Renormalization and the Kosterlitz-Thouless transition in a two-dimensional superconductor

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The nonlinear I-V characteristics of thin high-sheet-resistance Hg-Xe alloy films are examined within the context of the Kosterlitz-Thouless theory of the superconducting transition. In the regime below the vortex-unbinding temperature T_c, where logarithmically bound vortices can be broken apart by a transport current, we find that $V \sim I^{a(T)}$. Comparison with theory allows us to infer the value of T_c and the mean-field temperature T_{c0} from $a(T)$, and the dependence of these temperatures on $R_N^{\square}$ appears in approximate agreement with the microscopic theory of dirty superconductors. A systematic deviation appears to be consistent with renormalization of the vortex interaction close to T_c due to the presence of small polarizable vortex pairs, and can be described by an effective vortex dielectric constant $\epsilon_c = 1.2$. Further evidence for this renormalization, which is a key feature of the Kosterlitz-Thouless transition, is obtained by examining the curvature of the $\log V$ vs $\log I$ plot very close to T_c. The current dependence of $a(I,T) = d(\log V)/d(\log I)$ is a direct measure of the spatial dependence of the vortex interaction, allowing a direct comparison with the analytic predictions of the renormalization equations. Satisfactory agreement is obtained using physically reasonable parameters.

I. INTRODUCTION

Phase transitions in two dimensions (2D) have been a subject of much recent interest,[1] both theoretical and experimental. The Kosterlitz-Thouless (KT) transition[2] has been predicted to occur in many 2D systems, including superfluids, superconductors, X-Y spin systems, and melting of solids. A key feature of this transition is that it occurs via unbinding of pairs of topological defects, whose interaction energy exhibits a logarithmic dependence on separation. The other key element is the renormalization of this interaction near the transition due to the presence of polarizable pairs of bound defects. An experiment which provided clear direct evidence of these defects, their logarithmic interactions, and its renormalization near the transition, would be the strongest possible indication that the transition is of the Kosterlitz-Thouless type.

In a 2D superconductor, the topological defects are vortices, which interact with a logarithmic potential over a scale less than the transverse penetration depth $\lambda_\perp$. For a film with normal-state sheet resistance $R_N^{\square}$ of order 1 kΩ or greater, $\lambda_\perp$ is the order of the sample size (mm to cm) over the entire range of interest, so that the sample is an effective candidate for a KT transition to the superconducting state.[3–5] We have earlier shown that in thin quench-condensed Hg-Xe alloy films, the low-temperature electrical properties are dominated by logarithmically bound-vortex pairs,[6] and have presented some evidence for renormalization effects near the transition.[7] A number of other 2D superconducting systems, including granular films[8,9] and proximity-coupled arrays,[10,11] have been investigated by a variety of means, and have generally lent varying degrees of support to the concept of a KT transition in these samples, with one notable exception.[12]

In the course of our further analysis of the data on the superconducting Hg-Xe samples, we have obtained a clearer understanding of the predicted renormalization effects, and how they can be probed quite directly by considering the current-voltage characteristics very near the transition. In the present paper we will attempt to explain how this can be done, and show how some of the data taken earlier provides a preliminary indication of the renormalization of the vortex interaction, in reasonable agreement with the predictions of the KT theory. In this way we present some of the most direct evidence yet available for the existence of a KT transition in a 2D superconductor.

We continue in Sec. II with a brief discussion of the techniques of sample preparation and characterization, introducing data on the resistive transition of one particular sample (no. 4) which will be featured in the remainder of the paper. Section III proceeds to analyze the low-temperature nonlinear current-voltage characteristics within the KT theory, in terms of current-induced unbinding of bound vortex—antivortex pairs. This analysis demonstrates the logarithmic nature of the vortex interaction, and confirms the dependence of the transition width on the normal-state sheet resistance of the films. Section IV incorporates the effects of bound vortex—antivortex pairs into an effective dielectric constant for the medium, and shows how this is important near the vortex-unbinding temperature T_c. The vortex correlation length for $T > T_c$ is discussed briefly in Sec. V , and the associated fit to the resistive transition is established, in reasonable agreement with theory. Section VI completes the analysis of the renormalization of the vortex interaction, and demonstrates how its spatial dependence is directly reflected in a careful examination of the nonlinear I-V curves very close to T_c. The overall conclusions of the paper are summarized in Sec. VII.

II. EXPERIMENTAL PROCEDURE AND RESISTIVE TRANSITION

In the following we will be referring to data taken on samples consisting of ultrathin (100 Å) films of an alloy of Hg and Xe, quench-condensed from a molecular-beam oven onto substrates thermally sunk to the liquid-helium bath at 4.2 K. The condensation rate, less than about 1 Å/sec, was low enough so that heating from the incident atoms could be ignored, and oven shielding likewise made radiational heating negligible. The substrates were glazed alumina, where the glazing was necessary to eliminate microscratches that would make these films discontinuous, an earlier problem with crystalline sapphire and quartz substrates. We have not yet obtained structural analysis of these samples, since warming to room temperature clearly destroys them. However, their method of preparation, as well as a number of other factors, leads us to believe that they are likely to be amorphous as prepared, or if granular, with grains that are smaller than about 100 Å.[13–15]

We have studied the electrical characteristics of a number of samples, some of them directly as prepared by quench condensation, others "annealed" at temperatures between 10 and 40 K, still far too cold for Xe sublimation. Annealing always resulted in an irreversible lowering of the resistivity, perhaps because of clustering on the microscopic scale. This decrease in resistivity generally appeared to saturate after a film was maintained at a given temperature for a period of under 10 min, and no such decrease ever occurred if the sample was kept below 8 K. The annealed films appear to exhibit similar electrical properties to the as-prepared films, and both types of films will be treated together in this paper.

