Inelastic scattering time in two-dimensional disordered metals

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(Received 2 December 1982)

The particle-particle diffusion propagator is evaluated diagrammatically in two dimensions in the presence of inelastic scattering due to screened Coulomb interactions. As expected, an inelastic scattering rate $1/\tau_\epsilon \propto T \ln T$ cuts off the backscattering divergence responsible for localization. The τ_ϵ thus determined is the same as the quasiparticle lifetime determined from the self-energy of the one-particle Green's function.

I. INTRODUCTION

Interaction effects in the weakly localized regime in two-dimensions have been extensively investigated recently.[1–4] It has now been established that the prefactor of the logarithmic temperature dependence of the conductivity is affected by interactions, i.e.,

$$\sigma_I' = -\frac{e^2}{2\pi^2} g \ln \frac{1}{4\pi\tau T} , \tag{1.1}$$

where τ is the elastic scattering time and g is the dimensionless effective coupling constant. On the other hand, the localization theory leads to[5,6]

$$\sigma_L' = -\frac{e^2}{2\pi^2} \ln \frac{\tau_\epsilon}{\tau} , \tag{1.2}$$

where τ_ϵ is the inelastic scattering time, which has so far been introduced phenomenologically, and which has been assumed to be the same as the quasiparticle lifetime. Such an electron quasiparticle lifetime will simply be proportional to T^{-2} if the dominant interaction is electron-electron scattering in a pure system.[6] However, once $T < \tau^{-1}$, the neglect of impurity scattering will not be valid and a different temperature dependence is expected. Actually Schmidt[7] and Altshuler and Aronov[8] some time ago examined this kind of possibility for three dimensions and found that $\tau_\epsilon^{-1} \propto T^{3/2}$ in dirty systems. Recently this problem has been discussed in detail by Abrahams *et al.*,[9] who concluded that $\tau_\epsilon^{-1} \propto T \ln T$ in two dimensions.

Although it is quite natural and physical that the quasiparticle lifetime should enter Eq. (1.2), this has to be examined in detail, since τ_ϵ in Eq. (1.2) has to be determined by the particle-particle diffusion propagator $D(\sigma,\omega_\lambda)$, which will have the form

$$D(q,\omega_\lambda) = \frac{1}{2\pi N(0)\tau^2} \frac{1}{Dq^2 + |\omega_\lambda| + 1/\tau_\epsilon} . \tag{1.3}$$

Here $N(0)$ is the density of states at the Fermi energy, D is the diffusion constant, and ω_λ is a Matsubara frequency equal to $2\pi\lambda T$.

In the present paper we shall evaluate τ_ϵ in Eq. (1.3) diagrammatically for the screened Coulomb interaction in the same context[1–4] as the derivation of σ_I', Eq. (1.1). We take units such that $\hbar = k_B = 1$.

II. PARTICLE-PARTICLE DIFFUSION PROPAGATOR

Our model Hamiltonian is given as follows:

$$\mathscr{H} = \sum_i \left[\frac{p_i^2}{2m} + u(r_i) \right] + \frac{1}{2} \sum_{i \neq j} v(r_i - r_j)$$
$$= \mathscr{H}_0 + V . \tag{2.1}$$

Here $u(r)$ is the one-particle potential due to randomness whereas $\frac{1}{2}\sum_{i\neq j} v(r_i - r_j) = V$ is the Coulomb interaction.

The particle-particle diffusion propagator $D(q,\omega_\lambda)$, given by Eq. (1.3), is defined diagrammatically as in Fig. 1(a), where $\epsilon_n = (2n+1)\pi T < 0$ and $\epsilon_{n+} = (\epsilon_n + \omega_\lambda) > 0$. Here $D \equiv D(q,\omega_\lambda)$ includes full effects of randomness and Coulomb interactions. The effect of interactions can be treated perturbatively as is shown in Fig. 1(b), where the interaction block G is due to interactions and D_0 is given by the propagator in the absence of interactions:

