PHYSICAL REVIEW B
CONDENSED MATTER

THIRD SERIES, VOLUME 27, NUMBER 10 — 15 MAY 1983

Comparison of the Faraday rotation for the two- and three-dimensional models of the inversion layer in a metal-oxide-semiconductor system

R. F. O'Connell and G. Wallace
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803

(Received 11 January 1983)

We have previously calculated the Faraday rotation due to the inversion layer in a metal-oxide-semiconductor system, using both a three-dimensional model and a two-dimensional model for the inversion layer. However, in the limit of the inversion-layer thickness going to zero, the results for both models should be the same, but they are not. Here we show that if the three-dimensional model is extended to include *multiple-reflection* effects within the inversion layer, we do get agreement. We conclude that the two-dimensional model automatically takes multiple-reflection effects within the inversion layer into account and that this also explains the source of the dependence of the Faraday rotation on the oxide layer which we previously obtained in the case of the two-dimensional model.

Recently we calculated the ellipticity and Faraday rotation caused by the electron gas of the inversion layer in a metal-oxide-semiconductor (MOS) system, in the case where the directions of both the incident radiation of frequency ω and the external magnetic field $B>0$ are oriented normal to the insulator-semiconductor interface.[1] Our basic starting point was the two-dimensional (2D) gas model of Chiu *et al.*[2] This model has also been used to interpret cyclotron resonance data.[3] It was shown that there are striking differences between this model and the single-pass three-dimensional (3D) Drude model investigated in Ref. 4. One difference between these two models is that the rotation angle for the 2D model depends on the oxide layer through the index of refraction n_o, whereas the single-pass 3D model is independent of the oxide. To bring in the effect of the oxide in the 3D model, we extend the single-pass result to include boundary effects at the inversion-layer interfaces as well as multiple-reflection effects within the inversion layer, i.e., the *multiple-pass Faraday rotation.* With this result we show below that as the inversion-layer thickness goes to zero (compared to the wavelength of the incident radiation), we reproduce the 2D model results.

It has been shown[5] that the multiple-pass Faraday rotation Θ, in the case of three distinct media, has a decomposition of the form

$$\Theta=\theta+\theta_T+\theta_{\mathrm{MR}}\ , \tag{1}$$

where θ is the single-pass Faraday rotation, θ_T is due purely to boundary effects and is independent of the sample thickness d, and θ_{MR} is due purely to multiple-reflection effects. Denoting the complex refractive indices as

$$N_{j\pm}\equiv n_{j\pm}+i\kappa_{j\pm}\ ,\quad j=1,2,3 \tag{2}$$

and propagation from medium i to medium j by the pair of indices (ij), θ_T and θ_{MR} may be written in the form[5]

$$\theta_T\equiv\tfrac{1}{2}(\gamma_{12+}+\gamma_{23+}-\gamma_{12-}-\gamma_{23-})\ , \tag{3}$$

$$\theta_{\mathrm{MR}}\equiv\tfrac{1}{2}(\eta_+-\eta_-)\ , \tag{4}$$

where

$$\tan\gamma_{ij\pm}=\frac{n_{j\pm}\kappa_{i\pm}-n_{i\pm}\kappa_{j\pm}}{n_{i\pm}(n_{i\pm}+n_{j\pm})+\kappa_{i\pm}(\kappa_{i\pm}+\kappa_{j\pm})}\ , \tag{5}$$

$$\tan\eta_\pm=\frac{|r_{23\pm}r_{21\pm}|\,e^{-\alpha_\pm d}\sin(2\beta_\pm d+\xi_{21\pm}+\xi_{23\pm})}{1-|r_{23\pm}r_{21\pm}|\,e^{-\alpha_\pm d}\cos(2\beta_\pm d+\xi_{21\pm}+\xi_{23\pm})}\ , \tag{6}$$

$$\tan\xi_{ij\pm}=\frac{2(n_{i\pm}\kappa_{j\pm}-n_{j\pm}\kappa_{i\pm})}{n_{j\pm}^2-n_{i\pm}^2+\kappa_{j\pm}^2-\kappa_{i\pm}^2}\ , \tag{7}$$

and

$$t_{ij\pm}=\frac{2N_{i\pm}}{N_{j\pm}+N_{i\pm}}\equiv|t_{ij\pm}|e^{i\gamma_{ij\pm}}\ , \tag{8}$$

$$r_{ij\pm}=\frac{N_{j\pm}-N_{i\pm}}{N_{j\pm}+N_{i\pm}}\equiv|r_{ij\pm}|e^{i\xi_{ij\pm}} \tag{9}$$

are the Fresnel transmission and reflection coefficients, respectively, and where the complex phase shift due to medium 2, of thickness d, is written as

$$\delta_\pm\equiv\frac{\omega d}{c}(n_{2\pm}+i\kappa_{2\pm})\ ,$$

$$\equiv(\beta_\pm+i\alpha_\pm/2)d\ . \tag{10}$$

For the special case of the MOS system, since absorption is negligible in the oxide and semiconductor, we take $N_{1\pm}=n_o$ (oxide), $N_{2\pm}=n_\pm+i\kappa_\pm$ (inversion layer), and $N_{3\pm}=n_s$ (semiconductor). Making the further assumptions (a) $\kappa_\pm\ll n_\pm$ (weak absorption) and (b) $\omega d/c\ll 1$ (thin sample), Eqs. (8) and (9) become real so that θ_T does not contribute to Θ and Eqs. (6) and (7) reduce to

$$\eta_\pm=\frac{2r_{23\pm}r_{21\pm}\beta_\pm d}{1-r_{23\pm}r_{21\pm}}\ , \tag{11}$$

where by condition (b), we have used the small-angle approximations for the trigonometric functions.

