Larmor precession and the traversal time for tunneling

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Baz' and Rybachenko have proposed the use of the Larmor precession as a clock to measure the time it takes a particle to traverse a barrier. An applied magnetic field is confined to the barrier. The spin of the incident particles is polarized perpendicular to this field. The extent of the Larmor precession occurring during transmission is used as a measurement of the time spent traversing the barrier. However, the particles tunneling through an opaque barrier also acquire a spin component parallel to the field since particles with spin parallel to the field have a higher transmission probability than particles with spin antiparallel to the field. Similar effects are actually used to polarize electrons and neutrons. An interpretation of this experiment compares the results with an approach which determines the traversal time by studying transmission of particles through a time-modulated barrier. This leads to three characteristic times describing the interaction of particles with a barrier. A dwell time measures the average time interval during which a particle interacts with the barrier whether it is reflected or transmitted at the end of its stay, a traversal time measures the time interval during which a particle interacts with the barrier if it is finally transmitted, and a reflection time measures the interaction time of a reflected particle.

I. INTRODUCTION

In 1966, Baz'[1] proposed the use of the Larmor precession as a clock to measure the duration of quantum-mechanical collision events. Rybachenko[2] applied this method to the simpler case of particles in one dimension scattered at a barrier. It is this simpler case which we treat in this paper. We consider particles with mass m and kinetic energy $E=\hbar^2k^2/2m$ moving along the y axis and interacting with a rectangular barrier of height V_0 and width d, centered at $y=0$. The particles carry spin $s=\frac{1}{2}$ and the incident particles are polarized in the x direction [Fig. 1(a)]. A small magnetic field $\vec{B}_0$, pointing in the z direction, is confined to the barrier. As particles enter the barrier they start a Larmor precession with frequency $\omega_L=g\mu B_0/\hbar$. Here, g is the gyromagnetic ratio and μ the absolute value of the magnetic moment. When the particles leave the barrier the precession stops. The polarization of the transmitted (and reflected) particles is compared with the polarization of the incident particles. References 1–3 consider only the component of the polarization perpendicular to the field. The angle between the initial and final polarization perpendicular to the field is assumed to be given by the Larmor frequency multiplied with the time a particle spends in the barrier. For energies $E<V_0$ and an opaque barrier, $k_0d\gg 1$, where $k_0=(2mV_0)^{1/2}/\hbar$, the transmission probability, in the absence of a field, is given by

$$T=[16k^2\kappa^2/(k^2+\kappa^2)^2]e^{-2\kappa d}\ , \tag{1.1}$$

where $\kappa=(k_0^2-k^2)^{1/2}$. For this case, Rybachenko[2] finds for the spin of the transmitted particles to lowest order in the field B_0,

$$\langle S_x\rangle \cong \hbar/2\ , \tag{1.2}$$

$$\langle S_y\rangle \cong -(\hbar/2)\omega_L\tau_y\ , \tag{1.3}$$

where we have introduced the time

$$\tau_y=\hbar k/V_0\kappa\ . \tag{1.4}$$

The precession angle in the x-y plane is $\omega_L\tau_y$ and Rybachenko concludes that τ_y is the time a particle takes to traverse the barrier. Since Eqs. (1.2) and (1.3) also hold for the reflected particles Rybachenko concludes that the time a reflected particle spends in the barrier is equal to the time a transmitted particle spends in the barrier.

A particle tunneling through a barrier in a magnetic field does not actually perform a Larmor precession. The main effect of the magnetic field is to align the spin with the field. Thus incident particles polarized in the x direction will acquire a polarization component parallel to the field while tunneling through the barrier [Fig. 1(b)]. Transmission and reflection of neutrons on saturated ferromagnetic

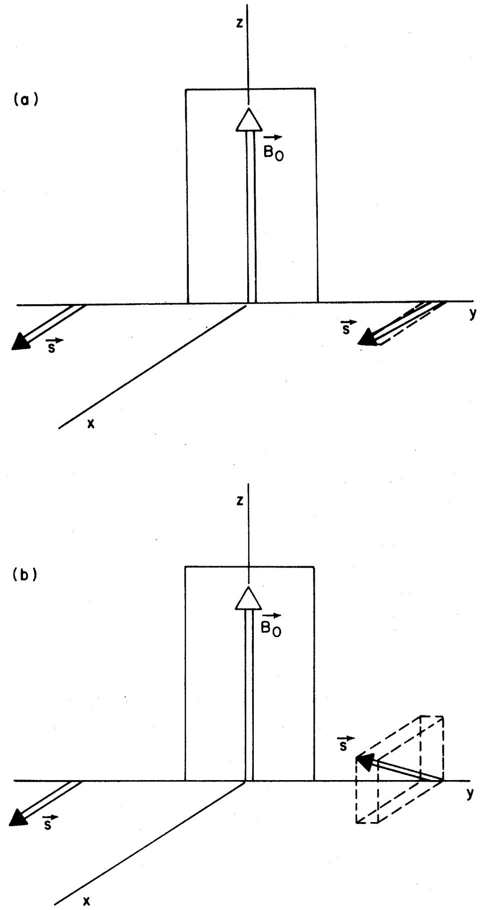

FIG. 1. (a) The quantum clock of Baz' and Rybachenko: A particle entering the barrier starts a Larmor precession with frequency ω_L in a magnetic field $\vec{B}_0$ confined to the barrier. The precession angle θ determines the time θ/ω_L a particle spends in the barrier. (b) Figure 1(a) is not correct: The spin of a particle tunneling through a barrier in a field is turned into the direction of the field.

samples is thus used to polarize neutrons.[4–6] In this case, the potential V_0 describes the interaction of the neutrons with the nuclei of the sample whereas the B_0 field arises both from the magnetization of the substrate and the externally applied field. Field emission from metals coated with a thin film of a ferromagnetic semiconductor is used to obtain highly polarized electrons.[7–9] The ferromagnetic exchange splitting of the conduction band of the semiconductor yields barriers of different height for electrons with spin parallel or antiparallel to the magnetization tunneling from the metal into the semiconductor. Therefore, it is surprising that Refs. 1–3 do not consider the appearance of a z component in the polarization of the scattered waves. In contrast to these practical applications, which require a strong magnetic field to be effective, Refs. 1–3 treat the case of a small magnetic field. Only in the limit of a small field can we expect the time determined by this method to be independent of the field. However, as we will now show, a z component in the polarization results even to first order in the field.