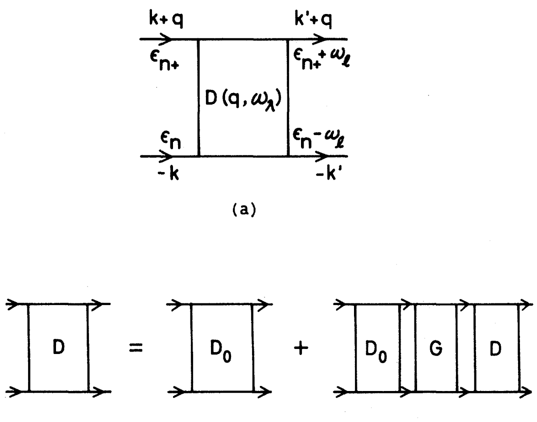

FIG. 1. Full particle-particle diffusion propagator D, with Coulomb interaction. (b) Equation for D in terms of that in the absence of interactions D_0 and the interaction vertex G.

$$D_0(q,\omega_\lambda)=\frac{1}{2\pi N(0)\tau^2}\frac{1}{Dq^2+|\omega_\lambda|} . \tag{2.2}$$

We confine ourselves to first order in the electron-electron interaction. Then G is given by the processes shown in Figs. 2(a), (b), and (c) where wavy lines are Coulomb interactions. In these figures we do not show the possible quantum corrections due to elastic impurity scattering. These are examined in detail later.

We now argue that for the leading contribution, it is possible to neglect graphs of the form of Fig. 2(c). Graphs of the form of Figs. 2(a) and 2(b) have $\omega_l=0$, which means that the two diffusion propagators D that enter Fig. 1(b) with G always have the same frequency ω_λ. This gives rise to a more singular contribution at $\omega_\lambda\to 0$ than that from Fig. 2(c), for which the two diffusion propagators of Fig. 1(b) differ in frequency by $2\omega_l$. The latter contribution can only be important if the interaction itself is singular at $\omega_l=0$, which in our case (in contrast to the case of paramagnetic impurities) it is not. This conclusion remains valid when the graphs for G are dressed with all possible quantum corrections.

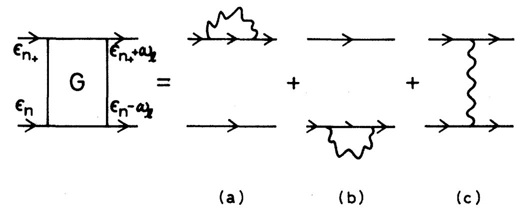

FIG. 2. Interaction vertex G in the linear order of the Coulomb interaction (wavy lines).

If we then ignore Fig. 2(c) we find that Fig. 1(b) can be represented by an algebraic equation whose solution is

$$D(q,\omega_\lambda)=\frac{1}{2\pi N(0)\tau^2}\frac{1}{Dq^2+|\omega_\lambda|+1/\tau_\epsilon} , \tag{2.3}$$

where

$$\frac{1}{\tau_\epsilon}=-\frac{1}{2\pi N(0)\tau^2}G . \tag{2.4}$$

Thus we need to evaluate G given by Figs. 2(a) and 2(b). In the absence of quantum corrections due to randomness, these processes result in familiar contributions G^0, consistent with the Fermi-liquid theory:

$$G^0=-T\sum_{\omega_l}\sum_{k,q}v(q,\omega_l)[\mathcal{G}^2(k,\epsilon_{n+})\mathcal{G}(-k,\epsilon_n)\mathcal{G}(k+q,\epsilon_{n+}+\omega_l)+\mathcal{G}(k,\epsilon_{n+})\mathcal{G}^2(-k,\epsilon_n)\mathcal{G}(k+q,\epsilon_n+\omega_l)] \tag{2.5a}$$

$$=-\frac{2}{\pi}\sum_{k,q}\Big[\mathcal{P}\int dx\, n(x)\mathrm{Im}v_R(q,x)\mathrm{Re}[G_R^2(k,\epsilon)G_A(k,\epsilon)G_R(k+q,\epsilon+x)]$$

$$-\int dx\, f(x)\mathrm{Im}G_R(k+q,x)\mathrm{Re}[v_R(q,x-\epsilon)G_R(k,\epsilon)G_A^2(k,\epsilon)]\Big] . \tag{2.5b}$$