In Fig. 1(a) we show the resistive transition (R vs T) into the superconducting state for one sample, with a composition of 0.60 mol fraction of mercury (0.40 mol fraction of Xe), which had been annealed at 18 K. The fit to this data in Fig. 1(b) will be discussed later. The normal-state sheet resistance at $T=5$ K was $R_N^{\square}=1.8$ kΩ, which taken together with the estimated thickness $d=150$ Å corresponds to a resistivity $\rho_N=2700$ $\mu\Omega$ cm, placing the material near the metal-insulator transition. Such a high resistivity was necessary in order to obtain a film with a sufficiently large $R_N^{\square}$ (of order 1 kΩ or greater) while maintaining sufficient thickness to assure a uniform film. We have elsewhere examined the composition dependence of the normal-state resisitivity of thick as-prepared films.[13]

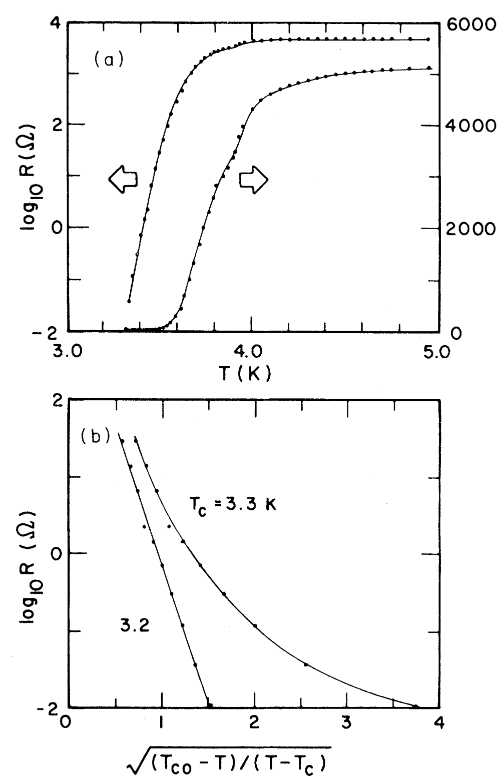

FIG. 1. (a) Resistive transition R (Ω) vs T (K) for sample no. 4, shown on a linear and a log scale. (b) Fit of the data from (a) for $T < 3.5$ K to the theoretical expression Eq. (40), using $T_{c0}=3.6$ K, and taking $T_c=3.3$ K (right) and $T_c=3.2$ K (left). The curve on the right is a guide to the eye; the straight line on the left corresponds to $A=0.5$ and $b=16.6$.

The resistances in Fig, 1 were measured in the low-current limit, typically using a measuring current of 1 μA or less, mechanically chopped at 37.5 Hz, with the resulting voltage detected with a lock-in amplifier with sensitivity to better than 10 nV. It was important to check the linearity of the I-V relation, particularly at the tail end of the transition, where the characteristics became strikingly nonlinear (see below) at somewhat higher currents. Another important concern was the possible presence of a magnetic field, since externally produced vortices will add to the resistance due to the thermally excited vortices. Although a μ-metal shield was present around the outside of the Dewar, residual magnetism in the structure of the apparatus necessitated the use of a small solenoid to cancel out the component of the field perpendicular to the film ($\lesssim 50$ mG). This was accomplished by minimizing the resistance in the tail of the transition, where it was highly sensitive to such fields.

III. CURRENT-INDUCED VORTEX UNBINDING

The primary measurements of the films under investigation are the dc current-voltage characteristics. In Fig. 2 we show the I-V curves for the same sample (no. 4) as in Fig. 1. The striking feature of this set of curves is that, on a log-log plot, they tend to fit a straight line over a large range of currents and voltages, particularly for the lower temperatures. This linear fit on a log-log plot means that the voltage varies as a power of the current, i.e.,

$$V \sim I^{a(T)} , \tag{1}$$

where the power $a(T)$ is a function of temperature, increasing as the temperature is lowered. We will later identify the temperature at which $a(T)=3$ as the location of the KT vortex-unbinding transition.

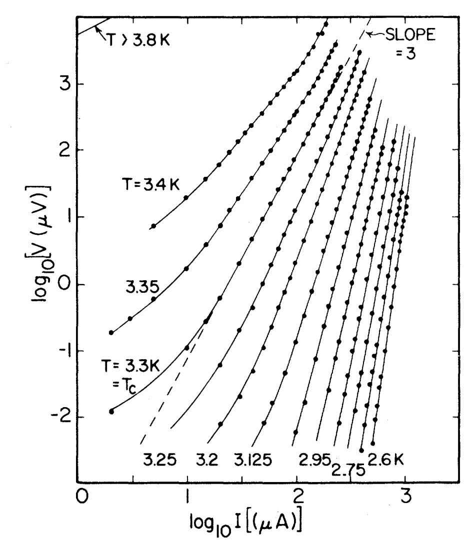

FIG. 2. Low-temperature I-V curves for sample no. 4, on a log-log plot, for various fixed temperatures. The dashed line with slope $=3$ corresponds to T_c.

Within the picture of the KT transition in a superconducting film, below the mean-field transition temperature T_{c0}, there exists a local-order parameter, but long-range order is destroyed by the presence of vortex excitations of both polarities, even in the absence of an external field. A current density $J_s = n_s e v_s$ exerts a force on a vortex (the Lorentz force), perpendicular to the direction of the current flow, of magnitude,

$$F_L = J_s \Phi_0 / c , \tag{2}$$

where $\Phi_0 = hc/2e$ is the flux quantum. In the absence of vortex pinning, this produces a steady dissipative vortex motion, which results in the presence of a nonzero resistance. A vortex and an antivortex (one of the opposite polarity) attract each other, however, and below a lower critical temperature $T_c = T_{\rm KT}$, they are effectively bound together as a pair. Since an applied current exerts an opposite force on each member of the pair, the net force is zero and the vortices do not move. Hence there is no resistance, and the major physical feature associated with superconductivity is established.

However, zero resistance is strictly found only in the limit of zero current. Precisely because an applied current exerts an opposite force on the members of a pair, a sufficiently large current will break apart the pair, yielding two vortices which are then free to move around, producing resistance, until they recombine.[6,16,17] We will show below that for *any* arbitrarily small current, *some* pairs will be broken apart, specifically the ones whose constituent vortices are very far apart. Therefore, in contrast to a three-dimensional (3D) superconductor, the critical current of a 2D superconductor is properly zero. This sensitivity to small perturbations is a general characteristic of quasi-long-range order in 2D. As the current is increased, a larger and larger fraction of the vortex pairs can be broken, yielding a current-dependent resistance $R(I)$. Since larger currents tear apart smaller vortex pairs, and thus probe smaller distances, one expects that the functional dependence of $R(I)$ should give some useful information on the way the vortex-antivortex interaction changes with scale. That is the fundamental idea behind what will follow.