Thus, from Eq. (4), the contribution to Θ due to multiple reflections is

$$\theta_{\rm MR}=\frac{\omega d(n_+-n_-)}{2c(n_o+n_s)}[(n_++n_-)-(n_o+n_s)]\ , \tag{12}$$

which in terms of the single-pass rotation

$$\theta=\frac{\omega d}{2c}(n_+-n_-) \tag{13}$$

becomes

$$\theta_{\rm MR}=\left[\frac{n_++n_-}{n_o+n_s}\right]\theta-\theta\ . \tag{14}$$

From Eq. (1), we obtain (recalling that $\theta_T=0$ here)

$$\Theta=\left[\frac{n_++n_-}{n_o+n_s}\right]\theta \tag{15}$$

for the multiple-pass Faraday rotation.

Using $\epsilon_\pm=n_\pm^2=\epsilon+i4\pi\sigma''_\pm/\omega$, where $\epsilon=n_s^2$, we may rewrite Eq. (15) in terms of the conductivity $\sigma_\pm$ as

$$\Theta=\frac{2\pi d}{c(n_o+n_s)}(\sigma''_--\sigma''_+)\ , \tag{16}$$

where the double prime denotes imaginary part.

Comparing Eq. (16) with Eq. (16) of Ref. 1, we conclude that we find agreement between the two-dimensional model of the inversion layer of Ref. 1 and the three-dimensional multiple-pass model if $\sigma_\pm d$ is identified as the two-dimensional conductivity.[6] Further, taking $n_\pm=n_s$ and using $n_o=1.95$ and $n_s=3.44$, Eq. (15) predicts an enhancement of 28% of the single-pass result.

Perhaps our most important conclusion is that the two-dimensional model of Chiu *et al.*[2] automatically takes into account multiple reflections within the inversion layer (resulting in a significant enhancement in the Faraday rotation, and similarly in the ellipticity). Whether such multiple reflections occur in practice is open to debate since the inversion layer is actually highly nonhomogeneous.[7] This nonhomogeneity is essentially averaged out by assuming a surface current, which is the essence of the Chiu *et al.* model. A definitive answer can only be obtained by carrying out the calculations using a nonhomogeneous three-dimensional model. Possible starting points are the models of Dahl and Sham[8] and of Equiluz and Maradudin.[9] These models consider a thin but finite layer and their possible application to Faraday rotation and ellipticity calculations is presently under study.

Note added in proof. Recently, we have discussed the optimum method for the determination of the parameters of the inversion layer (effective mass, collision frequency, and electron surface concentration) from the experimental data.[10] In addition, we have studied the effect of a finite semiconductor substrate (which is relevant when the semiconductor does not have a wedge to prevent multiple reflections from the semiconductor-vacuum interface) on the Faraday rotation and ellipticity in a MOS system.[11] Finally, we have presented a brief survey of magneto-optical experiments in the general area of two-dimensional systems in solid-state and surface physics.[12]

This research was partially supported by the U. S. Department of Energy, Division of Materials Science, under Contract No. DE-AS05-79ER10459.

---

[1] R. F. O'Connell and G. Wallace, Phys. Rev. B 26, 2231 (1982).

[2] K. W. Chiu, T. K. Lee, and J. J. Quinn, Surf. Sci. 58, 182 (1976).

[3] T. A. Kennedy, R. J. Wagner, B. D. McCombe, and J. J. Quinn, Solid State Commun. 18, 275 (1976).

[4] R. F. O'Connell and G. Wallace, Phys. Rev. B 25, 5527 (1982).

[5] R. F. O'Connell and G. Wallace, Can. J. Phys. 61, 49 (1983).

[6] R. F. O'Connell and G. Wallace, Physica 115B, 1 (1982).

[7] F. Stern and W. E. Howard, Phys. Rev. 163, 816 (1967).

[8] D. A. Dahl and L. J. Sham, Phys. Rev. B 16, 651 (1977).

[9] A. Eguiluz and A. A. Maradudin, Ann. Phys. (N.Y.) 113, 29 (1978).

[10] R. F. O'Connell and G. Wallace, J. Phys. Chem. Solids (in press).

[11] R. F. O'Connell and G. Wallace, Physica B (in press).

[12] R. F. O'Connell, J. Phys. (Paris) Colloq. C2, Suppl. 11, C2-81 (1982).