In Eq. (2.5a) $v(q,\omega_l)$ is the screened Coulomb interaction

$$v(q,\omega_l)=\frac{v_q^0}{1+v_q^0\Pi(q,\omega_l)} , \tag{2.6}$$

where $v_q^0=2\pi e^2/q$, $\Pi(q,\omega_l)$ is the polarization function,

$$\mathcal{G}(k,\epsilon_n)=(i\epsilon_n-\epsilon_k+i\,\mathrm{sgn}\epsilon_n/2\tau)^{-1} ,$$

$$\epsilon_k=k^2/2m-\epsilon_F .$$

In Eq. (2.5b), $\mathcal{P}$ denotes the principal part and

$$v_R(q,x)=v(q,-ix+0) .$$

Also,

$$G_{R(A)}(k,\epsilon)=\mathscr{G}(k,-i\epsilon\pm 0)\ ,$$
$$f(x)=(e^{\beta x}+1)^{-1}\ ,$$
$$n(x)=(e^{\beta x}-1)^{-1}\ ,$$
$$\beta=1/T\ .$$

In the limit of weak scattering $\epsilon_F\tau\rightarrow\infty$, we note that

$$G_R(k,\epsilon)G_A(k,\epsilon)=2\pi\tau\delta(\epsilon-\epsilon_k)\ , \tag{2.7}$$

and then

$$G_R(k,\epsilon)G_A^2(k,\epsilon)=i\pi(2\tau)^2\delta(\epsilon-\epsilon_k)\ , \tag{2.8a}$$

$$\begin{aligned}\mathrm{Re}G_A^2(k,\epsilon)G_A(k,\epsilon)G_R(k+q,\epsilon+x) &=\pi(2\tau)^2\delta(\epsilon-\epsilon_k)\mathrm{Im}G_R(k+q,\epsilon+x)\\ &=-(2\pi\tau)^2\delta(\epsilon-\epsilon_k)\delta(\epsilon+x-\epsilon_{k+q})\ .\end{aligned} \tag{2.8b}$$

Consequently, G^0 is given by

$$G^0=8\pi\tau^2\sum_q\int dx[n(x)+f(x+\epsilon)]A_g(x)\mathrm{Im}v_R(q,x) \tag{2.9a}$$

$$=8\pi\tau^2\sum_q\int dx\frac{1}{\sinh\beta x}A_q(x)\mathrm{Im}v_R(q,x)\ , \tag{2.9b}$$

where

$$A_q(x)=\sum_k\delta(\epsilon-\epsilon_k)\delta(\epsilon+x-\epsilon_{k+q})$$

and in Eq. (2.9b) we introduced the approximation $n(x)+f(x+\epsilon)=n(x)+f(x)=(\sinh\beta x)^{-1}$. In the clean limit

$$\mathrm{Im}v_R(q,x)=-\left[\frac{g_0}{2N(0)}\right]^2\pi x A_q(x)\ , \tag{2.10}$$

where we defined g_0 as

$$g_0=\left\langle\frac{2N(0)v_q^0}{1+\Pi_R(q,x)v_q^0}\right\rangle\ . \tag{2.11}$$

The average in Eq. (2.11) is over the scattering over the Fermi surface. As long as $\kappa\equiv 4\pi N(0)e^2$ is much larger than the Fermi momentum k_F, $\kappa/k_F\gg 1$, we have $g_0\sim 1$. For free electrons $A_q(x)$ is given by

$$A_q(x)=\frac{mN(0)}{k_Fq}\left[1-\left[\frac{q}{2k_F}\right]^2\right]^{-1/2}\Theta(2k_F-|q|)\ , \tag{2.12}$$

where x, which is at most of the order of temperature, can be ignored compared to the Fermi energy. Finally, τ_ϵ in the clean limit τ_ϵ^0 is given by

