Low-temperature scaling for systems with random fields and anisotropies

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Random fields (or anisotropies) shift the lower critical dimensionality of spin systems from d_c^0 to d_c. For dimensionalities $d_c^0 < d < d_c$ at low temperatures T, the magnetization has a discontinuity (from zero to M) when Δ (the average square field) approaches zero. The correlation length ξ diverges as $\Delta^{-\nu_\Delta}$. The structure factor is shown to scale as $S(q,\xi)=\xi^d\bar{S}(q\xi)$. Simple assumptions on scaling near $T=0$ yield $\nu_\Delta=1/(d_c-d)$, with $d_c=4$ for continuous symmetry spins and $d_c=2$ for Ising spins.

The critical behavior of magnetic systems with random quenched fields, of the form $\sum_i \vec{H}_i\cdot\vec{S}_i$, with $[\vec{H}_i]_{av}=0$, $[\vec{H}_i\cdot\vec{H}_j]_{av}=\Delta\delta_{ij}$, has been the subject of much recent discussion.[1–5] Arguments based on diagrammatic expansions valid at dimensionalities $4<d<6$ showed that the leading singular behavior in d dimensions is exactly the same as that of the nonrandom system in $(d-2)$ dimensions.[3] However, this left the detailed behavior of the experimentally relevant cases $d<4$ unresolved. For Heisenberg-type systems (with $n\geqslant 2$ spin components), it is widely accepted that the lower critical dimensionality (below which there is not long-range ferromagnetic order) is shifted by the random fields from $d_c^0=2$ to $d_c=4$. Details of the behavior of thermodynamic functions at $d<4$ have not been calculated.

At low temperatures, random uniaxial anisotropies of the form $D\sum_i(\hat{n}_i\cdot\vec{S}_i)^p$, where $\hat{n}_i$ is a unit vector of random direction and $p\geqslant 2$,[6,7] and random off-diagonal exchange interactions of the form $\sum_{\alpha\beta}J_{ij}^{\alpha\beta}S_i^\alpha S_j^\beta$ (Ref. 8) generate local random fields which also destroy long-range order at $d<4$. We have recently shown[7,9] that, to leading order in D, one has a phase with an infinite susceptibility. However, the effects of higher orders in D remained unclear.

The situation for the Ising model ($n=1$) in a random field is even less clear. Domain arguments suggest that the lower critical dimension is shifted from $d_c^0=1$ to $d_c=2$.[2] This result is supported by a recent interface model,[4] but disagrees with earlier interface models[5] which gave $d_c=3$. The Ising model in a random field may be easily studied experimentally by applying a uniform field to dilute antiferromagnets.[10] Recent experiments[11] showed that such antiferromagnets exhibit modified properties and no long-range order at $d=2$, but left many quantitative details, especially concerning the structure form factor, unexplained.

In the present Rapid Communication we formulate a *low-temperature–scaling theory* for systems with random fields (or random anisotropies) *below their lower critical dimensionality*. We consider the behavior of the random field system in the limit of small T, h, and Δ, where T is the temperature (in units of J/k_B, J being the exchange coupling), h is a uniform magnetic field (in units of J), and $\Delta=[|\vec{H}_i|^2]_{av}/J^2$. For convenience, we refer to the ferromagnetic case.

For $d<d_c$, the zero-field spontaneous magnetization M is zero for all $\Delta>0$, while $M\neq 0$ for $\Delta=0$, $d>d_c^0$, and $T<T_c^0$, where T_c^0 is the ordering temperature of the nonrandom system. Thus there must occur for $d_c^0<d<d_c$ a *first-order transition* when $\Delta\to 0$ for $T<T_c^0$ at which the magnetization changes discontinuously from zero to M. In what follows we use known scaling properties near discontinuity transitions[12] to analyze this transition.

