Resistivity dependence of $1/f$ noise in metal films

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The $1/f$ noise of a number of different types of metal films has been studied at room temperature. We find that the noise of nominally identical samples can vary by as much as a factor of 10, which is well outside the estimated experimental uncertainties for the quantities thought to be relevant in determining the noise. This suggests that some other variables play an important role in the noise process. Despite these sample-to-sample variations, we find that the minimum level of $1/f$ noise for a given metal is a fairly well-defined quantity. Moreover, the minimum noise level exhibits a systematic resistivity dependence which has not been observed previously, and which cannot be accounted for by the semiempirical formula of Hooge. These results are compared with those of previous workers, and a modification of Hooge's formula is suggested.

I. INTRODUCTION

When a direct current is passed through a resistor, it is often found that, in addition to the Johnson noise, there is an "excess noise" which has a power spectral density proportional to $1/f^{\alpha}$, where f is the frequency and $\alpha \approx 1$. This is commonly known as "$1/f$ noise." There is much debate concerning the origin of this noise; indeed, in most cases of interest there is no theory which can account for the observed properties of the noise.[1-3] However, it is widely accepted that the $1/f$ noise of continuous metal films at room temperature is in order-of-magnitude agreement with the semiempirical formula of Hooge[4]:

$$S_v = \gamma V^2 / N_c f \ . \tag{1}$$

Here S_v is the power spectral density of the noise in excess of the Johnson noise, V is the average voltage across the resistor, N_c is the number of free carriers,[5] and γ is a dimensionless constant of order 2×10^{-3}. In addition, Hooge and co-workers have shown that (1) must be modified to account for the noise of thin bismuth films[6] and heavily doped semiconductors.[7] In these cases they find

$$S_v = \left[\frac{\mu}{\mu_{\rm ph}}\right]^2 \frac{\gamma V^2}{N_c f} \ , \tag{2}$$

where μ is the total mobility, and $\mu_{\rm ph}$ is the contribution to the mobility from electron-phonon scattering. Hence they conclude that electron-phonon scattering is intimately related to $1/f$ noise. Although neither (1) nor (2) is consistent with the observed temperature dependence of the $1/f$ noise in metal films,[8,9,2] nearly all theories of the noise which have been developed have Hooge's formula as an exact or approximate limiting form.[1,2] It is therefore important to determine the nature and extent of any limitations of (2) as a description of the noise at room temperature. We have performed an extensive study of the $1/f$ noise of several different types of metal films. We find that the magnitude of the noise exhibits a systematic dependence on the resistivity of the metal film, which cannot be accounted for by the Hooge formula.

II. EXPERIMENTAL DETAILS AND RESULTS

We have studied the noise of a number of pure metals (see Table I), and of gold-silver alloys (Au_xAg_{1-x}) of various concentrations. All films were evaporated at a pressure of $\approx 10^{-6}$ Torr, except platinum which was deposited by ion-beam sputtering. The Au_xAg_{1-x} were formed by coevaporating Au and Ag, and then annealing the films in a vacuum at 675 K. Both glass and sapphire substrates were used. For the noise measurements, the films were cut with a tungsten needle to form strips (50—2000)-μm long and (8—40)-μm wide. Typical film resistivities and thicknesses are given in Table I. The resistivity of the Au_xAg_{1-x} varied as expected with alloy concentration. We have also studied a number of Au and Ag films which were annealed similarly to the Au_xAg_{1-x}; these had resistivities 20—30% lower than films of the same thickness which were not annealed. All measurements were made at room temperature. The experimental setup used to measure the noise was the same as that described previously.[10] It was possible to perform the usual[10] measurements of the noise power without excess contact noise, and without heating the samples significantly.

TABLE I. Resistivities (ρ), thicknesses (t), and excess-noise frequency dependencies of the metals studied.

Metal	ρ ($\mu\Omega$ cm)	t (Å)	α[d]
Ag	2.5–7[a]	500–1100	1.19±0.07
Cu	2.5–7[a]	400–700	1.17±0.09
Au	2.5–10[a]	300–600	1.08±0.08
Au_xAg_{1-x}	3–17[b]	600–900	1.20±0.08
In	10–16[a]	700–1400	1.14±0.08
Sn	13–20[a]	600–1000	1.16±0.11
Pb	22–30[a]	800	1.15±0.10
Pt	40–60[c]	80–200	1.15±0.07

[a]Values given are measured resistivities of the samples. These resistivities are near the bulk values, allowing for boundary scattering.
[b]The resistance increased "parabolically" both as $x \to 0.5$, and as $1-x \to 0.5$.
[c]Resistivities of the platinum films were nearly 4 times the bulk value, due to the increased impurity scattering commonly found in sputtered films. Some boundary scattering was also evident in the thinner ($t < 150$ Å) films.
[d]Values for α shown are averages over all samples measured ± one standard deviation.