We will begin by considering only the unrenormalized interaction. The vortex-antivortex pair energy, for zero applied current, can be written in the form

$$U_0(r)=2E_c+q^2\ln(r/\xi)\ , \qquad (3)$$

where E_c is the vortex core energy and $\xi=\xi_{\rm GL}$ is the effective vortex core radius. By analogy with the 2D Coulomb gas, $q=(\pi n_s\hbar^2/2m)^{1/2}$ is the effective vortex charge, and $n_s\equiv n_s^{\rm 2D}=n_s^{\rm 3D}d$ is the 2D superelectron density, where d is the film thickness. In the presence of an applied supercurrent, this interaction energy is modified by the Lorentz force as follows:

$$\begin{aligned}U(r)&=U_0(r)-F_L\cdot r\\&=2E_c+q^2[\ln(r/\xi)-2mv_sr/\hbar]\ .\end{aligned} \qquad (4)$$

This potential has a saddle point for vortices oriented such that the vector connecting them is at right angles to the direction of current flow. The distance to the saddle, which corresponds to a maximum in the potential in that direction, is

$$r_c=\hbar/2mv_s=\hbar n_se/2mJ_s\ , \qquad (5)$$

with an energy at the saddle of

$$\begin{aligned}U(r_c)&=2E_c+q^2[\ln(r_c/\xi)-1]\\&\approx U_0(r_c),\ \ r_c>>\xi\\&\approx 2E_c-q^2\ln(J_s/J_0),\ \ J_s<<J_0\end{aligned} \qquad (6)$$

where the current $J_0=\hbar n_se/2m\xi$ is essentially (to a factor of order unity) the Ginzburg-Landau critical current. If we then assume a simple classical escape over this saddle point, the rate of production of free vortices Γ_e goes as

$$\begin{aligned}\Gamma_e&\sim\exp[-U(r_c)/k_BT]\\&\sim(r_c/\xi)^{(-q^2/k_BT)}\\&\sim(J_s/J_0)^{+(q^2/k_BT)}\ .\end{aligned} \qquad (7)$$

The density of free vortices N_F will be determined by the balance of the rate of production and the rate of recombination. Since recombination is a two-body process, and there are equal numbers of vortices and antivortices, the rate equation can be written as

$$\dot{N}_F=\Gamma_e-\alpha N_F^2\ , \qquad (8)$$

so that in steady state, $N_F\sim\Gamma_e^{1/2}$. Then, since in the absence of flux pinning, the electrical resistance should be proportional to the density of free vortices, one has[4]

$$\begin{aligned}R&=(2\pi\xi^2R_N)N_F\\&\sim(J_s/J_0)^{(q^2/2k_BT)}\ ,\end{aligned} \qquad (9)$$

and the voltage is then of the form $V\sim I^a$, where

$$a=1+q^2/2k_BT=1+\pi n_s\hbar^2/4mk_BT \qquad (10)$$

is the exponent that we should associate with the experimentally determined slopes of Fig. 2. From this analysis it is evident that measurements of the temperature-dependent *I-V* characteristics of 2D superconducting films permit the determination of $n_s(T)$.

For all such systems with logarithmically interacting vortices, application of renormalization-group technology to the KT model predicts a universal relation between the superelectron density at the critical temperature T_c, and T_c itself, the so-called "universal jump condition",[18]

$$q^2(T_c)=\pi n_s(T_c)\hbar^2/2m=4k_BT_c\ . \qquad (11)$$

This suggests that one can determine T_c experimentally by the relation

$$a(T_c)=1+q^2(T_c)/2k_BT_c=3\ . \qquad (12)$$

Outside of the critical region close to T_c, renormalization effects should be small and the superelectron density should be given by its unrenormalized value n_s^0. This "bare," unrenormalized n_s^0 goes linearly to zero at T_{c0} as $n_s^0\sim1-T/T_{c0}$, for T not too small. Thus in this regime the exponent $a(T)$ should have the approximate form

$$a_{\rm lin}(T)\approx1+{\rm const}(1-T/T_{c0})\ . \qquad (13)$$

It is then possible to extrapolate from this region to determine T_{c0} using the relation $a_{\rm lin}(T_{c0})=1$ which follows from Eq. (13). In Fig. 3 the temperature dependence $a(T)$ is shown for three samples of varying sheet resistance. There is, in fact, a low-temperature linear region, and at a higher temperature $a(T)$ crosses the value 3. Thus it is possible to define both T_c and T_{c0}.

The values of T_{c0} inferred depend in part on which points are included in the straight-line fit. In an earlier work, points close to T_c were included in this linear fit, and the resulting values of T_{c0} were somewhat smaller than those which we now claim. As we will discuss later, the points close to T_c should fall *below* the linear fit due to renormalization effects. Another aspect to the fit is that given the theoretical temperature dependence of n_s^0/T, one would expect deviations from the linear relation of Eq. (13) at sufficiently low T, corresponding to upward curvature above the straight line. However, the data points for the lowest temperatures in Fig. 3

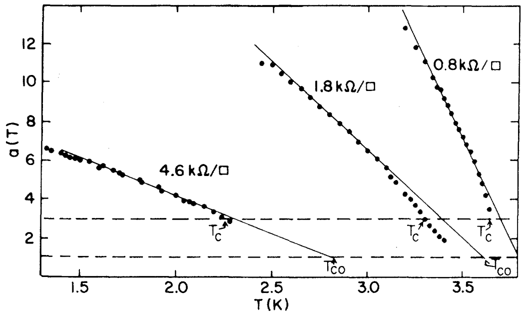

FIG. 3. Temperature dependence $a(T)$ of the exponent from the low-temperature I-V curves, for samples nos. 1, 4, and 5 (left to right). The straight lines $a_{\rm lin}(T)$ are drawn to fit the data below the critical region close to T_c [determined by $a(T_c)=3$], and $a_{\rm lin}(T_{c0})=1$ determines T_{c0}.

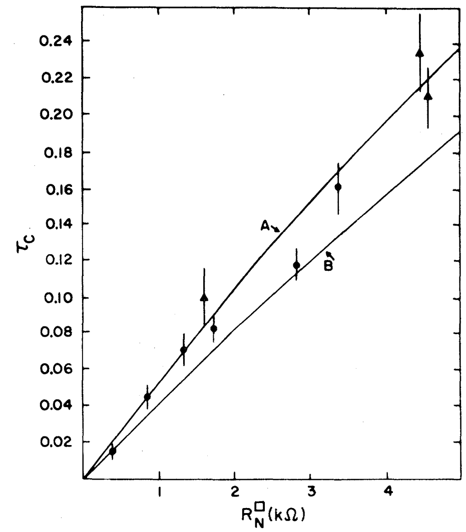

FIG. 4. Dependence of transition width $\tau_c = 1 - T_c/T_{c0}$ on the normal-state sheet resistance $R_N^{\square}$ for nine samples. The vertical error bars represent the uncertainty in determining T_{c0} by extrapolation. Curves A and B are theoretical curves using Eq. (37). Curve A uses a parameter $\epsilon_c = 1.2$, and curve B, $\epsilon_c = 1.0$, i.e., the result of BMO (Ref. 3).

appear to be below the straight-line fits. This apparent discrepancy remains unresolved.

