\delta_{\alpha\nu}\delta_{\beta\mu})S_{i\beta}^{(0)}S_{j\mu}^{(0)}\lambda_{ij}\lambda_{ik} . \quad (A7)$$

If we take $\delta S_{i\alpha}\propto e^{-i\omega t}$, then (A6) becomes

$$M_{ik,\alpha\nu}\delta S_{k\nu}=\omega^2\delta S_{k\alpha} , \quad (A8)$$

which has the appearance of a conventional eigenvalue problem. However, M is not Hermitian. Even defining

$$\begin{aligned}\tilde{M}_{ik,\alpha\nu}&\equiv M_{ik,\alpha\nu'}\delta_{\nu'\nu}^{Tk} ,\\ \delta_{\nu'\nu}^{Tk}&=\delta_{\nu'\nu}-S_{k\nu'}^{(0)}S_{k\nu}^{(0)} ,\end{aligned} \quad (A9)$$

so that $\tilde{M}$ is assured to give zero when multiplied on the right, does not give a Hermitian operator $\tilde{M}$. Hence the conventional theorems about eigenvalues and eigenfunctions do not go through (see Ref. 3).

The approach that Ginzburg takes, on the other hand, gives an eigenvalue problem with a Hermitian operator and a weight function. The key to this approach is to work with both $\delta\vec{\mathrm{S}}_i$ and $\delta\vec{\theta}_i$, where $\delta\vec{\theta}_i\cdot\vec{\mathrm{S}}_i^{(0)}=0$ and

$$\delta\vec{\mathrm{S}}_i=\delta\vec{\theta}_i\times\vec{\mathrm{S}}_i^{(0)} ,\quad \delta\vec{\theta}_i=\vec{\mathrm{S}}_i^{(0)}\times\delta\vec{\mathrm{S}}_i , \quad (A10)$$

where we have assumed unit spins, so $\vec{\mathrm{S}}_i^{(0)}\cdot\vec{\mathrm{S}}_i^{(0)}=1$. Thus (A4) can be rewritten as

$$\begin{aligned}\delta\dot{\vec{\theta}}_i&=\vec{\mathrm{S}}_i^{(0)}\times\delta\dot{\vec{\mathrm{S}}}_i\\ &=\vec{\mathrm{S}}_i^{(0)}\times(-\vec{\mathrm{S}}_i^{(0)}\times\sum_j\lambda_{ij}\delta\vec{\mathrm{S}}_j)\\ &=\sum_j\lambda_{ij}(\delta\vec{\mathrm{S}}_j)_{\perp i} .\end{aligned} \quad (A11)$$

We can write this in a form which guarantees that $\delta\vec{\theta}_i\cdot\vec{\mathrm{S}}_i^{(0)}=\delta\vec{\mathrm{S}}_i\cdot\vec{\mathrm{S}}_i^{(0)}=0$ by defining

$$W_{ij,\alpha\beta}\equiv\lambda_{ij}\delta_{\alpha\gamma}^{Ti}\delta_{\gamma\beta}^{Tj}=W_{ji,\beta\alpha} . \quad (A12)$$

(Note that, properly, W has no inverse.) Then (A11) can be written as

$$\delta\dot{\theta}_{i\alpha}=W_{ij,\alpha\beta}\delta S_{j\beta} . \quad (A13)$$

Inverting this in the subspace of transverse spin components, we find that

$$\delta S_{j\beta}=(W^{-1})_{jk,\beta\gamma}\delta\dot{\theta}_{k\gamma} . \quad (A14)$$