In all cases, the excess-noise power spectral density was proportional to $1/f^{\alpha}$ over the entire frequency range investigated (0.3 Hz $< f <$ 100 Hz). Values of α obtained for the various metals are listed in Table I, where we give the average value obtained for a large number of samples (typically 40) of each metal.[11] None of the materials, with the possible exception of Au, exhibited noise with α "exactly" equal to unity. Accurate values of α for In, Sn, Pb, Pt, or Au_xAg_{1-x} have not, to our knowledge, been reported previously in the literature. The values of α we find for Ag and Cu are in good agreement with those reported by Eberhard and Horn.[9] For Au at room temperature these authors found $\alpha \simeq 1.28 \pm 0.1$, which is somewhat higher than the average value that we find: $\alpha = 1.08 \pm 0.08$. Since the difference between these values is only slightly larger than the combined uncertainties, it may not be significant. However, of the 50 Au samples we studied, none exhibited $\alpha > 1.25$. We are therefore inclined to believe that the different values of α observed reflect a real difference between the samples studied, although we have no explanation for its cause.[12] We should also note that the only other reported value of α for Au of comparable accuracy appears to be that of Scofield *et al.*,[13] who find $1.0 < \alpha < 1.1$, in agreement with our result.

While the observed values of α did not vary greatly from metal to metal, the magnitude of the noise varied widely. In Fig. 1 we show $\gamma \equiv S_v N f / V^2$ as a function of the resistivity for the different metals studied. We have chosen to plot this quantity, since, according to (1), normalizing the noise power in this way should remove the effects of sample size N[14] and measuring voltage V, and thus allow a direct comparison of the results for different films.[15] Two features of Fig. 1 are noteworthy. First, there is a large sample-to-sample variation of γ, even for nominally identical samples made out of the same material. Similar variations have been observed by previous workers.[9,16] In our measurements, experimental uncertainties, in parameters such as N, can account for sample to sample differences of about ±40%, corresponding to a total variation of a factor of 2. The observed values of γ for a given material often vary by as much as a factor of 10. Hence,

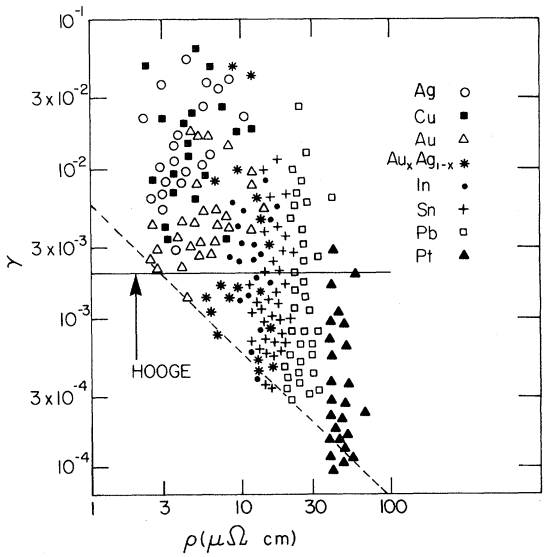

FIG. 1. $\gamma \equiv S_v N f / V^2$ at $f = 10$ Hz as a function of resistivity for several metals. Overlapping points have been omitted for clarity. Here N is the number of atoms in the sample. The solid line is the prediction of the Hooge model (1) with $\gamma = 2 \times 10^{-3}$. The dashed line represents (3).

these variations appear to be indicative of real differences between samples. Such differences are not consistent with (1) or (2), as these relations assume that $1/f$ noise is of purely "bulk" origin, i.e., that the values of N and V completely determine the magnitude of the noise. Our results seem to suggest that this is not entirely the case. One is therefore faced with at least two possibilities: (1) some other properties of the samples, besides N (and V), which are not under direct experimental control play an important role in the noise process, or (2) some property of the environment of the film, which also is not under experimental control, is important. Our recent results[10] for Sn appear to be an example of the latter case. For Sn we found that the magnitude of the noise depends upon the type of substrate used, and on the nature of the surface layer between the film and the substrate. We found that varying these parameters could change the noise level by about an order of magnitude, and this is the variation exhibited in Fig. 1. We note that for Sn samples on the same type of substrate, with the same surface preparation, the sample-to-sample variations were much smaller,[10] and could be accounted for by the uncertainties in N. The sample-to-sample variations found for In seem to have a similar origin: the noise of the In films depended upon the choice of substrate in the same manner as found for Sn.[17] Thus, in a sense, the sample-to-sample variations shown in Fig. 1 for Sn and In are "understood." However, for the other metals studied, the noise level did not appear to depend upon the choice of substrate or the surface preparation.