Note that in Fig. 3, the samples with higher sheet resistances have broader transitions, as characterized by the parameter $\tau_c = (T_{c0} - T_c)/T_{c0}$. This correlation can be compared to theory using the approach of Beasley, Mooij, and Orlando (BMO).[3] Since

$$n_s = dmc^2/(4\pi e^2 \lambda^2) \tag{14}$$

the universal jump condition [Eq. (12)] can be expressed in terms of the magnetic penetration depth $\lambda(T)$ as

$$\Phi_0^2 d/2\pi^2\lambda^2(T_c) = 4k_B T_c \ . \tag{15}$$

One can then use the dirty-limit BCS formula for $\lambda(T)$ to obtain

$$(T_c/T_{c0})/\{[\Delta(T_c)/\Delta(0)]\tanh[\Delta(T_c)/2k_B T_c]\} = 2.18 R_c/R_N^{\square} \ , \tag{16}$$

or in the simplified form appropriate when T_c is close to T_{c0},

$$T_c/T_{c0} \approx (1 + 0.17 R_N^{\square}/R_c)^{-1} \ . \tag{17}$$

Here $R_c = \hbar/e^2 = 4100\ \Omega$ is the characteristic resistance in two dimensions.

In Fig. 4 we have plotted the inferred value of the parameter $\tau_c = 1 - T_c/T_{c0}$ for nine different samples as a function of the sheet resistance $R_N^{\square}$, as well as the BMO theoretical expression Eq. (16). There is an approximate fit between experiment and theory, especially inasmuch as there are no adjustable parameters in this agreement. The vertical error bars reflect the uncertainty of determining the value of T_{c0} by extrapolation. Since all the points but one lie above the line, as do most of the error bars, there is apparently a systematic deviation of the experimental values above the BMO prediction. As we will show below, this appears to be due to renormalization effects not properly taken into account in the universal jump condition given in Eq. (15).

IV. RENORMALIZATION OF THE VORTEX INTERACTION

An important aspect of the renormalization-group theory of the KT transition involves the screening of the vortex-antivortex interaction due to the presence of a background of polarizable vortex pairs located in between the given test vortices. For $T > T_c$, the interaction is totally screened at large distances; even below T_c there should still be an effect. In order to understand the renormalization of the interaction in a conceptually simple form it is useful to incorporate the effect of these intervening vortex pairs into a dielectric constant ϵ of an effective medium, following in part the approach of Young.[17,19] In our discussions of the problem, we will not attempt to provide a completely rigorous derivation, but rather to develop a coherent picture of the underlying physics.

Because vortex-antivortex pairs are thermal excitations of the system, for $T \ll T_c$, there are likely to be few vortex pairs present. Therefore, the space between a given pair is unlikely to contain other vortex pairs, so that a simple bare (unrenormalized) interaction should be largely correct. At higher temperatures, at $T \approx T_c$, there are likely to be many more vortex pairs, including some which are located between and around the constituent vortices of other, larger pairs, as shown schematically in the inset of Fig. 5. Since vortices of opposite polarity attract, these smaller pairs will tend to align in the field produced by the larger pairs, although the degree of polarization indicated is exaggerated for emphasis. As one can see by analogy with the Coulomb problem, or by consideration of the current flows surrounding the vortices, this polarization will act to reduce the strength of the vortex interaction, i.e., to reduce the size of the effective charge $q(r)$ below the intrinsic (unrenormalized) charge q_0. This can be characterized in terms of an effective vortex dielectric constant $\epsilon(r) \gtrsim 1$, which takes account of the effect of vortex pairs of the size less than or equal to r on the vortex interaction. In terms of $\epsilon(r)$, the vortex *force* can be expressed as

$$F(r) = q_0^2 / r\epsilon(r) = q^2(r)/r \ , \tag{18}$$

and the pair energy is therefore

$$U(r) = 2E_c + \int_{r'=\xi}^{r'=r} q^2(r') d(\ln r') \ . \tag{19}$$

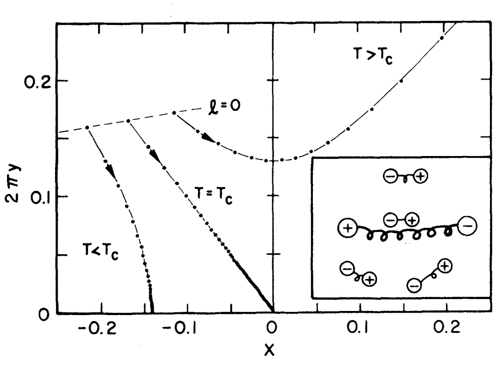

FIG. 5. Renormalization contours of the vortex interaction, in terms of the quantities x and y from Eq. (29). Actual values of the solution are shown, using Eqs. (44)–(46), with $\epsilon_c = 1.2$, $R_N^{\square} = 2000\ \Omega$ (parameters approximately appropriate for sample no. 4) and with curves shown for $T/T_c = 0.995$, 1.0, and 1.005 (from left to right). The points correspond to intervals of $l = \ln(r/\xi) = 0.5$. The inset represents schematically the physical origin of the screening of the vortex interaction.

We also define the fully renormalized quantities $q_R = q(r \to \infty)$ and $n_s^R = n_s(r \to \infty)$. Since $q(r)$ decreases with increasing r, as the temperature is increased, vortex unbinding will first occur at $r \to \infty$. The universal jump condition is then given, in terms of these fully renormalized quantities, as

$$\begin{aligned} q_R^2(T_c^-) &= \pi\hbar^2 n_s^R(T_c^-)/2m = 4k_B T_c \ , \\ q_R(T_c^+) &= 0 = n_s^R(T_c^+) \ . \end{aligned} \tag{20}$$

As we will point out again later, this discontinuity at $T = T_c$ only appears in the limit of an infinite system.

The problem then becomes one of calculating the screening due to the background of bound-vortex pairs. This involves the derivation of the renormalization equations, which we will sketch briefly below.[17,19] We can write the dielectric constant $\epsilon(r)$ in the standard way,

$$\epsilon(r) = 1 + 4\pi\chi(r) \ , \tag{21}$$

in terms of the susceptibility $\chi(r)$, which in turn can be expressed as

$$\chi(r) = \int_\xi^r dr' \int_0^{2\pi} d\theta\, r' n_p(r',\theta)\alpha(r') \ , \tag{22}$$

where

$$\alpha(r) = q_0^2 r^2 / 4k_B T \tag{23}$$

is the pair polarizability and

$$n_p(r,\theta) = (N_0^2/\xi^4)\exp[-U(r)/k_B T] \tag{24}$$

is the density of thermally excited vortex pairs. The quantity N_0 is expected to be of order unity (quite possibly less than one), and represents the number of independent sites to place a vortex core in a cell of area ξ^2. It can also be introduced from the point of view of a configurational entropy $2k_B \ln N_0$ of the vortex cores, and hence included in the exponential as part of the free energy.[20] The derivation implicitly assumes a rather low density of vortices, in particular that there are no pair-pair interactions. This is evident, for example, in Eq. (23), where the unrenormalized effective charge is used to determine the polarizability. This approximation is necessary to prevent the equations from becoming intractable.

