PHYSICAL REVIEW B VOLUME 27, NUMBER 10 15 MAY 1983

Quantized magnetoresistance in two-dimensional electron systems

F. F. Fang
I.B.M. Thomas J. Watson Research Center, Yorktown Heights, New York 10598

P. J. Stiles
Brown University, Providence, Rhode Island 02912
(Received 12 January 1983)

In a quantizing magnetic field, the resistance value of a two-dimensional electron gas between any well-resolved magnetic levels in any open geometry is given by multiple fractions of h/e^2. Measurements in various geometrical configurations of Si(100) (metal-oxide-semiconductor) transistor yield a well-defined and accurate plateau of these values.

Since the first report of the quantized Hall resistance in a Si MOS (metal-oxide-semiconductor) system by v. Klitzing *et al.*,[1] there has been considerable interest in understanding the physical basis on which this measurement yields the Hall resistance ρ_{xy} with remarkable accuracy.[2–4] For example, Laughlin[2] explained this phenomenon as a consequence of gauge invariance and the existence of a mobility gap. Halperin[3] extended Laughlin's analysis to investigate the existence of the extended states in a weakly disordered two-dimensional (2D) system between magnetic states and discussed the role of the edge states in the quantized Hall conductance. All of the explanations involve a coherent Hall current in a nearly localized regime.

In this paper, we report a much simpler measurement which gives exactly the same value as the quantized Hall resistance of v. Klitzing *et al.* and Tsui *et al.*[5] We measured two-terminal resistance of a pair of contacts in any configuration of an MOS inversion layer. For instance, in a conventional Hall bar, resistance between any pair of contacts was measured while the gate is biased against any of the inversion layer contacts. Figure 1 shows two terminal resistances between several pairs of these contacts at $H = 15$ T and 1.5 K as a function of gate voltage. The sample is a Si(100) MOS field-effect Hall bar structure. The Si substrate is 100-Ω cm p type with gate oxide thickness of about 1 μm. The peak mobility is over 10^4 cm^2/V s. The contact configurations are shown in the inset of the figure. Prominent resistance steps are shown with values

$$R = \frac{h}{ie^2} \quad , \tag{1}$$

where i is the number of filled magnetic states. The values are reproducible and accurate for any pair of terminals, in a limited step range to the precision of our measurements, a few parts per 10^5. Between steps, resistance values vary among different pair of contacts. Presumably it is due to the difference in various current paths when the system is not completely quantized. This result owes its origin to the quantized Hall effect and may shed some light toward the understanding of the phenomenon.

Consider the Hall bar geometry shown in the inset of Fig. 1. If one applies a constant voltage between the current probes (source and drain) at the quantized Hall resistance step the entire voltage is measured across the Hall probes 1,5 or 2,4. Figure 2 shows such a measurement together with the Hall resistance ρ_{xy} at 15 T as a function of gate voltage. As reported by Englert and v. Klitzing[6] and by Tsui and Gossard[7] there is no potential difference between resistivity probes 1 and 2 or 4 and 5. At the quantized resistance steps, the externally applied voltage will totally appear between any two probes on opposite sides of the sample. As all the current passes between the latter two probes, it follows that the ter-

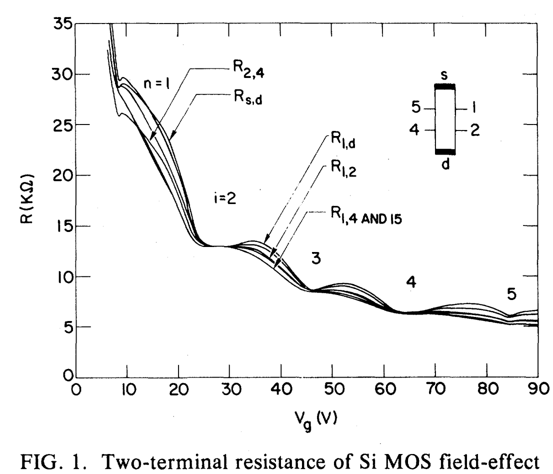

FIG. 1. Two-terminal resistance of Si MOS field-effect Hall bar at $H = 15$ T. The terminal pairs are marked on each curve. The designations of the terminals are shown in the inset for the sample sketch. The resistance steps for all samples are shown to have values h/ie^2 where the integer is noted at the steps.