While the cause of the sample-to-sample variations shown in Fig. 1 is, except for Sn and In, not known, these results do suggest that much of the $1/f$ noise observed in metal films is not "intrinsic" to the metal. That is, the noise level is not determined solely by the properties of the bulk metal, independent of the sample environment, etc. Since we would expect that noise due to the sample environment, or other "extrinsic" causes, will only increase the sample noise level, the best experimental measure of the $1/f$ noise intrinsic to a metal should be the *minimum* noise observed for a given metal. Examination of Fig. 1 shows that this minimum noise level is in fact a fairly well-defined quantity. Moreover, the minimum noise level exhibits a systematic dependence on the sample resistivity. While Hooge's formula, as discussed above, cannot account for the sample-to-sample variations for a given metal, let us now consider if this formula can account for the variation of the minimum noise. For the pure metals (except Pt, which is considered below), (1) is applicable. The only sample-dependent quantity in (1) which can vary is N_c, the number of charge carriers. In Fig. 1 we have used N, the number of atoms in the sample, to normalize the noise power, since this quantity is determined readily. In attempting to convert this to N_c we are faced with a difficult problem—namely, how does one accurately determine the number of charge carriers in a metal? All of the metals studied have fairly complicated Fermi surfaces, and it is not at all clear how to estimate N_c in a realistic way. Perhaps the best one can do is retreat to a nearly-free-electron approach, and simply use the atomic valence to convert from N to N_c. This is the course we will now follow. Using standard valences,[18] one finds that Pb and Sn have 4 times more carriers per atom than does Au. Thus, according to Hooge's formula,[1] the noise level as plotted in Fig. 1 for Pb and Sn should be 4 times smaller than that of Au. From Fig. 1 we see that the minimum noise levels for Pb and Sn are about 7 and 6 times, respectively, less than that of Au. Given the uncertainties in the various parameters, especially in converting from N to N_c, this level of agreement is satisfactory. However, let us now consider the results for Pt and Au_xAg_{1-x}. We treat these separately from the other metals, since here most of the resistivity arises from elastic scattering, as opposed to phonon scattering, so (2) is now applicable. The resistivity of our Pt films was typically 40 $\mu\Omega$ cm, which is 4 times the bulk value for Pt. This implies that $\mu/\mu_{ph}\approx\frac{1}{4}$. Thus (2) predicts that the noise level for Pt should be 16 times smaller than that of, for example, Au. This is in reasonable agreement with the difference between the minimum noise levels of Pt and Au.[19] However, for the $Au_{0.5}Ag_{0.5}$, which had $\rho\approx 16$ $\mu\Omega$ cm, $\mu/\mu_{ph}\approx\frac{1}{8}$. According to (2), $Au_{0.5}Ag_{0.5}$ should exhibit 4 times less noise than the Pt. In fact, $Au_{0.5}Ag_{0.5}$ is 4 times noisier. There is thus a factor-of-16 discrepancy, which is probably too large to be accounted for by the various uncertainties.

We therefore conclude that, while Hooge's formula (1) is roughly consistent with the variation of the minimum noise levels observed for the pure metals in which phonon scattering dominates the resistivity, (2) does not account for the variation that we observe in Pt and Au_xAg_{1-x}. Rather, the results for *all* of these materials display a very simple dependence on the resisitivity ρ. The dashed line in Fig. 1 is proportional to $1/\rho$, and the minimum noise levels are seen to be quite consistent with this dependence. Incorporating this into (1), we find that the minimum noise level $S_{v,\min}$ is given by

$$S_{v,\min}=\frac{\rho_0}{\rho}\frac{V^2}{Nf^\alpha}\ , \tag{3}$$

where ρ_0 is a constant with a value of approximately

6×10^{-3} $\mu\Omega$ cm.[20] Based on our results, (3) gives an accurate estimate of the *minimum* levels of $1/f$ noise to be expected, and which have so far been observed, in metal films and alloys. However, it is not at all clear when or why one should expect to observe the minimum noise level experimentally. In preliminary studies of the high-resistivity alloys manganin and Au-Pd, for example, we have consistently observed noise levels which are 2 orders of magnitude higher than the values derived from (3). While this is not inconsistent with (3), it would be very interesting to understand why the noise levels are so high. In this regard we should note that for manganin Voss and Clarke[16] found S_vNf/V^2 to be less than 1×10^{-4}, which is in good agreement with (3).[21] The reason why we observe a different noise level for manganin than do Voss and Clarke is not known. This difference underscores our earlier assertion that at least some of the quantities which influence the noise are not yet under experimental control.