That particles polarized in the x direction acquire a z component when tunneling through the barrier can be understood in the following way. A beam of particles polarized in the x direction can be represented as a mixture of particles which have a z component $\hbar/2$ with probability $\frac{1}{2}$ and a z component $-\hbar/2$ with probability $\frac{1}{2}$. Outside the barrier the particles have kinetic energy E independent of the spin. But in the barrier the kinetic energy differs by the Zeeman contribution $\pm\hbar\omega_L/2$, giving rise to a different exponential decay for the wave functions within the barrier,

$$\kappa_\pm=(k_0^2-k^2\mp m\omega_L/\hbar)^{1/2}\ , \tag{1.5}$$

where the sign indicates whether the z component of the spin is parallel or antiparallel to the field. We will primarily consider the case of a small field, so that

$$\kappa_\pm\cong\kappa\mp m\omega_L/2\hbar\kappa\ . \tag{1.6}$$

Since $\kappa_+<\kappa_-$, particles with spin $\hbar/2$ will penetrate the barrier more easily than particles with spin $-\hbar/2$. The transmission probability for particles with spin $\pm\hbar/2$ in a field is found by replacing κ in Eq. (1.1) by $\kappa_\pm$ given by Eq. (1.6). Neglecting corrections in the preexponential factors, this yields a transmission probability $T_\pm=Te^{\pm\omega_L\tau_z}$, where

$$\tau_z=md/\hbar\kappa\ , \tag{1.7}$$

is the time a particle with real velocity $v=\hbar\kappa/m$ would take to traverse the barrier. The z component of the spin of the transmitted particles is determined by the imbalance of the flux of the transmitted particles with spin components $\hbar/2$ and $-\hbar/2$, respectively, divided by the total flux,

$$\langle S_z\rangle=\frac{\hbar}{2}\frac{T_+-T_-}{T_++T_-}=\frac{\hbar}{2}\tanh\omega_L\tau_z\cong\frac{\hbar}{2}\omega_L\tau_z\ . \tag{1.8}$$

To obtain the last expression in Eq. (1.8), we have assumed a small field so that $\omega_L\tau_z\ll 1$. For energies $E<V_0$, the y component Eq. (1.3) is much smaller than the z component Eq. (1.8). We have $\tau_y\ll\tau_z$ and, therefore, Eq. (1.8) describes the major effect of the magnetic field on the spin of a

transmitted particle.

The paper is organized in the following way. In Sec. II, we calculate the polarization of the transmitted particles and the reflected particles for all values of the kinetic energy E of the incident particles. This is a textbook calculation.[10] We emphasize the limit of a small magnetic field, as Baz' and others do.[1–3] In Sec. III, we reinterpret the Baz' clock through comparison with other methods. We find that the interaction of the particles with the barrier is determined by three characteristic times; a dwell time, a traversal time, and a reflection time. In Appendix A, we show that the magnetic field need not necessarily be confined to the barrier as we will assume for simplicity throughout the paper. In Appendix B, we consider particles with a spin $s>\frac{1}{2}$ tunneling through a barrier in a magnetic field.

II. TUNNELING THROUGH A BARRIER IN A MAGNETIC FIELD

We have to solve the scattering problem for the Hamiltonian

$$H=\begin{cases}(p^2/2m+V_0)1-(\hbar\omega_L/2)\sigma_z\,, & |y|\le d/2\,,\\ (p^2/2m)1, & |y|\ge d/2\,,\end{cases} \tag{2.1}$$

where 1 is the unit 2×2 matrix and $\sigma_z,\sigma_y,\sigma_x$ are the Pauli spin matrices. H acts on spinors

$$\psi=\begin{bmatrix}\psi_+(y)\\ \psi_-(y)\end{bmatrix}. \tag{2.2}$$

Here $|\psi_\pm(y)|^2dy$ is the probability of finding a particle with spin $\pm\hbar/2$ in the interval $y,y+dy$. The incident beam is polarized in the x direction,

$$\psi=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}e^{iky}\,. \tag{2.3}$$

Since H is diagonal in this spinor basis, we solve the scattering problem for particles with $\hbar/2$ and $-\hbar/2$ separately. For each spinor component, we assume $e^{iky}+A_\pm e^{-iky}$ to the left of the barrier, $y\le -d/2$, and transmitted waves $D_\pm e^{iky}$ to the right of the barrier, $y\ge d/2$. These waves are matched to $B_\pm e^{\kappa_\pm y}+C_\pm e^{-\kappa_\pm y}$ in the barrier. Here, $\kappa_\pm$ is given by Eq. (1.5). The effect of the magnetic field B_0 is to change the height V_0 of the barrier to $\tilde{V}_0=V_0\pm\hbar\omega_L/2$. Thus, it is sufficient to solve the scattering problem for the barrier in the absence of the magnetic field. The coefficients $A_\pm$, $B_\pm$, $C_\pm$, and $D_\pm$ are then found by replacing κ in the field-free problem by $\kappa_\pm$. For $\omega_L=0$ ($B_0=0$), the coefficient D multiplying e^{iky} of the transmitted wave is given by

$$D=T^{1/2}e^{i\Delta\phi}e^{-ikd}\,, \tag{2.4}$$

where

$$T=\{1+[(k^2+\kappa^2)^2/4k^2\kappa^2]\sinh^2(\kappa d)\}^{-1}\,, \tag{2.5}$$

is the transmission probability and the phase increase across the barrier, $\Delta\phi$, is determined by

$$\tan(\Delta\phi)=\frac{k^2-\kappa^2}{2\kappa k}\tanh(\kappa d)\,. \tag{2.6}$$

The coefficient multiplying the reflected wave is

$$A=R^{1/2}e^{-i\pi/2}e^{i\Delta\phi}e^{-ikd}\,, \tag{2.7}$$

where $R=1-T$ is the reflection probability. The coefficients B and C, which determine the wave in the barrier, are related to D by

$$B=\frac{\kappa+ik}{2\kappa}e^{ikd/2}e^{-\kappa d/2}D\,, \tag{2.8}$$

$$C=\frac{\kappa-ik}{2\kappa}e^{ikd/2}e^{\kappa d/2}D\,. \tag{2.9}$$

This completes the solution of the scattering problem. We can now discuss the results in detail.