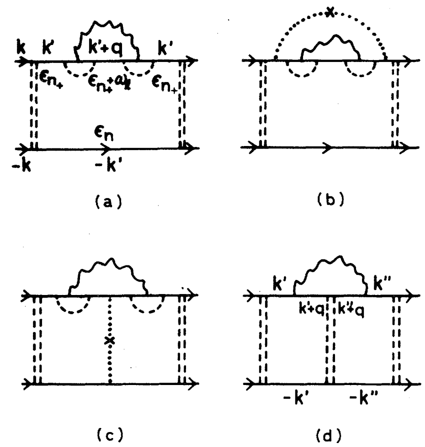

FIG. 3. Quantum corrections to G, where broken and double-broken lines are particle-hole and particle-particle propagators, respectively, defined in Fig. 4.

$$\frac{1}{\tau_\epsilon^0}=\frac{\pi}{2}g_0^2\frac{T^2}{\epsilon_F}\ln\frac{1}{\delta}\ , \tag{2.13}$$

where δ is the cutoff parameter of the order of either $(\epsilon_F\tau)^{-1}$ or T/ϵ_F, which is introduced to take into account the smearing of the Fermi surface due to disorder or temperature.

Next we examine the quantum corrections to $1/\tau_\epsilon$. As has been discussed elsewhere,[3] there exist four different kinds of contributions to the self-energy function of electrons, depending on either Hartree or Fock type of process with either particle-particle or particle-hole diffusion propagators. In the present case of screened Coulomb interaction, τ_ϵ^{-1} is seen to be determined by g_1 processes, i.e., those with small energy and momentum transfers. These processes are shown in Figs. 3(a)–3(d), where broken and double-broken lines are the particle-hole and particle-particle diffusion propagators shown in Figs. 4(a) and 4(b). These propagators are singular when $\epsilon_n\epsilon_{n'}<0$ and in such a case these are given as

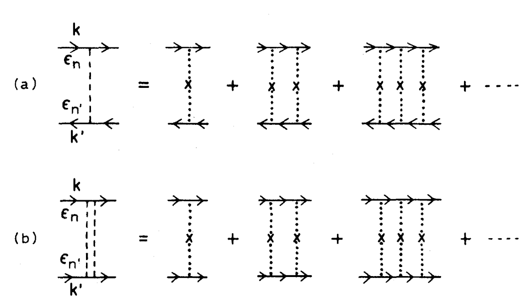

FIG. 4. (a) Particle-hole diffusion propagators. (b) Particle-particle diffusion propagators.

follows. Figure 4(a) is equal to

$$\frac{1}{2\pi N(0)\tau^2}\frac{1}{D(k-k')^2+|\epsilon_n-\epsilon_{n'}|} \equiv \Gamma_{p\text{-}h}(k-k',\epsilon_n-\epsilon_{n'}) , \tag{2.14a}$$

and Fig. 4(b) is equal to

$$\frac{1}{2\pi N(0)\tau^2}\frac{1}{D(k+k')^2+|\epsilon_n-\epsilon_{n'}|+1/\tau_\epsilon} \equiv \Gamma_{p\text{-}p}(k+k',\epsilon_n-\epsilon_{n'}) . \tag{2.14b}$$

In Eq. (2.14b), we introduced τ_ϵ which is to be evaluated and determined self-consistently. By use of $\Gamma_{p\text{-}h}(q,\omega_l)$ the sum of the contributions from Figs. 3(a), 3(b), and 3(c) is equal to

$$-[2\pi N(0)]^3\tau^6 T \sum_{\epsilon_{n+}+\omega_l>0} v(q,\omega_l)\Gamma^2_{p\text{-}h}(q,\omega_l) \times(Dq^2+|\omega_l|) , \tag{2.15a}$$

which simplifies to

$$-[2\pi N(0)\tau^2]^2 T \sum_{\omega_l>\epsilon_{n+}} v(q,\omega_l)\Gamma_{p\text{-}h}(q,\omega_l) . \tag{2.15b}$$