From these equations, it follows directly that the spatial derivative of the dielectric constant has the following form:

$$d\epsilon/dr = 2\pi^2 q_0^2 r^3 n_p(r)/k_B T \ . \tag{25}$$

The nature of the coupled equations becomes more clearly evident if we define several new variables,

$$K(r) \equiv q^2(r)/2\pi k_B T = q_0^2/2\pi\epsilon(r) k_B T \ ,$$
$$y^2(r) \equiv r^4 n_p(r), \quad l = \ln r(\xi) \ . \tag{26}$$

In terms of these, one has the Kosterlitz recursion relations[18]

$$dK^{-1}/dl = 4\pi^3 y^2 \ ,$$
$$dy^2/dl = y^2(4 - 2\pi K) \ . \tag{27}$$

These, taken together with the boundary conditions at $l=0$ (the core interaction is not renormalized), provides in principle the complete solution to the problem.

Further simplification can be obtained very near the transition, where $T \approx T_c$ and $K \approx 2/\pi$. If we define the quantity x such that

$$x \equiv (2/\pi K) - 1 \approx 1 - \pi K/2 \ ,$$
$$\pi K = 2/(1-x) \approx 2 - 2x \ , \tag{28}$$

then the above recursion relations become

$$dx/dl = 8\pi^2 y^2 \ ,$$
$$dy/dl = 2xy \ . \tag{29}$$

In this form, one can easily see that the quantity

$$x^2 - 4\pi^2 y^2 = C = x_0^2 - 4\pi^2 y_0^2 \tag{30}$$

is an invariant of this system, i.e.,

$$dC/dl = 0 \quad [x_0 \equiv x(l=0), \ \ y_0 \equiv y(l=0)] \ .$$

Since the only other variable in this system is the temperature, $C = C(T)$. This establishes that the solutions to these equations lie on a set of hyperbolas, which describe three distinct regimes, depending on the temperature, as illustrated in Fig. 5.

First, for $C=0$, which corresponds to $T=T_c$, the hyperbola is the degenerate case of a straight line going to the origin. The point $x=0, y=0$ represents a fixed point of the system as $r \to \infty$. Physically, $x_R = 0$ corresponds to the universal jump condition $K_R = 2/\pi$, and $y_R = 0$ means that there are no free vortices at infinite separation. For $C<0$ $(T>T_c)$ the curve turns away from the origin and approaches infinity at infinite separation. Here $x_R = \infty$ corresponds to $K_R = 0$, i.e., the interaction has been totally screened, and $y_R = \infty$ means that there are plenty of free vortices at large distances to do the screening. In the other limit, for $C>0$ $(T<T_c)$, there is again a fixed point at $y^R = 0$, but now with a value $x_R = -\sqrt{C}$, corresponding to a somewhat stronger interaction. This line of fixed points for $T \le T_c$ is a novel but well-known feature of the Kosterlitz-Thouless model.

From this, one can obtain the approximate temperature dependence of K_R. For low temperatures, $x_R \approx x_0$, as the renormalization does not have too much of an effect, so that

$$K_R \approx K_0 \sim n_s^0 \sim (1 - T/T_{c0})$$

(provided the temperatures are not *too* low). Very close to T_c, since $C(T_c)=0$, $C(T) \approx b(1-T/T_c)$, and

$$\pi K_R = 2 + 2[b(1-T/T_c)]^{1/2} \ . \tag{31}$$

This square-root cusp is another well-known feature of the KT transition. These two dependences are shown in Fig. 6, together with several other "partially renormalized" plots of $K(l,T)$ to be discussed later. Since the experimental exponent $a(T)$ in Fig. 3 is to be associated with $1+\pi K_R$, the curves in these two fiqures should be compared and have several qualitative similarities, among these being the downward curvature below the extrapolation from the linear regime.

This downward curvature is a reflection of the increased value of the effective vortex dielectric constant ϵ as $T \to T_c$. A particularly useful parameter to describe this is

$$\epsilon_c \equiv \epsilon(r \to \infty, \ T \to T_c)$$
$$= \pi K_0(T_c)/2 = [1 + x_0(T_c)]^{-1}$$
$$= [1 - 2\pi y_0(T_c)]^{-1} \ , \tag{32}$$

which represents the fully renormalized value of ϵ just below the transition.

In contrast to some other aspects of the KT tran-

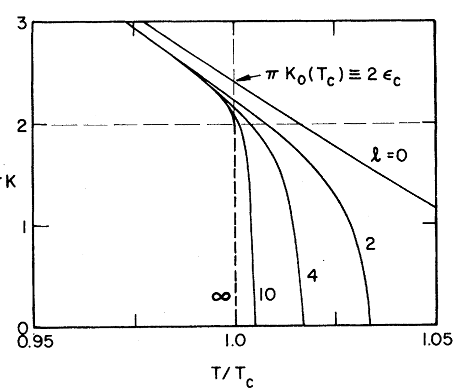

FIG. 6. Temperature dependence of the theoretical quantity $\pi K(l,T)$ $[=a(T)-1]$ for several fixed values of the scale l. The same parameters are used as for Fig. 5. The line $l=0$ corresponds to the bare, unrenormalized interaction πK_0; the curve for $l=\infty$ to the fully renormalized $\pi K_R(T)$.

sition, the value of $y_0(T_c)$ appears to be model dependent, and therefore nonuniversal. From Eq. (28) it can be expressed in terms of the quantities N_0 and the core energy E_c as

$$y_0(T_c)=N_0\exp[-E_c(T_c)/k_BT_c] . \quad (33)$$

We can express E_c in a simple model, as simply the superconducting condensation energy lost in a vortex core of radius ξ,

$$\begin{aligned} E_c(T_c) &= [\pi\xi^2(T_c)d][H_c^2(T_c)/8\pi] \\ &= \Phi_0^2 d/64\pi^2\lambda^2 \\ &= \pi n_s(T_c)\hbar^2/16m = k_BT_c/2 , \end{aligned} \quad (34)$$

where we have used the universal jump condition Eq. (11) and an identity of superconductivity theory.[21] The fact that E_c is of order kT_c should not by itself be surprising, since $2E_c$ is the minumum excitation energy of a vortex pair, an elementary excitation of the system, and these excitations are fairly common at T_c. Substituting this into Eq. (33), one obtains

$$\epsilon_c=(1-3.8N_0)^{-1}\approx 1+3.8N_0 , \quad (35)$$

where N_0 is an unknown quantity. For these approximations to work, one must have $N_0 \ll 1$. It should probably be calculable from first principles, but we are not aware of such a calculation. In that case, it makes sense for us to include the parameter ϵ_c in the analysis of the experimental data, and thus to infer a value for ϵ_c and hence for N_0.