A. The strong-field limit

The orientation of the spin of the transmitted particles is determined by the spinor

$$\psi=(|D_+|^2+|D_-|^2)^{-1/2}\begin{bmatrix}D_+\\ D_-\end{bmatrix}, \tag{2.10}$$

where D_+ and D_- are found by replacing κ in Eqs. (2.5)–(2.7) by κ_+ or κ_-, respectively. We find for the expectation values

$$\langle S_z\rangle=\frac{\hbar}{2}\langle\psi|\sigma_z|\psi\rangle=\frac{\hbar}{2}\frac{|D_+|^2-|D_-|^2}{|D_+|^2+|D_-|^2}\,, \tag{2.11a}$$

$$\langle S_y\rangle=\frac{\hbar}{2}\langle\psi|\sigma_y|\psi\rangle=\frac{\hbar}{2}i\frac{D_+D_-^*-D_+^*D_-}{|D_+|^2+|D_-|^2}\,, \tag{2.11b}$$

$$\langle S_x\rangle=\frac{\hbar}{2}\langle\psi|\sigma_x|\psi\rangle=\frac{\hbar}{2}\frac{D_+D_-^*+D_+^*D_-}{|D_+|^2+|D_-|^2}\,. \tag{2.11c}$$

For $D_\pm$ we now invoke Eq. (2.4) with κ replaced by $\kappa_\pm$. This yields

$$\langle S_z\rangle=\frac{\hbar}{2}\frac{T_+-T_-}{T_++T_-}\,, \tag{2.12a}$$

$$\langle S_y \rangle = -\hbar \sin(\Delta\phi_+ - \Delta\phi_-) \frac{(T_+ T_-)^{1/2}}{T_+ + T_-} , \tag{2.12b}$$

$$\langle S_x \rangle = \hbar \cos(\Delta\phi_+ - \Delta\phi_-) \frac{(T_+ T_-)^{1/2}}{T_+ + T_-} . \tag{2.12c}$$

Equations (2.12) are correct for arbitrary magnetic field. $T_\pm$ depends exponentially on the magnetic field, $T_+ \sim e^{-2\kappa_+ d}$ and $T_- \sim e^{-2\kappa_- d}$. Since $\kappa_- > \kappa_+$, we conclude that for a nonvanishing field and a sufficiently opaque barrier $T_+ \gg T_-$. Therefore, $\langle S_z \rangle \cong \hbar/2$ and $\langle S_y \rangle = \langle S_x \rangle \cong 0$. The transmitted beam is completely polarized in the z direction. T_- remains small as long as $E < V_0 + \hbar\omega_L/2$ and thus the transmitted wave is polarized in the z direction up to this energy.

The orientation of the spin of the reflected wave is determined by

$$\psi_R = (|A_+|^2 + |A_-|^2)^{-1/2} \begin{bmatrix} A_+ \\ A_- \end{bmatrix} . \tag{2.13}$$

We use an index R to distinguish the properties of the reflected wave from the transmitted wave. Proceeding as above, we find

$$\langle S_z \rangle_R = \frac{\hbar}{2} \frac{R_+ - R_-}{R_+ + R_-} , \tag{2.14a}$$

$$\langle S_y \rangle_R = -\hbar \sin(\Delta\phi_+ - \Delta\phi_-) \frac{(R_+ R_-)^{1/2}}{R_+ + R_-} , \tag{2.14b}$$

$$\langle S_x \rangle_R = \hbar \cos(\Delta\phi_+ - \Delta\phi_-) \frac{(R_+ R_-)^{1/2}}{R_+ + R_-} . \tag{2.14c}$$

We can express these results in terms of the spin of the transmitted wave in the following way. For the z component, we find

$$\langle S_z \rangle_R = -\langle S_z \rangle \frac{T_+ + T_-}{R_+ + R_-} , \tag{2.15a}$$

which expresses the conservation of angular momentum. The incident wave has $\langle S_z \rangle = 0$ and Eq. (2.15a) expresses the fact that the magnetic moment carried by the reflected wave is opposite to the moment carried by the transmitted wave. For the other two components we find

$$\langle S_y \rangle_R = \langle S_y \rangle \left[\frac{R_+ R_-}{T_+ T_-} \right]^{1/2} \frac{T_+ + T_-}{R_+ + R_-} , \tag{2.15b}$$

$$\langle S_x \rangle_R = \langle S_x \rangle \left[\frac{R_+ R_-}{T_+ T_-} \right]^{1/2} \frac{T_+ + T_-}{R_+ + R_-} . \tag{2.15c}$$

B. Infinitesimal field

We will now study the polarization of the transmitted wave and the reflected wave in the limit of an infinitesimal field. Consider $\langle S_z \rangle$ as given in Eq. (2.12a). Invoking Eq. (1.6) we find for the imbalance of the transmission coefficients

$$T_+ - T_- = T(\kappa_+) - T(\kappa_-) = -(m\omega_L/\hbar\kappa)\partial T/\partial\kappa , \tag{2.16}$$

where T is given by Eq. (2.5). In Eq. (2.16), the Larmor frequency ω_L is multiplied by a time $(m/\hbar\kappa)\partial T/\partial\kappa$. This suggests the definition of characteristic times τ_z, τ_y, and τ_x such that

$$\langle S_z \rangle = (\hbar/2)\omega_L \tau_z , \tag{2.17a}$$

$$\langle S_y \rangle = -(\hbar/2)\omega_L \tau_y , \tag{2.17b}$$

$$\langle S_x \rangle = (\hbar/2)(1 - \omega_L^2 \tau_x^2/2) . \tag{2.17c}$$

To find $\langle S_z \rangle$ we have, according to Eq. (2.12a), to divide the result of Eq. (2.16) by $T_+ + T_- \cong 2T$. This yields