Ginzburg makes this point in a slightly different fashion, reasoning that $\delta\dot{\theta}_i$ is like a local magnetic field $\vec{\mathrm{h}}_i$, and therefore that the form

$$\delta S_{j\beta}=\chi_{jk,\beta\gamma}\delta\dot{\theta}_{k\gamma} \tag{A15}$$

should hold, where χ is the transverse local susceptibility. Comparison of (A14) and (A15) immediately yields an explicit expression for χ:

$$\chi_{ij,\alpha\beta}=(W^{-1})_{ij,\alpha\beta}\ . \tag{A16}$$

[Note that Ref. 23 studies a modified problem, that of the transverse response of spin i to an arbitrary (rather than transverse) field at site j. To study this modified problem in our formalism we would modify (A12) to read $W_{ij,\alpha\beta}\equiv\lambda_{ij}\delta^{Ti}_{\alpha\beta}(\neq W_{ji,\beta\alpha})$. For (2.10) of Ref. 23 to agree with this, the factor of $\delta_{\alpha\beta}$ on the left-hand side of (2.10) would have to be replaced by $\delta^{Ti}_{\alpha\beta}$.]

In the spirit of Ginzburg's approach, we rewrite (A4) using (A10):

$$\begin{aligned}\delta\dot{S}_{i\alpha}&=-\sum_{\beta,\gamma}\epsilon_{\alpha\beta\gamma}S^{(0)}_{i\beta}\sum_{\mu,\nu}\sum_{j}\lambda_{ij}\epsilon_{\gamma\mu\nu}\delta\theta_{j\mu}S^{(0)}_{j\nu}\\&=-\sum_{\nu}\sum_{j}U_{ij,\alpha\nu}\delta\theta_{j\nu}\ ,\end{aligned} \tag{A17}$$

$$\begin{aligned}U_{ij,\alpha\nu}&\equiv-\sum_{\beta,\gamma,\mu}\epsilon_{\alpha\beta\gamma}\epsilon_{\mu\nu\gamma}S^{(0)}_{i\beta}S^{(0)}_{j\mu}\lambda_{ij}\\&=\sum_{\beta,\mu}(\delta_{\alpha\nu}\delta_{\beta\mu}-\delta_{\alpha\mu}\delta_{\beta\nu})S^{(0)}_{i\beta}S^{(0)}_{j\mu}\lambda_{ij}\\&=\sum_{\mu}(\delta_{\alpha\nu}(\vec{S}^{(0)}_i\cdot S^{(0)}_j)-S^{(0)}_{i\nu}S^{(0)}_{j\mu})\lambda_{ij}\\&=U_{ji,\nu\alpha}\ .\end{aligned} \tag{A18}$$

Use of (A15) in (A17) then yields Eq. (13) of Ref. 25:

$$\chi_{ij,\alpha\beta}\delta\ddot{\theta}_{j\beta}=-U_{ij,\alpha\beta}\delta\theta_{\beta}\ . \tag{A19}$$

Letting $\delta\vec{\theta}_j\alpha e^{-i\omega t}$, Eq. (A19) becomes

$$U_{ij,\alpha\beta}\delta\theta_{j\beta}=\omega^2\chi_{ij,\alpha\beta}\delta\theta_{j\beta}\ . \tag{A20}$$

This is an eigenvalue problem with the weight function $\chi_{ij,\alpha\beta}$ [$=\chi_{ji,\beta\alpha}$ by (A12) and (A16)]. Because U and χ are real and symmetric, the eigenvalues ω^2 are real. (They are also positive if the system is locally stable.) The orthogonality of two modes m and n satisfying, e.g.,

$$U_{ij,\alpha\beta}\delta\theta^{(m)}_{j\beta}=\omega^{2(m)}\chi_{ij,\alpha\beta}\delta\theta^{(m)}_{j\beta}\ ,$$

is expressed as

$$0=\delta\theta^{(m)}_{i\alpha}\chi_{ij,\alpha\beta}\delta\theta^{(n)}_{j\beta}\quad(\omega^{(m)}\neq\omega^{(n)})\ . \tag{A21}$$

This can be rewritten on multiplying by $-i\omega^{(n)}$ and with (A15), as

$$0=\sum_i\delta\vec{\theta}^{(m)}_i\cdot\vec{\delta S}^{(n)}_i\ , \tag{A22}$$

where we have used $\delta\dot{\theta}^{(n)}_{j\beta}=-i\omega^{(n)}\delta\theta^{(n)}_{j\beta}$.