We can do this by modifying the result of Beasley, Mooij, and Orlando to take account of renormalization. Using ϵ_c as a parameter, the universal jump condition Eq. (11) can be expressed as

$$\begin{aligned} 4k_BT_c &= q_R^2(T_c)=q_0^2(T_c)/\epsilon_c \\ &= \Phi_0^2 d/[8\pi^2\lambda^2(T_c)\epsilon_c] . \end{aligned} \quad (36)$$

If one then uses the BCS microscopic-theory value for the penetration depth $\lambda(T_c)$ as before, but retains ϵ_c, one finds that ϵ_c occurs only in the combination $R_N^{\square}\epsilon_c$, yielding

$$(T_c/T_{c0})/[\Delta(T_c)/\Delta(0)]\tanh[\Delta(T_c)/2k_BT] = 2.18R_c/R_N^{\square}\epsilon_c , \quad (37)$$

in general, and for T_c near T_{c0} the approximation form

$$T_c/T_{c0}\approx[1+0.17R_N^{\square}\epsilon_c/R_c]^{-1} . \quad (38)$$

Then we can go back to the results of Fig. 4. If we assume that ϵ_c is independent of $R_N^{\square}$, then using ϵ_c as a single fitting parameter for all the samples yields a best-fit value $\epsilon_c=1.2\pm0.1$, which corresponds to $N_0=0.05\pm0.03$.

This value of $\epsilon_c=1.2$ is quite comparable to those predicted and measured in other systems,[18,12] especially inasmuch as it is expected to be a nonuniversal feature of the transition. The value of $N_0=0.05$ seems rather small, but if vortices can be packed at the upper critical field H_{c2} to a density of $1/2\pi\xi^2$, then perhaps one might expect $N_0\approx 1/2\pi$. If our inferred value of N_0 is too small, it is also possible that our simple model of the vortex core may underestimate the core energy E_c. In any case, $y_0(T_c)=0.03 \ll 1$, as it must be for the validity of the lowest-order theory that we have been applying.

There is a possibility that the Hg-Xe films used in this study may be strong-coupling superconductors, inasmuch as pure Hg is well known to be one. In such a circumstance, Eq. (37), which is based on BCS theory, would no longer be valid. To a first approximation, it might be corrected by replacing the ratio $2\Delta(0)/k_BT_c=3.5$ used in its evaluation by the value appropriate to the actual material.[23] The effect of such an alteration would be to permit larger values of T_c for given values of T_{c0} and $R_N^{\square}$ than those predicted by BMO. This is opposite to the effect of renormalizing the interaction, and would require an even larger value of ϵ_c than that estimated above. For example, if we take the value $2\Delta(0)/k_BT_c=4.0$, appropriate to pure Hg, as an upper-limit strong-coupling correction for the Hg-Xe system, then one obtains $\epsilon_c=1.4$. There is currently insufficient evidence to resolve this issue, but in any case the renormalization effects would appear to be substantial.

V. THE VORTEX CORRELATION LENGTH AND THE RESISTIVE TRANSITION

One aspect of the vortex-unbinding transition that we have not been emphasizing is the shape of the resistive transition itself, in the regime for $T>T_c$ (Fig. 1). This is in part because our analysis suggests that the low-temperature *I-V* characteristics are more closely related to the essential quantities of the KT transition. The resistive transition is, however, expected to relate to one important parameter of the KT transition that we have not discussed until now, the vortex correlation length $\xi_+(T)$, which is defined for $T>T_c$ and diverges as $T\rightarrow T_c$. It should be clearly distinguished from the usual Ginzburg-Landau coherence length $\xi(T)$, which diverges not at T_c but at the mean-field temperature T_{c0}. Physically, ξ_+ represents the scale at which vortices begin to unbind. For smaller scales, vortices are still bound in pairs, even though the temperature is above the nominal vortex-unbinding temperature T_c. In part because the effective scale in the KT problem is logarithmic, the theoretical expression for the correlation length ξ_+ takes the unusual exponential form[4]

$$\xi_+(T)=C\xi(T)\exp\{[b(T_{c0}-T_c)/(T_c-T)]^{1/2}\}\ , \tag{39}$$

where the constants C and b are expected to be of order unity.

A further refinement has been suggested by Minnhagen[24] (and used by Abraham *et al.*[11]), whereby the expression $T_{c0}-T_c$ in the numerator of Eq. (39) should be replaced by $T_{c0}-T$, to take into account the fact that in this problem, the effective charge $q\sim n_s^{1/2}$ is a function of temperature. An alternative extended form of Eq. (39) has been proposed by Halperin and Nelson,[4] which interpolates between Eq. (39) close to T_c and the form appropriate to fluctuation resistance above T_{c0}. Since the physical justification for this appears to be somewhat uncertain, and in any case we will be examining the data only for $T<T_{c0}$, we will use only the former correction factor. Another generalized expression for $\xi_+(T)$ has been derived for the two-dimensional Coulomb gas,[25] but has not yet been adapted to the case of a 2D superconductor.

Since all vortices within the clusters of size ξ_+ will be paired, except for those few of a single sign in excess, the effective density of free, unpaired vortices should go as $N_f\sim 1/\xi_+^2$.[4] This will produce resistance in the low-current limit of the form

$$\begin{aligned} R&=R_N 2\pi\xi^2 N_F \\ &=AR_N\exp\{-2[b(T_{c0}-T)/(T_c-T)]^{1/2}\} \end{aligned} \tag{40}$$

where $A=0(1)$. In Fig. 1(b) we show a plot of $\log(R)$ vs $[(T_{c0}-T)/(T_c-T)]^{1/2}$ for the same points as in Fig. 1(a) from $T=3.3-3.5$ K. The parameters $T_{c0}=3.6$ K and $T_c=3.2$ K were determined for this sample from the low-temperature *I-V* characteristics, and are used for points on the right of Fig. 1(b). However, the points follow a straight line much more closely for $T_c=3.2$ K, as is shown on the left. This fit corresponds to $A=0.5$ and $b=16.6$ in Eq. (40). The value of b may seem high, but similar fits to other superconducting systems have also given values in this range or greater.

Finally, there are reasons to expect that the data obtained in the resistive transition, particulary in its low-temperature tail, may be somewhat less reliable than the data obtained from the *I-V* curves. One such reason is that the actual resistance levels, and hence the number of vortices being detected, is typically much greater in the low-temperature, high-current regime, than in the higher-temperature regime in the low-current limit. The former data is thus less likely to be perturbed by the presence of stray magnetic fields, edge effects, etc. Nevertheless, the fact that the resistive transition is roughly in agreement with that predicted by theory is encouraging.