$$\tau_z = -(m/\hbar\kappa)\partial \ln T^{1/2}/\partial\kappa , \tag{2.18a}$$

and similar calculations lead to

$$\tau_y = -(m/\hbar\kappa)\partial\Delta\phi/\partial\kappa , \tag{2.18b}$$

and

$$\tau_x = (m/\hbar\kappa)[(\partial\Delta\phi/\partial\kappa)^2 + (\partial \ln T^{1/2}/\partial\kappa)^2]^{1/2} = (m/\hbar\kappa)\,|D^{-1}\partial D/\partial\kappa| , \tag{2.18c}$$

where D and $\Delta\phi$ are given by Eqs. (2.4) and (2.6). To obtain Eq. (2.18c) we must expand the terms in Eq. (2.12c) to second order in B_0 (to order ω_L^2). Since

$$\langle S_x \rangle^2 + \langle S_y \rangle^2 + \langle S_z \rangle^2 = \hbar^2/4 ,$$

we must have

$$\tau_x = (\tau_y^2 + \tau_z^2)^{1/2} . \tag{2.19}$$

In Eqs. (2.18) we have expressed the charateristic times as derivatives of T and $\Delta\phi$ with respect to κ. Evaluating these derivatives from Eqs. (2.5) and (2.6) we find for $E < V_0$,

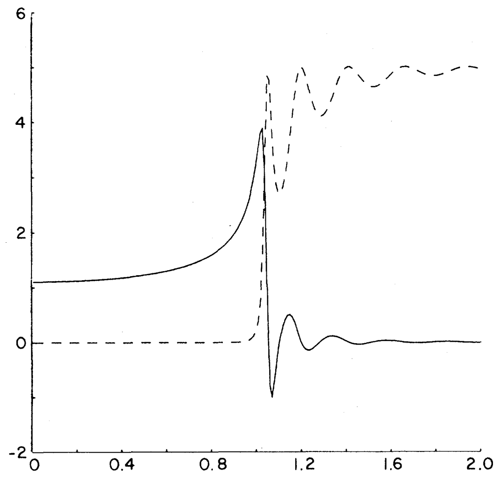

FIG. 2. τ_z as a function of $k=(2mE/\hbar)^{1/2}$ for a barrier of strength $k_0d=3\pi$, $k_0=(2mV_0)^{1/2}/\hbar$, $\tau_0=md/\hbar k_0$. τ_z determines the expectation value of the z component of the transmitted particles $\langle S_z\rangle=(\hbar/2)\omega_L\tau_z$. The broken line is the transmission probability for this barrier (magnified five times).

$$\tau_z=\frac{mk_0^2}{\hbar\kappa^2}\frac{(\kappa^2-k^2)\sinh^2(\kappa d)+(\kappa d k_0^2/2)\sinh(2\kappa d)}{4k^2\kappa^2+k_0^4\sinh^2(\kappa d)}, \tag{2.20a}$$

and

$$\tau_y=\frac{mk}{\hbar\kappa}\frac{2\kappa d(\kappa^2-k^2)+k_0^2\sinh(2\kappa d)}{4k^2\kappa^2+k_0^4\sinh^2(\kappa d)}. \tag{2.20b}$$

For $E>V_0$, we have to replace κ by iK, where $K=[2m(E-V_0)]^{1/2}/\hbar$. τ_z is shown in Fig. 2 as a function of k in units of $k_0=(2mV_0)^{1/2}/\hbar$ for a barrier with strength $k_0d=3\pi$. A detailed discussion of Eqs. (2.20) which are the central results of this paper will be given in Sec. III.

Consider the reflected wave. We define times $\tau_{zR},\tau_{yR},\tau_{xR}$ such that

$$\langle S_z\rangle_R=(\hbar/2)(\omega_L\tau_{zR}), \tag{2.21a}$$

$$\langle S_y\rangle_R=-(\hbar/2)(\omega_L\tau_{yR}), \tag{2.21b}$$

$$\langle S_x\rangle_R=(\hbar/2)(1-\omega_L^2\tau_{xR}^2/2). \tag{2.21c}$$

With the help of Eqs. (2.15a) and (2.15b) we find

$$\tau_{zR}=-\tau_zT/R, \tag{2.22a}$$

$$\tau_{yR}=\tau_y, \tag{2.22b}$$

and

$$\tau_{xR}=(\tau_{yR}^2+\tau_{zR}^2)^{1/2}=(\tau_y^2+\tau_z^2T^2/R^2)^{1/2}, \tag{2.22c}$$

with τ_x,τ_y given by Eqs. (2.20). These results will be discussed in detail in the next section.

All the above calculations assume that the field is confined to the barrier and that the particles carry spin $s=\frac{1}{2}$. In Appendixes A and B, we show that these two assumptions are unnecessary and merely simplified our calculations.

III. DWELL, TRAVERSAL, AND REFLECTION TIME

In this section, we give a physical interpretation of the Gedanken experiment of Baz'.[1–3] We do this by a comparison with other attempts to find the interaction time of particles with a barrier.