This relationship can also be derived from the conventional treatments [it follows, after some algebra, from (3.21) of Ref. 3]. If we consider that $\delta\vec{\theta}^{(m)}_i$ is due to the uniform rotation modes (at zero frequency), we may set $\delta\vec{\theta}^{(m)}_i=\delta\vec{\alpha}$, with $\delta\vec{\alpha}$ arbitrary, so (A22) yields

$$0=\sum_i\delta\vec{S}^{(n)}_i\quad(\omega^{(n)}\neq0)\ . \tag{A23}$$

This says that finite frequency modes possess no net magnetization. With (A15) this becomes

$$0=\sum_{i,j}\chi_{ij,\alpha\beta}\delta\dot{\theta}^{(n)}_{j\beta}\quad(\omega^{(n)}\neq0) \tag{A24}$$

or

$$0=\sum_{i,j}\chi_{ij,\alpha\beta}\delta\theta^{(n)}_{j\beta}\quad(\omega^{(n)}\neq0)\ . \tag{A25}$$

It is (A25) which enables us to determine the macroscopic spin-space rotation angles $\delta\theta_\beta$, associated with zero frequency. Since $\delta\theta_{j\beta}$ is due both to $\delta\theta_\beta$, and to the $\delta\theta^{(n)}_{j\beta}$'s, by (A25) we have

$$\sum_\beta\sum_{i,j}\chi_{ij,\alpha\beta}\delta\theta_{j\beta}=\sum_\beta\left[\sum_{i,j}\chi_{ij,\alpha\beta}\right]\delta\theta_\beta=\chi\delta\theta_\alpha\ . \tag{A26}$$

Here we have employed

$$\begin{aligned}\chi_{\alpha\beta}&\equiv\partial m_\alpha/\partial H_\beta=\partial\left[\sum_i\delta S_{i\alpha}\right]\Big/\partial H_\beta\\&=\partial\left[\sum_{i,j}\sum_\gamma\chi_{ij,\alpha\gamma}\delta H_\gamma\right]\Big/\partial H_\beta\\&=\sum_{i,j}\chi_{ij,\alpha\beta}\ ,\end{aligned} \tag{A27}$$

where $\chi_{\alpha\beta}=\chi\delta_{\alpha\beta}$ follows from the macroscopic isotropy of the spin-glass state.

Thus (A26) gives us our definition of the spin-space rotation angle $\delta\vec{\theta}$

$$\delta\theta_\alpha=\chi^{-1}\sum_{i,j}\sum_\beta\chi_{ij,\alpha\beta}\delta\theta_{j\beta}\ . \tag{A28}$$

Note that, by (A15),

$$\begin{aligned}\delta\dot{\theta}_\alpha&=\chi^{-1}\sum_{i,j}\sum_\beta\chi_{ij,\alpha\beta}\delta\dot{\theta}_{j\beta}\\&=\chi^{-1}\sum_i\delta S_{i\alpha}=\chi^{-1}\delta m_\alpha\ ,\end{aligned} \tag{A29}$$

in agreement with Eq. (2.2) of Ref. 4. We remark that Ref. 4 took $\delta\theta_\alpha\propto\sum_i\delta\theta_{i\alpha}$, in contrast to (A28). Such a definition of $\delta\theta_\alpha$ will likely not satisfy (A29).

The variable $\delta\theta_\alpha$ satisfies the commutation relation

$$[m_\gamma,\delta\theta_\alpha]=\chi^{-1}\sum_{i,j,k}[\delta S_{k\gamma},\sum_\beta \chi_{ij,\alpha\beta}\delta\theta_{j\beta}]$$
$$=\chi^{-1}\sum_{i,j,k}\sum_{\beta,\mu,\nu}\chi_{ij,\alpha\beta}[\delta S_{k\gamma},\epsilon_{\beta\mu\nu}S_{j\mu}^{(0)}\delta S_{j\nu}]$$
$$=i\chi^{-1}\sum_{i,j}\sum_{\beta,\mu,\nu}\epsilon_{\beta\mu\nu}\chi_{ij,\alpha\beta}S_{j\mu}^{(0)}\epsilon_{\gamma\nu\lambda}S_{j\lambda}\ , \quad \text{(A30)}$$