Here we noted that the leading contribution from each process is canceled. From Eq. (2.15a) to (2.15b) we made use of the fact that $v(q,\omega_l)$ is an even function of ω_l. On the other hand, the process of Fig. 3(d) yields the following contribution:

$$[2\pi N(0)\tau^2]^2 T \sum_{\epsilon_{n+}+\omega_l>0} v(q,\omega_l)\Gamma_{p\text{-}p}(q,\omega_\lambda+\omega_l) . \tag{2.16}$$

By use of Eqs. (2.15b) and (2.16) and by taking the contributions from the processes where the mutual interactions are on the lower Green's function, we obtain the following result for $1/\tau'_\epsilon$, the quantum correction to $1/\tau^0_\epsilon$:

$$\frac{1}{\tau'_\epsilon}=2\pi N(0)\tau^2\sum_q\left[T\left[\sum_{\omega_l>\epsilon_{n+}}+\sum_{\omega_l>-\epsilon_n}\right]v(q,\omega_l)\Gamma_{p\text{-}h}(q,\omega_l)-T\left[\sum_{\omega_l>-\epsilon_{n+}}+\sum_{\omega_l>\epsilon_n}\right]v(q,\omega_l)\Gamma_{p\text{-}p}(q,\omega_\lambda+\omega_l)\right] \tag{2.17}$$

By analytic continuation the summation over ω_l in Eq. (2.17) may be transformed to the following integration:

$$\frac{1}{\tau'_\epsilon}=-2N(0)\tau^2\sum_q\int_{-\infty}^{\infty}dx\,\mathrm{Im}v_R(q,x)\{[f(x-\epsilon)+f(x+\epsilon)]\Gamma^R_{p\text{-}h}(q,x)+2n(x)\Gamma^R_{p\text{-}p}(q,x)\}$$
$$+iN(0)\tau^2\sum_q\int dx\,[f(x-\epsilon)+f(x+\epsilon)]v_A(q,x)[\Gamma^R_{p\text{-}h}(q,x)-\Gamma^R_{p\text{-}p}(q,x)] \tag{2.18}$$
$$\simeq -4N(0)\tau^2\sum_q\int_{-\infty}^{\infty}dx\frac{\Gamma^R_{p\text{-}p}(q,x)}{\sinh\beta x}\mathrm{Im}v_R(q,x) , \tag{2.19}$$

where $\Gamma^R_{p\text{-}h}(q,x)=\Gamma_{p\text{-}h}(q,-ix+0)$, $\Gamma^R_{p\text{-}p}(q,x)=\Gamma_{p\text{-}p}(q,-ix+0)$. In obtaining Eq. (2.19) from Eq. (2.18) we have retained the most singular contribution. Note that only $\Gamma^R_{p\text{-}p}(q,x)$ remains in Eq. (2.19). At small q and x the screening in $v_R(q,x)$ is of diffusive type,[9] i.e.,

$$\mathrm{Im}v_R(q,x)=\frac{2\pi e^2}{q}\mathrm{Im}\frac{Dq^2-ix}{D\kappa q-ix}\simeq-\frac{2\pi e^2}{q}\frac{D\kappa qx}{(D\kappa q)^2+x^2} , \tag{2.20}$$

and $\Gamma^R_{p\text{-}p}(q,x)$ is given by

$$\Gamma^R_{p\text{-}p}(q,x)=\frac{1}{2\pi N(0)\tau^2}\frac{1}{Dq^2-ix+1/\tau_\epsilon} . \tag{2.21}$$

Consequently, we perform the sum over q and estimate Eq. (2.19) as follows:

$$\frac{1}{\tau'_\epsilon}=\frac{2e^2}{\pi D\kappa}\int_0^\infty dx\frac{x}{\sinh\beta x}\mathrm{Re}\left[\frac{1}{ix+1/\tau_\epsilon}\ln\left[\frac{D\kappa^2(ix+1/\tau_\epsilon)}{x^2}\right]\right] . \tag{2.22}$$