A. The phase-delay time

A time delay for scattering processes can be calculated by following the peak of a wave packet via the method of the stationary phase.[11,12] The time it takes for the peak of the transmitted wave packet and of the reflected wave packet to appear, measured from the moment the peak of the incident packet strikes the barrier at $y=-d/2$, is given by

$$\tau_\phi=\hbar d\Delta\phi/dE=(m/\hbar k)d\Delta\phi/dk. \tag{3.1}$$

Taking into account that κ is a function of k, we find from Eq. (2.6)

$$\tau_\phi=\frac{m}{\hbar k\kappa}\frac{2\kappa d k^2(\kappa^2-k^2)+k_0^4\sinh(2\kappa d)}{4k^2\kappa^2+k_0^4\sinh^2(\kappa d)}, \tag{3.2}$$

for $E<V_0$, respectively, $k\le k_0$. For $E>V_0$ we have to replace κ in Eq. (3.2) by iK. For small k, we find that

$$\tau_\phi\cong(2m/\hbar k\kappa)\tanh\kappa d \tag{3.3}$$

diverges as the kinetic energy of the incident particles tends to zero. This is in contrast to the characteristic times $\tau_z,\tau_y,\tau_x,\tau_{zR},\tau_{yR},\tau_{xR}$ found in the previous section which all remain finite as k tends to zero. Also note that for an opaque barrier, $k_0d\gg1$, the time τ_ϕ becomes independent of the width d of the barrier. The strong deformation of a wave packet when it interacts with the barrier makes the procedure of following the peak of the packet not meaningful.[13,14]

Equations (3.1)—(3.3) are correct for a wave packet characterized by a narrow momentum distribution.[12] If a wave packet with a wide momentum distribution strikes a barrier, the transmitted wave packet will exhibit a distribution displaced to higher momenta. Thus the transmitted packet moves faster than the incident packet. Since momentum is conserved the reflected wave packet exhibits a distribution which is shifted to lower momenta.

B. The dwell time

An approach by Smith[15] and others[16,17] yields a collision time τ_d for scattering events. τ_d is defined as the ratio of the number of particles within the barrier to the incident flux $j=\hbar k/m$,

$$\tau_d = N/j \; . \tag{3.4}$$

This approach does not distinguish whether, at the end of their stay, particles are reflected or transmitted. This time is the average *dwell* time of the particles in the barrier. The wave function in the barrier is given by $Be^{\kappa x}+Ce^{-\kappa x}$, where B and C are given by Eqs. (2.8) and (2.9). We find for the number of particles in the barrier,

$$N = \int_{-d/2}^{d/2} |\psi|^2 dx = \frac{k^2}{\kappa} \frac{2\kappa d(\kappa^2-k^2)+k_0^2 \sinh(2\kappa d)}{4k^2\kappa^2+k_0^4\sinh^2(\kappa d)} \; . \tag{3.5}$$

Dividing N by the incoming flux, we find

$$\tau_d = \tau_y \; , \tag{3.6}$$

where τ_y is given by Eq. (2.20b). Therefore, the extent to which a spin precesses around the z axis is determined by the average dwell time of the particle in the barrier.

Figure 3 compares the dwell time $\tau_d=\tau_y$ and the time τ_ϕ. Whereas $\tau_d=\tau_y$ tends to zero as k tends to zero [Eq. (1.4)], τ_ϕ diverges. This is in contrast to statements in Refs. 15—17, claiming that τ_d and τ_ϕ differ only by small oscillations. A simple calculation shows that $\tau_\phi \geq \tau_d$, for all k and $\tau_\phi=\tau_d$ only for $K=K_n$, where $K_n d = n\pi$ determines the energies

$$E_n = \hbar^2 K_n^2/2m + V_0$$

for which the barrier is transparent, $T=1$. For $E=E_n$ we find from Eqs. (3.2) and (2.20b),

$$\tau_\phi = \tau_d = \left[\frac{md}{\hbar k_n} \right] \left[\frac{K_n^2+k_n^2}{2K_n^2} \right] \; . \tag{3.7}$$

In summary, we have shown that the extent to which the spin undergoes a Larmor precession is determined by the dwell time of a particle in the barrier. The dwell time is not related to the phase delay as often claimed in the literature. Only if tunneling is unimportant, that is if the barrier is almost transparent, are the time τ_ϕ and the dwell time τ_d comparable.

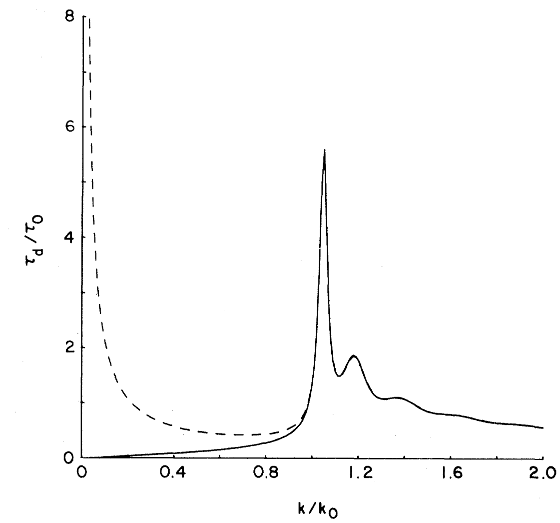

FIG. 3. The dwell time $\tau_d=\tau_y$ (full line), and the phase time τ_ϕ (broken line) as a function of k for a barrier with strength $k_0 d = 3\pi$, as in Fig. 2. The dwell time measures the average time a particle spends in the barrier, whether it is reflected or transmitted at the end of its stay.

C. Traversal time

To determine the time τ_T a particle takes to traverse the barrier Büttiker and Landauer[18] have considered a time-modulated barrier, $V(t)=V_0+V_1\cos\omega t$, with V_1 a small perturbation. For a *slowly* varying potential, $\omega \ll 1/\tau_T$, the tunneling particle sees an effectively static barrier of height $V(t)$. The time dependence of the transmitted wave can be found by replacing κ for the static barrier in Eq. (2.4) by $\kappa_t=\{2m[V(t)-E]\}^{1/2}/\hbar$ for the oscillating barrier. For a slowly varying potential, the additional time dependence of the transmitted wave is caused by the variation of the transmission probability with the height of the barrier. If the potential oscillates *fast* compared to the traversal time $\omega \gg 1/\tau_T$, the particles see a barrier of average height V_0. Particles absorb or emit modulation quanta $\hbar\omega$. The absorption of a modulation quantum is more likely because a particle of higher energy has a higher probability of transmission through the barrier. Thus, we can identify a characteristic behavior both at frequencies small and large compared to the reciprocal traversal time. The crossover between these two types of behavior yields the traversal time τ_T. For an opaque barrier and for $E < V_0$, $\hbar\omega \ll E$, the intensity of the transmitted particles which have absorbed or emitted a modulation quantum is given by[18]