where we have employed $[\delta S_{k\gamma},\delta S_{j\nu}]=[S_{k\gamma},S_{j\nu}]=i\sum_\lambda \delta_{kj}\epsilon_{\gamma\nu\lambda}S_\lambda$. Taking the expectation value of (A30), and using the fact that $\sum_\beta \chi_{ij,\alpha\beta}S_{j\beta}^{(0)}=0$ (a local field along the equilibrium direction of the jth spin causes no spin rearrangements) yields

$$\langle[m_\gamma,\delta\theta_\alpha]\rangle=-i\chi^{-1}\sum_{i,j}\chi_{ij,\alpha\gamma}=-i\delta_{\alpha\gamma}\ . \quad \text{(A31)}$$

This is as expected.[4] Further,

$$[\delta\theta_\alpha,\delta\theta_\gamma]=\chi^{-2}\sum_{i,j}\sum_{k,l}[\sum_{\beta,\mu,\nu}\chi_{ij,\alpha\beta}\epsilon_{\beta\mu\nu}S_{j\mu}^{(0)}\delta S_{j\nu},\sum_{\lambda,\eta,\delta}\chi_{kl,\gamma\lambda}\epsilon_{\lambda\delta\eta}S_{l\delta}^{(0)}\delta S_{l\eta}]$$
$$=i\chi^{-2}\sum_{i,j,k}\sum_{\beta,\mu,\nu}\sum_{\lambda,\eta,\zeta}\chi_{ij,\alpha\beta}\chi_{kj,\gamma\lambda}\epsilon_{\beta\mu\nu}\epsilon_{\lambda\delta\eta}\epsilon_{\nu\eta\zeta}S_{j\mu}^{(0)}S_{j\delta}^{(0)}S_{j\zeta}\ .$$

Taking the expectation value of this, with

$$(\chi_{j\alpha})_\beta\equiv\sum_i\chi_{ji,\beta\alpha}=\sum_i\chi_{ij,\alpha\beta} \quad \text{(A32)}$$

giving the response at the jth site in the αth direction to a unit field in the βth direction, we have

$$\langle[\delta\theta_\alpha,\delta\theta_\gamma]\rangle=i\chi^{-2}\sum_j\sum_{\beta,\mu,\nu,\lambda}(\chi_{j\alpha})_\beta(\chi_{j\gamma})_\lambda\epsilon_{\beta\mu\nu}(S_{j\nu}^{(0)}S_{j\lambda}^{(0)}-\delta_{\lambda\nu})S_{j\mu}^{(0)}$$
$$=-i\chi^{-2}\sum_j\sum_{\beta,\mu,\lambda}(\chi_{j\alpha})_\beta(\chi_{j\gamma})_\lambda\epsilon_{\beta\mu\lambda}S_{j\mu}^{(0)}$$
$$=i\chi^{-2}\sum_j\vec{S}_j^{(0)}\cdot(\vec{\chi}_{j\alpha}\times\vec{\chi}_{j\gamma})\ . \quad \text{(A33)}$$

Clearly this is antisymmetric in α and γ, so we may write

$$\langle[\delta\theta_\alpha,\delta\theta_\beta]\rangle=\sum_\gamma\epsilon_{\alpha\beta\gamma}T_\gamma\ , \quad \text{(A34)}$$

where T_γ must be proportional to some vector property of the SG (i.e., $\vec{m}$ or $m_r\hat{n}$). Since the magnetization of a SG is usually small, we thus have $\langle[\delta\theta_\alpha,\delta\theta_\beta]\rangle\approx 0$, as expected.[4]

To summarize, we have found a definition of the macroscopic spin-space rotation angle which satisfies the expected commutation relations *and* contains no contributions from localized modes. This definition differs from the one we employed earlier (Ref. 4), which did permit contributions from localized modes. Because their commutation relations are the same, the hydrodynamics is unaffected. Only in microscopic calculations of macroscopic quantities (e.g., the dissipative coefficients of Sec. VI) would the difference become apparent.

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