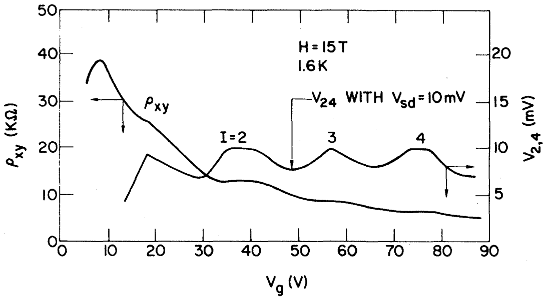

FIG. 2. Hall voltage as a function of gate voltage at $H = 15$ T for supply voltage $V_{ab} = 10$ mV. Also shown is the Hall resistance ρ_{xy} with the steps at the corresponding integer as marked.

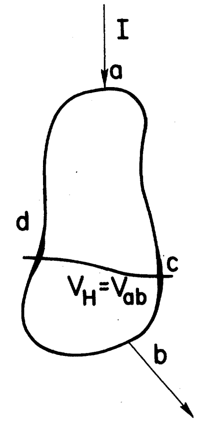

FIG. 3. Two-dimensional system of arbitrary shape and arbitrary boundary contacts. The current I is applied between a and b. Hall voltage V_H equal to the applied voltage V_{ab} at the quantum steps is developed between c and d.

minals which supply the external driving current and voltage are precisely defined by the quantized Hall resistance ρ_{xy}.

In a more general sense, consider an arbitrary shape of 2D systems shown in Fig. 3. The current I is supplied through a pair of contacts a and b. At the quantized Hall steps the Hall voltage $V_H = I\rho_{xy}$ must be developed along any arbitrary line between the current contacts, i.e.,

$$\int_c^d E\, dl = V_H = V_{ab} \quad .$$

Since V_H at ith step is also given by

$$V_H = \frac{H}{iN_L e} \int_c^d j_0\, dl = \frac{hc}{ie^2} I \quad , \tag{2}$$

where $N_L = eH/hc$ is the degeneracy of each magnetic level and j_0 is the current density in the absence of magnetic field, we conclude that the two-terminal resistance

$$R_{ab} = \frac{V_{ab}}{I} = \frac{V_H}{I} = \frac{hc}{ie^2} \quad . \tag{3}$$

The observed results of the quantized two-terminal resistance of 2D electrons suggest a state of the system at which the resistance between any pair of terminals on the periphery is quantized and given by h/ie^2 between i^{th} magnetic levels as long as the structure does not short out the Hall voltage (as is the case for the Corbino disk geometry). The equivalence of this to the Hall resistance not only makes measurements in this effect simple but shows a special property of the state between magnetic levels. The system at this state is completely quantized and has zero-dimensional freedom. A well-defined and discrete current-voltage relationship between any two points at the 2D boundary exists. The universality of this property may well offer a clue to the origin of the quantized Hall effect.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the technical assistance of M. Thomas with thanks in this experiment. The work of P. J. S. is supported in part by a grant from the National Science Foundation.

[1]K.v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. 45, 494 (1980).
[2]R. B. Laughlin, Phys. Rev. B 23, 5632 (1981).
[3]B. I. Halperin, Phys. Rev. B 25, 2185 (1982).
[4]Y. Imry, J. Phys. C 15, L221 (1982).
[5]D. C. Tsui, H. L. Störmer, and A. C. Gossard, Phys. Rev. B 25, 1405 (1982).
[6]T. H. Englert and K.v. Klitzing, Surf. Sci. 73, 70 (1978).
[7]D.C. Tsui and A. C. Gossard, Appl. Phys. Lett. 38, 550 (1981).