$$T_{E\pm\hbar\omega}/T_E = (V_1\tau_T/2\hbar)^2[(e^{\pm\omega\tau_T}-1)^2/\omega^2\tau_T^2] , \tag{3.8}$$

with a traversal time $\tau_T = md/\hbar\kappa$. In the limit of small frequencies Eq. (3.8) reduces to

$$T_{E\pm\hbar\omega}/T = (V_1\tau_T/2\hbar)^2 . \tag{3.9}$$

In this limit the intensities of particles which have emitted or absorbed a quantum is independent of the modulation frequency. From Eq. (3.8) we find for the relative imbalance of the intensities of the sidebands,

$$\frac{T_{E+\hbar\omega}-T_{E-\hbar\omega}}{T_{E+\hbar\omega}+T_{E-\hbar\omega}} = \tanh(\omega\tau_T) . \tag{3.10}$$

Equation (3.10) shows that τ_T determines the crossover from the low-frequency behavior, where $T_{E+\hbar\omega} \cong T_{E-\hbar\omega}$, to the high-frequency behavior, where $T_{E+\hbar\omega} \gg T_{E-\hbar\omega}$. It is τ_x in the Baz'-Rybachenko experiment which has the same low-energy and high-energy limit as τ_T.

For the reflected intensities, we find for $E < V_0$ and $\hbar\omega \ll E$,

$$R_{E\pm\hbar\omega}/R = (V_1\tau_R/2\hbar)^2 , \tag{3.11}$$

where $\tau_R = (\hbar/V_0)k/\kappa$ is the time a reflected particle interacts with the barrier. Note that Eq. (3.13) is independent of ω, so we have not had to assume $\omega \ll 1/\tau_R$. This is so since $\hbar/\tau_R \gg V_0$ for $E < V_0$, i.e., τ_R is very short.

D. Reinterpretation of the Baz'-Rybachenko experiment

The low-frequency results Eqs. (3.9) and (3.11) are reminiscent of a two-level system[19] with states $|1\rangle$ and $|2\rangle$ and energies E and $E\pm\hbar\omega$ driven at resonance by an off-diagonal perturbation $V_1\cos\omega t$. If the whole population is in state $|1\rangle$ at $t=0$, then the population in level $|2\rangle$ grows initially as $(V_1 t/2\hbar)^2$. Thus τ_T and τ_R, in Eqs. (3.9) and (3.11), play the role of an effective interaction time. Similarly, for a two-level system with states $|1\rangle$ and $|2\rangle$ at the same energy, an off-diagonal perturbation V_1, switched on at $t=0$, causes the population of level 2 to grow as $(V_1 t/2\hbar)^2$ if the whole population was initially in level 1. We show now that the Baz' experiment can be considered in this way leading to effective interaction times τ_T and τ_R, where

$$\tau_T = \tau_x = (\tau_y^2+\tau_z^2)^{1/2} , \tag{3.12}$$

is the interaction time with the barrier of a particle which finally is transmitted, and

$$\tau_R = \tau_{xR} = (\tau_y^2+\tau_z^2 T^2/R^2)^{1/2} \tag{3.13}$$

is the interaction time of a particle with the barrier which is finally reflected. τ_z and τ_y are given by Eqs. (2.20).

Consider, for a moment, a new coordinate frame $\hat{x},\hat{y},\hat{z}$, in which the quantization direction $\hat{z}$ is parallel to the old x axis. The magnetic field $\vec{B}_0$ points along the minus $\hat{x}$ direction, and $\hat{y}=y$. The polarization of the incident beam, is then described by

$$\psi = \begin{bmatrix} 1 \\ 0 \end{bmatrix} . \tag{3.14}$$

In the new frame all incoming particles have spin up. The spins of the transmitted particles are then found from Eq. (2.10) through a $\pi/2$ rotation around the y axis. This yields

$$\psi = 2^{-1/2}(|D_+|^2+|D_-|^2)^{1/2}\begin{bmatrix} D_+ + D_- \\ D_- - D_+ \end{bmatrix} , \tag{3.15}$$

where D_+ and D_- are defined as in Sec. II. Therefore,

$$|C|^2 = \frac{1}{2}\frac{|D_+ - D_-|^2}{|D_+|^2+|D_-|^2} = \tfrac{1}{4}\omega_L^2\tau_x^2 , \tag{3.16}$$

is the fraction of particles whose spin has flipped from $+\hbar/2$ to $-\hbar/2$ along the $\hat{z}$ direction (the old x direction). In Eq. (3.16) we have taken the small-field limit so that

$$|C|^2 = (1/4)\,|(1/D)\partial D/\partial\kappa|^2(\omega_L m/\hbar\kappa)^2$$

and have used Eq. (2.18c). Equation (3.16) has the same form as Eq. (3.9) and shows that τ_x gives the interaction time for transmitted particles. We can, in a similar way, consider the fraction of reflected particles whose spin is flipped while interacting with the field and obtain

$$|C_R|^2 = (1/4)\omega_L^2\tau_{xR}^2$$

with τ_{xR} determined by Eq. (2.22c).

Figure 4 shows the traversal time $\tau_T(=\tau_x)$ [Eq. (2.19)] in comparison to the dwell time τ_d $(=\tau_y)$ [Eq. (2.20b)] for a barrier with strength, $k_0 d = 3\pi$. The traversal time τ_T is much longer than the dwell time for $E < V_0$. On the other hand for $E > V_0$ and increasing energy, the traversal time τ_T approaches

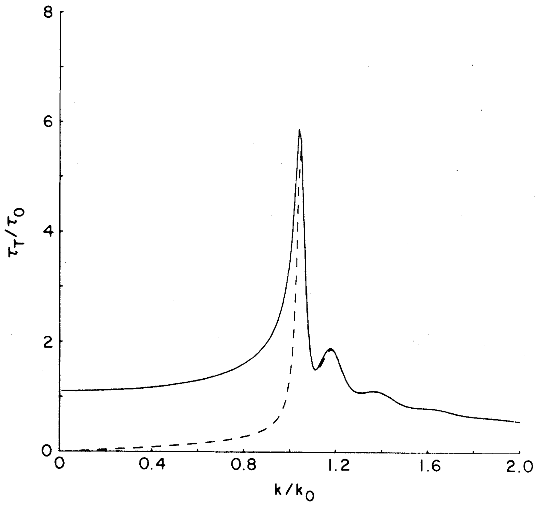

FIG. 4. The traversal time τ_T ($=\tau_x$) (full line) compared to the dwell time τ_d ($=\tau_y$), (broken line) for the same barrier as in Fig. 2. The traversal time measures the time a particle spends in the barrier if its finally transmitted.