III. SUMMARY AND CONCLUSIONS

We have performed an extensive study of $1/f$ noise in a number of different metals and alloys. The magnitude of the noise in nominally identical samples has been found to vary by up to a factor of 10. For Sn and In this variation appears to be due to effects of the environment of the film. However, the source of the variation for the other metals studied is not known. Despite these sample-to-sample variations, the minimum noise level of a given material is a fairly well-defined quantity. The minimum noise level was found to vary as $1/\rho$, where ρ is the resistivity of the sample. Such a variation does not appear to be consistent with Hooge's relation, and certainly warrants further study.

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[1]F. N. Hooge, T. G. M. Kleinpenning, and L. K. J. Vandamme, Rep. Prog. Phys. 44, 479 (1981), and references therein.

[2]P. Dutta and P. M. Horn, Rev. Mod. Phys. 53, 497 (1981).

[3]M. B. Weissman, in *Proceedings of the Sixth International Conference on Noise in Physical Systems, Gaithersburg, 1981,* Natl. Bur. Stand. (U.S.) Special Publ. No. 614, edited by P. H. E. Meijer, R. D. Mountain, and R. J. Soulen, Jr. (GPO, Washington, D.C., 1981), p. 133.

[4]F. N. Hooge, Physica (Utrecht) 60, 130 (1972). Note that it is generally found that the frequency dependence of the noise is not exactly $1/f$, but rather is of the form $1/f^{\alpha}$, $0.9\leq\alpha\leq1.4$.

[5]A number of workers believe that N_c in (1) should be replaced by N, the number of atoms in the sample. We shall discuss this further below.

[6]F. N. Hooge, J. Kedzia, and L. K. J. Vandamme, J. Appl. Phys. 50, 8087 (1979).

[7]F. N. Hooge and L. K. J. Vandamme, Phys. Lett. 66A, 315 (1978).

[8]J. W. Eberhard and P. M. Horn, Phys. Rev. Lett. 39, 643 (1977).

[9]J. W. Eberhard and P. M. Horn, Phys. Rev. B 18, 6681 (1978).

[10]D. M. Fleetwood and N. Giordano, Phys. Rev. B 25, 1427 (1982).

[11]To within experimental error, the values found for α did not depend upon the choice of substrate, or the surface preparation of the substrate prior to the deposition of the film (see Ref. 10).

[12]One difference between the Au films studied by Eberhard and Horn (Ref. 9) and the samples we have studied is that most of our samples were not annealed prior to performing the noise measurements. However, as noted above, we did study a number of annealed films of both Au and Ag. Within experimental error the values of α and the overall noise levels were the same for both the annealed and the unannealed samples.

[13]J. H. Scofield, D. H. Darling, and W. W. Webb, Phys. Rev. B 24, 7450 (1981).

[14]Note that here we have normalized S_v by the number of atoms in the sample N, not the number of charge carriers, as is discussed later.

[15]We have checked the dependence of S_v on V in a number of samples and find $S_v\sim V^{2.0\pm0.1}$ in good agreement with (1). As for the dependence on N, we have investigated Pt samples with N ranging from 10^7 to 10^{14} atoms [D. M. Fleetwood, J. T. Masden, and N. Giordano (unpublished)], and we find that S_v does indeed vary as $1/N$. Thus, this normalization procedure should remove the dependence on V and N.

[16]R. F. Voss and J. Clarke, Phys. Rev. B 13, 556 (1976).

[17]D. M. Fleetwood and N. Giordano (unpublished).

[18]See, for example, N. W. Ashcroft and N. D. Mermin,

Solid State Physics (Holt, Rinehart and Winston, New York, 1976), p. 5.

[19]We should remark that it is not clear what the appropriate number of carriers per atom should be for Pt even in a nearly-free-electron approximation. Here, we assume a valence of 1 ($N_c = N$).

[20]Note that if the units of ρ_0 are $\mu\Omega$ cm, then (3) is not dimensionally correct when $\alpha \neq 1$. The dependence of $S_{v,\min}$ on (V^2/ρ) suggests that for metal films $S_{v,\min} \sim (I^2R)\, R_0$ instead of $S_{v,\min} \sim V^2 = I^2R^2$.

[21]The bulk resistivity for manganin is ~ 44 $\mu\Omega$ cm, so from (3) we expect that the minimum noise of the manganin should be comparable to that of the Pt we report.