VI. RENORMALIZATION AND THE *I-V* CHARACTERISTICS

One can proceed even further in this renormalization analysis, if one realizes that the experimentally measured quantities are in fact *not* fully renormalized. As discussed earlier, measurement of the nonlinear *I-V* characteristics at a current I corresponds to probing the vortex interaction at a distance $r_c\sim 1/I$, or $l_c=\ln(r_c/\xi)$ on the logarithmic scale. For the range of currents in our experiments, l_c will turn out to be in the range from 2 to 5, which is certainly not in the large-distance limit.

In particular, if we reconsider our earlier derivation of the *I-V* curves for $T<T_c$ [Eqs. (7)–(9)], the current-dependent resistance $R(I)$ depended exponentially on the pair in interaction energy $U(r_c)$:

$$R(I)\sim\exp[-U(r_c)/2k_BT]\ . \tag{41}$$

Then the derivative on a plot of $\log V$ vs $\log I$ can be expressed as

$$\begin{aligned} a(I,T)&\equiv\frac{d\ln V}{d\ln I}=1+\frac{1}{k_BT}\left.\frac{dU}{d\ln r}\right|_{r=r_c} \\ &=1+\pi K(r_c)\ . \end{aligned} \tag{42}$$

This derivative $a(I,T)$ should be independent of I only for $T<T_c$ in the limit of arbitrarily small current ($r_c\to\infty$). In fact, the lines on the log-log plot in Fig. 2 are not all perfectly straight, particularly for $T>T_c$, but even for $T\lesssim T_c$. It should be possible to analyze this curvature to provide evidence for the spatial renormalization of the vortex interaction.

To obtain a more detailed comparison between theory and experiment, a full solution of the set of Eqs. (2)–(7) is necessary. Fortunately an analytic solution is available to the approximation set of Eqs. (29),[16] valid sufficiently close to T_c. Since $x^2-4\pi^2y^2=C$, the first equation can be rewritten

$$dx/dl=2(x^2-C) \tag{43}$$

and is directly integrable. The solutions take three different forms for the three different regimes, and are listed here for reference:

(i) $T=T_c$: $C=0$, $x_0=-2\pi y_0$

$$x=-2/(l-2/x_0)\ ,$$
$$2\pi y=-x\ . \tag{44}$$

(ii) $T<T_c$: $C>0$, $|x_0|>2\pi y_0$

$$x=-A\coth(2Al+B)\ ,$$
$$2\pi y=A\,\mathrm{csch}(2Al+B)\ , \tag{45}$$
$$A=C^{1/2},\ \ B=\cosh^{-1}(x_0/2\pi y_0)\ .$$

(iii) $T>T_c$: $C<0$, $|x_0|<2\pi y_0$

$$x=D\tan(2Dl+E)\ ,$$
$$2\pi y=D\sec(2Dl+E)\ , \tag{46}$$
$$D=|C|^{1/2},\ \ E=\sin^{-1}(x_0/2\pi y_0)\ .$$

We have used this analytic solution to calculate the expected temperature dependence $\pi K(l,T)=a(l,T)-1$ for various fixed values of l. In carrying this out, we have used parameters $T_{c0}/T_c=1.1$, $\epsilon_0=1.2$, and a linear unrenormalized dependence $K_0(T)\sim n_s^0(T)\sim(1-T/T_{c0})$, all of which may be appropriate for the data shown in Fig. 2. As one can see in Fig. 6, the universal jump condition shows sharply only for infinite scale ($l=100$ is quite sufficient in this case). For finite l, the jump is smeared out over a range of temperatures, and the position of the transition, as determined by where $a(T)$ crosses the value 3 is increased slightly above the actual T_c. Therefore, the lack of a clear square-root cusp and a sharp jump at T_c in the experimental data should not be surprising.

The fits to determine $a(T)$ from Fig. 2 were taken typically over currents in the range from 10 to 300 μA. The resulting values of $a(T)$ go continuously through $a(T_c)=3$ (see Fig. 3). However, the resistive transition [Fig. 1(a)] was measured using currents of 1 μA, and the linearity of the I-V curves was checked, corresponding to $a(T)=1$ for $T>T_c$. Therefore, if we had chosen to determine the exponent $a(T)$ by fitting at the lowest current still yielding a voltage above the limits of resolution and background (as appears to have been done in Ref. 11), we would have seen a sharper apparent transition in Fig. 3. Nevertheless, the comparison between theory and experiment for finite I is in reasonable agreement with $l=\ln(I_0/I)\approx 2-5$.

One can obtain a more quantitative comparison by examining the I-V curves themselves. From Eq. (24), the density of thermally excited pairs is

$$n_p(r)=y^2(r)/r^4\sim\exp[-U(r)/k_BT]\ . \tag{47}$$

In terms of this, and using the kinetic-equation argument we made earlier, one has that the resistance goes as

$$R(I)\sim[n_p(r_c)]^{1/2}=y(r_c)/r_c^2\sim yI^2\ . \tag{48}$$

Therefore, we can make the association $y\sim V/I^3$. This suggests that the experimental data for $T\approx T_c$ should be replotted in the form $\ln(V/I^3)$ vs $\ln(I)$, and compared to the analytic solutions for $\ln(y)$ vs $-l=\ln(I/I_0)$. On a log-log plot, the constants of proportionality amount to horizontal and vertical shifts, and the slopes should be the same:

$$d\ln(V/I^3)/d\ln I=-d\ln y/dl=-2x\ , \tag{49}$$

so that this analysis involves both x and y, the variables of the renormalization equations. Furthermore, since $V\sim I^3$ near T_c, this treatment removes some of the many orders of magnitude of variation, and makes the curvature in the characteristics more obvious.[26] Some of the data for the sample shown earlier in Fig. 2 is plotted in this way in Fig. 7(a), and a set of theoretical curves, which have been approximately scaled to match the experimental curves, are shown in Fig. 7(b). In this scaling, we assumed that I scales with $I_0(T)\sim(1-T/T_{c0})^{3/2}$, and that V scales with $V_0(T)=AI_0(T)R_N$ such that

$$y(l)=[V/V_0(T)]/[I/I_0(T)]^3\ . \tag{50}$$

The fit was obtained by first setting the temperature dependence into the set of theoretical curves, then shifting the whole set of curves horizontally and vertically to get the best visual match of the entire set of curves. In this way, we obtained $I_0(T_c)=2.4$ mA, with the constant $A=0.25$. Given the roughness of the fitting procedure, these are very reasonable. With this fit, l ranges from 2 to 6, and $\ln y$ from -5 to $+1$.