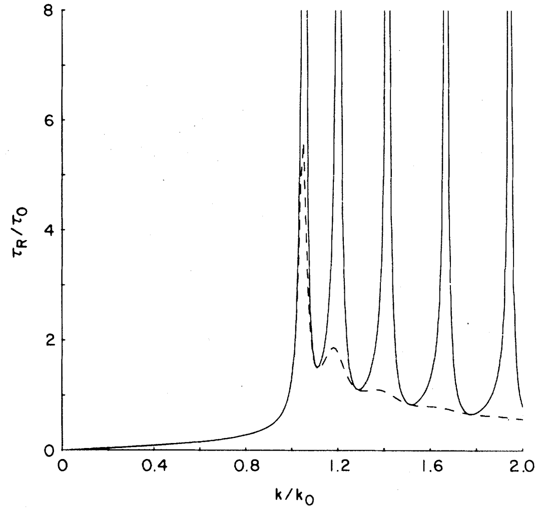

FIG. 5. The reflection time τ_R (full line) in comparison with the dwell time τ_d (broken line) for the same barrier as in Fig. 2. The reflection time measures the time a particle interacts with the barrier if it is finally reflected.

rapidly the dwell time τ_d since the barrier is now increasingly transparent. In the limit that the kinetic energy tends to zero, τ_y vanishes and the traversal time

$$\tau_T=\tau_x=(\tau_y^2+\tau_z^2)^{1/2}$$

is determined by τ_z. We find from Eq. (2.20a) for $k=0$,

$$\tau_T=\tau_z=\frac{1}{2}\frac{\hbar}{V_0}\left[1+\frac{k_0 d}{\tanh(k_0 d)}\right]. \tag{3.17}$$

For a wide barrier Eq. (3.17) yields $\tau_T=md/\hbar k_0$, proportional to the width of the barrier. For a very thin barrier $\tau_T=\hbar/V_0$. V_0, the barrier height, is the minimal energy a particle has to "borrow" for classical barrier traversal. For an opaque barrier and $E<V_0$ we find from Eq. (3.12) that over a wide range of k, $\tau_T=md/\hbar\kappa$, as found by Büttiker and Landauer.[18]

Figure 5 shows the reflection time τ_R in comparison with the dwell time τ_d for a barrier of strength $k_0 d=3\pi$. For the reflection time $\tau_R=\tau_{xR}=(\tau_y^2+\tau_z^2T^2/R^2)^{1/2}$ we find for small k to lowest order in k

$$\tau_R=\tau_y=\frac{\hbar k}{2V_0 k_0}\frac{2k_0 d+\sinh(2k_0 d)}{\sinh^2(k_0 d)}, \tag{3.18}$$

since T tends to zero proportional to the energy $E\sim k^2$. The reflection time becomes equal to the dwell time since, for small k, almost all particles are reflected. For a wide barrier $\tau_R=\hbar k/V_0 k_0$ and for a thin barrier $\tau_R=2\hbar k/V_0 k_0^2 d$. For an opaque barrier and $E<V_0$, we find from Eq. (3.13) that over a wide range of $k<k_0$, $\tau_R=\hbar k/V_0\kappa$. Consider now the singularities of τ_R for $E>V_0$. At the energies of complete transparency of the barrier $E=E_n=\hbar^2K_n^2/2m+V_0$, where $K_n d=n\pi$, the z component of the transmitted particles changes from the positive direction in the negative direction (see Fig. 2). We find from Eq. (2.20a),

$$\tau_z=-\frac{m}{\hbar}\frac{k_0^4 d^2}{4k_n^2K_n^3}(K-K_n). \tag{3.19}$$

Thus if $T=1$, we have $\tau_z=0$ and hence the traversal time τ_T is equal to the dwell time $\tau_d=\tau_y$. This has to be so since at this energy no particles are reflected. The reflection probability tends to zero like

$$R\cong k_0^4(K-K_n)^2d^2/4k_n^2K_n^2. \tag{3.20}$$

Now using Eqs. (3.19) and (3.20) we find that near energies for which the barrier is completely transparent, the z component of the reflected beam determined by

$$\tau_{zR}\cong-\tau_z T/R=\frac{m}{\hbar}\frac{1}{K_n(K-K_n)}, \tag{3.21}$$

diverges. To understand this behavior, we consider for a moment $\langle S_z \rangle_R$ for a finite field determined by Eq. (2.14a). We see that if the barrier is completely transparent for particles with spin parallel to the field, the reflected beam is completely polarized antiparallel to the field. At a higher energy the barrier will be completely transparent for the particles with spin antiparallel to the field and the reflected beam will be completely polarized in the direction of the field. As the field is made smaller and smaller, these two completely polarized states come closer and closer together on the energy scale and finally give rise to the singularity described by Eq. (3.21).

The energies where τ_z crosses from negative to positive values are determined by

$$\frac{Kdk_0^2}{k^2+K^2}=\frac{Kdk_0^2}{2K^2+k_0^2}=\tan(Kd) . \tag{3.22}$$

At these energies the traversal time and the reflection time are both equal to the dwell time $\tau_T=\tau_R=\tau_d(=\tau_y)$. The energies at which this happens are smaller than the energies which correspond to the local minimas in the transmission probability which occur at $Kd_n=(\pi/2)(2n+1)$.