We evaluate the integral in Eq. (2.22) approximately, using the fact that the major contribution comes from $0<x<\beta^{-1}=T$. This permits the replacement

$$\frac{1}{\sinh\beta x}=\begin{cases}\frac{1}{\beta x}, & x<T\\ 0, & x>T\ .\end{cases} \tag{2.23}$$

Since the scale of variation of x is $1/\tau_\epsilon$ which is of order[9] $T/\epsilon_F\tau \ll T$, we may extend the upper limit of the x integration [given by Eq. (2.3) as T] to infinity. We then find

$$\frac{1}{\tau'_\epsilon}=\frac{1}{2\epsilon_F\tau}T\ln(D\kappa^2T\tau_\epsilon^2)\ . \tag{2.24}$$

Here we have used the two dimensional relations $D=v_F^2\tau/2$ and $\kappa=4\pi N(0)e^2=2me^2$. From Eq. (2.24) we see that we need to determine τ_ϵ self-consistently.

It is clear that for $T<1/\tau$, $1/\tau'_\epsilon$ dominates $1/\tau_\epsilon^0$ of Eq. (2.13). Then we can use $1/\tau_\epsilon$ for $1/\tau'_\epsilon$ in Eq. (2.24) and find the leading logarithmic behavior

$$\tau/\tau_\epsilon=(T/2\epsilon_F)\ln(T_1/T)\ ,$$
$$T_1=4(\epsilon_F\tau)^2D\kappa^2\ . \tag{2.25}$$

This result is the same as that found previously by Abrahams *et al.*[9]

III. RESULTS AND DISCUSSIONS

We have seen that in two dimensions the inelastic scattering time τ_ϵ, defined by the particle-particle diffusion propagator, is given as follows if we assume the dynamically screened Coulomb interaction:

$$\frac{1}{\tau_\epsilon}=\begin{cases}\frac{\pi T^2}{2\epsilon_F}\ln\frac{\epsilon_F}{T}, & T>1/\tau & (3.1)\\ \frac{T}{2\epsilon_F\tau}\ln\frac{T_1}{T}, & T<1/\tau\ . & (3.2)\end{cases}$$

In Eq. (3.1) we assumed that the inverse screening length κ is much larger than $2k_F$ and then g_0, defined by Eq. (2.11), is approximated as $g_0=1$. Equation (3.2) indicates that, once $T<1/\tau$, the smearing of the Fermi surface affects the inelastic processes as has been first noted by Schmidt[7] and recently elaborated by Abrahams *et al.*[9] These latter authors evaluated the lifetime of the quasiparticle from the self-energy of the one-particle electron Green's function. In this paper we evaluated the lifetime of the particle-particle diffusion propagator in the momentum representation, since the inelastic lifetime appearing in the localization problem is defined through this propagator. We have seen that τ_ϵ in the former is essentially determined by the particle-particle propagator $\Gamma_{p\text{-}p}$ [see Eq. (2.19)], while in the latter it is governed by the particle-hole propagator $\Gamma_{p\text{-}h}$. Actually, we have found that τ_ϵ in the present scheme is determined self-consistently, since it depends on $\Gamma_{p\text{-}p}$, which in turn has the same τ_ϵ to be determined. Surprisingly, the result of this self-consistency leads to the same result as that determined by the self-energy.[9]

A different discussion of the electron-electron scattering inelastic cutoff has been formulated by Altshuler, Aronov, and Khmelnitsky.[10] Their results are rather similar to ours, although in some parameter ranges the $\ln T$ is absent.

The present diagrammatic procedures clearly indicate that the dynamical interactions with different symmetries will result in τ_ϵ much different from the one we obtained here. This will be discussed elsewhere.

ACKNOWLEDGMENTS

We acknowledge the hospitality of the Aspen Center for Physics, where much of this work was carried out. This work was supported in part by National Science Foundation Grant No. DMR-79-21360. We thank P. A. Lee for many discussions and B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitsky for correspondence and preprints of their work.

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