As for the comparison between the experimental and theoretical curves, there is reasonable semiquantitative agreement in the curvature and the rough-temperature dependence, with T_c here corresponding to about 3.29 K, very close to the previous estimate of 3.3. There are substantial deviations between the two sets of curves, particularly away from the center of the graph, but many of these deviations can be understood. First, the analytic solutions to Eq. (29) are physically valid only for small x and y. In the upper left-hand corner, y is getting larger than unity; in the lower right-hand corner, x is getting large. For large currents, most of the experimental data curves up slightly. We believe this may be due to heating of the sample slightly above the nominal temperature. For currents not too much larger than these, thermal runaway occurs. Finally, in the lower left quadrant of the experimental data, problems of minimum resolution and background effects start to

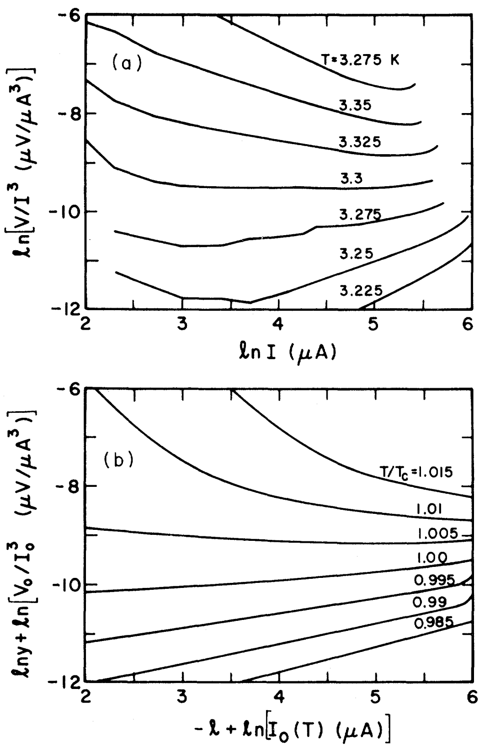

FIG. 7. (a) Plot of $\ln(V/I^3)$ vs $\ln(I)$ from the data of sample no. 4, for a range of temperatures very close to $T_c \approx 3.3$ K. The lines are drawn connecting the experimental points. (b) Set of theoretical curves systematically shifted to match the experimental curves of (a). $\ln(y)+\ln[V_0(T)/I_0^3(T)]$ is plotted against $-l+\ln[I_0(T)]$. The fit corresponds to $I_0(T_c)=2.4$ mA, $I_0(T) \sim (T-T_{c0})^{3/2}$, and $V_0(T)=AI_0(T)R_N$, where $A=0.25$.

become important.

An additional check on the consistency of this approach comes from considering the magnitude of $I_0(T)=2.4$ mA inferred from the fit. From the definition of I_0, if one uses the universal jump condition [Eq. (11)] to evaluate $n_s(T_c)$, one obtains

$$I_0(T_c)=4ek_BT_cw/\pi\hbar\xi(T_c)$$
$$=880 \text{ mA}/\xi(T_c) \text{ (Å)} \qquad (51)$$

(w is the film width $=1$ mm) which gives $\xi(T_c)=370$ Å for the inferred value of 2.4 mA for $I_0(T_c)$. We were unable to obtain a direct measurement of $\xi(T_c)$ [e.g., from a measurement of the perpendicular critical field $H_{c2}(T)$], but one can obtain an approximate value by using the theory of dirty superconductors. The estimated normal-state film resistivity is $\rho_N=2700$ $\mu\Omega$ cm. If we estimate $\rho_N l=1\times10^{-11}$ Ω cm^2, then $l=0.4$ Å. This is already clearly outside the regime on conventional transport, but we earlier used the same theory of dirty superconductors to determine the dependence of $R_N^{\square}$, with reasonable results.

The BCS coherence length is estimated to be $\xi_0=5000$ Å, taking $T_{c0}=3.6$ K and $v_F=1.5\times10^8$ cm/sec,[27] and assuming the weak coupling formula[21] (which may not be appropriate). From these values and the dirty-limit formula[21] we obtain the Ginzburg-Landau coherence length $\xi(T_c)=130$ Å. Given the approximations made, this may be satisfactory agreement with the value inferred above from Eq. (51). However, back in Eq. (6), in the derivation of the vortex excitation rate over the saddle point, the approximation was made that

$$\ln(r_c/\xi)-1 \approx \ln(r_c/\xi) \ .$$

Therefore, a more careful analysis would replace $\xi(T_c)$ in Eq. (51) by $e\xi(T_c)$ [$e=\exp(1)=2.7\cdots$], yielding a corrected estimate from Eq. (51) of $\xi(T_c)=140$ Å. It would be desirable to have a more direct check on $\xi(T_c)$, but all in all the agreement is highly encouraging.

This analysis, while not in itself proving that the experimental data follows the renormalized theory, certainly points the way toward a more definitive test. What is needed on the experimental side are a set of careful measurements of the I-V curves very close to T_c, over a wide range of currents, for a well-characterized and uniform sample. An independent measurement of the vortex core size $\xi(T_c)$ would also be useful. On the theoretical side, it would be desirable to extend the form of the renormalized interaction farther into the regime for $T>T_c$, for larger values of $y(l,T)$. Work in these areas is continuing.

VII. CONCLUSIONS

To summarize what has been shown in this paper, we have fabricated high-sheet-resistance homogeneous films of Hg-Xe alloys, and examined the nonlinear I-V characteristics near and below the superconducting critical temperature. The fact that the I-V curves are linear on a log-log plot is a rather direct indication of the logarithmic nature of the vortex-antivortex interaction. The slope $a(T)$ on this log-log plot is a measure of the prefactor of this logarithmic interaction, and we can unambiguously infer the BCS critical temperature by extrapolating $a_{\text{lin}}(T_{c0})=1$, and the temperature T_c at which the Kosterlitz-Thouless vortex-unbinding transition should occur by taking $a(T_c)=3$. The transition width $\tau_c=1-T_c/T_{c0}$ depends on the sheet resistance of the films $R_N^{\square}$ in approximate agreement with the theory of dirty superconductors, as derived by Beasley, Mooij, and Orlando. This provides additional confirmation that the low-temperature transport properties are dominated by logarithmically interacting vortex pairs. However, there is a systematic deviation from the simple unrenormalized interaction, characterized by an effective dielectric constant at the transition $\epsilon_c = n_s^0/n_s^R = 1.2 \pm 0.1$, in reasonable accord with expectations of theory, although this parameter is nonuniversal. The shape of the resistive transition for $T > T_c$ is also in reasonable agreement with predictions of the KT theory. Furthermore, the apparent lack of a sharp jump in $a(T)$ is consistent with its measurement at a finite current and hence a finite scale. Finally, detailed analysis of $V(I)$ near T_c provides reasonable agreement with theoretical predictions of the spatial renormalization of the vortex interaction and related quantities. Taken together, these appear to provide fairly strong evidence for some sort of vortex-unbinding transition at T_c, which may be of the KT universality class.

ACKNOWLEDGMENTS

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