To summarize: We have presented a reinterpretation of the Baz'-Rybachenko experiment. We pointed out that there are three characteristic times associated with the interaction of particles with a barrier: a dwell time, a traversal time, and a reflection time. None of these characteristic times is related to the phase-delay time. We treated only the rectangular barrier. We hope that this paper stimulates further investigation of barriers of arbitrary shape as well as scattering problems in higher dimensions.

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APPENDIX A: PENETRATION OF A FIELD

We have assumed that the magnetic field is precisely confined to the barrier. This is an unnecessary assumption, made only for simplicity. If the magnetic field extends beyond the barrier, the particles undergo a Larmor precession in these additional regions and the effects due to the barrier alone are easily separated. To show this, we consider the case where the barrier potential vanishes and particles enter a region of field of width d.

We have to solve the scattering problem for the Hamiltonian Eq. (2.1) with $V_0=0$. As above, we assume that the incident particles are polarized in the x direction. We are interested in the effect of an infinitesimal field. Thus, we consider the case where $E>\hbar\omega_L/2$. In the region of the field, the wave is described by $B_\pm e^{iK_\pm x}+C_\pm e^{-iK_\pm x}$, where

$$K_\pm=(k^2\pm m\omega_L/\hbar)^{1/2} . \tag{A1}$$

The transmission probability

$$T_\pm=\frac{4k^2(k^2\pm m\omega_L/\hbar)}{4k^2(k^2\pm m\omega_L/\hbar)+(m\omega_L/\hbar)^2\sin^2[(k^2\pm m\omega_L/\hbar)^{1/2}d]} , \tag{A2}$$

is found by replacing κ in Eq. (2.5) by $iK_\pm$. In the limit of a small field, $\omega_L\to 0$, we obtain

$$T_\pm=1-\frac{(\hbar\omega_L)^2}{16E^2}\sin^2(kd)+O(\omega_L^3) . \tag{A3}$$

Thus the transmission probability changes only to second order in the field. Moreover, to this order, the transmission coefficient does not depend on the sign of the spin. Using Eq. (2.12a), we find a spin component in the z direction,

$$\langle S_z\rangle=\frac{\hbar}{2}\left[\frac{\hbar\omega_L}{E}\right]^3[2\sin^2(kd)-kd\sin(2kd)] , \tag{A4}$$

only to *third* order in ω_L. This is in contrast to the first-order effect produced by a barrier *and* a small magnetic field [Eqs. (1.8) and (2.17a)]. For the y component, we find by substituting $iK_\pm$ [Eq. (A1)] for κ in Eq. (2.6) and using Eq. (2.18b),

$$\langle S_y\rangle=-\frac{\hbar}{2}\omega_L\frac{md}{\hbar k} . \tag{A5}$$

To order ω_L^2, we have

$$\langle S_x\rangle=(\hbar/2)[1-(1/2)(md/\hbar k)^2\omega_L^2] .$$

Thus to lowest order in the field B_0 the particles traversing a small magnetic field perform a Larmor precession as expected.

APPENDIX B: PARTICLES WITH SPIN $s\geq\frac{1}{2}$

Consider the case of a particle with spin $s\geq\frac{1}{2}$ tunneling through a barrier in a field. We calculate

$\langle S_z \rangle$ for $s > \frac{1}{2}$. The polarization of the incident particles is determined by

$$\psi = \sum_{m_z=-s}^{m_z=+s} C_{m_z} \,|\, s,m_z \rangle \,, \tag{B1}$$

where

$$\sum_{m_z=-s}^{m_z=+s} |\, C_{m_z} \,|^2 = 1$$

and $|\, s,m_z \rangle$ are the eigenstates of S_z with eigenvalues $\hbar m_z$. For each eigenvalue m_z, the exponential decay of the wave function in the barrier is different and given by

$$\kappa_{m_z} = (k_0^2 - k^2 - m_z 2m\omega_L/\hbar)^{1/2} \,. \tag{B2}$$

The spinor of the transmitted wave is

$$\psi = \left[\sum_{m_z=-s}^{m_z=+s} |\, C_{m_z} \,|^2 \,|\, D(\kappa_{m_z}) \,|^2 \right]^{-1/2} \left[\sum_{m_z=-s}^{m_z=+s} C_{m_z} D(\kappa_{m_z}) \,|\, s,m_z \rangle \right] , \tag{B3}$$

since H is diagonal in the basis of the $|\, s,m_z \rangle$. The z component of the spin is then given by

$$\langle S_z \rangle = \hbar \sum_{m_z=-s}^{m_z=+s} m_z \,|\, C_{m_z} \,|^2 \,|\, D(\kappa_{m_z}) \,|^2 \Big/ \sum_{m_z=-s}^{m_z=+s} |\, C_{m_z} \,|^2 \,|\, D(\kappa_{m_z}) \,|^2 \,. \tag{B4}$$

In the limit of a small magnetic field where

$$\kappa_{m_z} = \kappa - m_z (m\omega_L/\hbar\kappa) \,, \tag{B5}$$

we obtain from (B4),

$$\langle S_z \rangle = -\hbar \left[\sum_{m_z=-s}^{m_z=+s} m_z^2 \,|\, C_{m_z} \,|^2 \right] \left[\frac{2\omega_L m}{\hbar\kappa} \right] \frac{\partial}{\partial\kappa} (\ln T^{1/2}) \,. \tag{B6}$$

Using Eq. (2.18a), we find

$$\langle S_z \rangle = \hbar \left[\sum_{m_z=-s}^{m_z=+s} m_z^2 \,|\, C_{m_z} \,|^2 \right] 2\omega_L \tau_z \,,$$

with τ_z determined by Eq. (2.20a). If the spin of the incident particles is polarized completely in the x direction, the coefficients in (B1) are given by[20]

$$C_{m_z} = \frac{1}{2^s} \left[\frac{2s!}{(s+m_z)!(s-m_z)!} \right]^{1/2} . \tag{B7}$$

We find $\sum_{m_z=-s}^{m_z=+s} m_z^2 C_{m_z}^2 = s/2$ so that

$$\langle S_z \rangle = \hbar s \omega_L \tau_z \,. \tag{B8}$$

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