Since M is discontinuous as $\Delta\to 0$, it may be written as $M=M_0(1-\Delta^{\beta_\Delta})$ in the limit $\beta_\Delta=0$. We next consider the correlation function $\langle S_i^\mu S_j^\mu\rangle$. The correlation length ξ, associated with this function,[13] is infinite at $\Delta=0$, $T<T_c^0$. At finite Δ, ξ is related to the size of the domains, and is thus expected to diverge, e.g., as $\xi\propto\Delta^{-\nu_\Delta}$. If we assume the usual scaling form

$$[\langle S_i^\mu S_j^\mu\rangle]_{av}=r_{ij}^{-(d-2+\eta_\Delta)}f(r_{ij}/\xi)\quad ,$$

then the scaling relation $\beta_\Delta=\frac{1}{2}\nu_\Delta(d-2+\eta_\Delta)$ implies that $\eta_\Delta=2-d$. Fourier transforming $[\langle S_i^\mu S_j^\mu\rangle]_{av}$ we thus obtain the scaling form of the *structure factor*,[14]

$$S^{\mu\mu}(q,\xi)=\xi^d\bar{S}(q\xi)\quad . \tag{1}$$

This result is independent of the way in which ξ diverges. If one assumes for S a Lorentzian squared form, $S=A/(\kappa^2+q^2)^2$, as observed experimentally,[11] then by Eq. (1) A will be proportional to κ^{4-d}, where $\kappa=\xi^{-1}$.

Rescaling lengths by a factor b a discontinuity in M implies by the usual scaling relations that the corresponding ordering field h/T scales like b^d. From its definition, a discontinuity in the Edwards-Anderson order parameter[15] $Q=[\langle S_i\rangle^2]_{av}$ is expected to accompany that in M. This in turn implies[12] that its ordering field Δ/T^2 also scales like b^d.

For $n \geq 2$ the basic excitations at low T are spin waves. Simple dimensional counting shows[16] that T scales like b^{2-d}. Collecting these results gives the following low-temperature recursion relations,

$$T' = b^{2-d}T \;,\quad \left(\frac{h}{T}\right)' = b^d\left(\frac{h}{T}\right) \;,\quad \left(\frac{\Delta}{T^2}\right)' = b^d\left(\frac{\Delta}{T^2}\right) \;. \tag{2}$$

At $\Delta=0$, the ferromagnetic fixed point $T=h=0$ is stable for $d>2$ and unstable for $d<2$. This identifies $d_c^0=2$. Since Eq. (2) implies that $\Delta' = b^{4-d}\Delta$, this fixed point is unstable against $\Delta>0$ for $d<4$, i.e., $d_c=4$.

Using these recursion relations, we conclude that the singular term in the free energy density will be of the form

$$f(T,\Delta,h) = F_s/T = \xi^{-d}\bar{f}(T\xi^{2-d}, h\xi^2) \;, \tag{3}$$

where (for low temperatures) $\xi = \Delta^{-\nu_\Delta}$ with $\nu_\Delta = 1/(4-d)$, consistent with $d_c=4$. For $d=4$, $\xi = e^{1/\Delta}$. Equation (3) also follows directly from fixed length spin (spin-wave) renormalization-group calculations.[17,18] From Eq. (3) the susceptibility takes the form

$$\chi(T,h,\Delta) = \xi^2\bar{\chi}(T\xi^{2-d}, h\xi^2) \;. \tag{4}$$

When $T > \Delta^{(2-d)/(4-d)}$ we expect a crossover from the "random" behavior to the "thermal" behavior. This thermal behavior implies, e.g., $\xi = T^{1/(d-2)}$ for $d<2$. Similarly Eq. (1) may be written more generally as

$$S(\Delta,T,h,q) = \xi^d\bar{S}(q\xi, T\xi^{2-d}, h\xi^2) \;. \tag{5}$$

All the quantities of interest have been calculated explicitly in the limit $n\to\infty$.[1,19] In particular, it was found that the transverse spin structure factor is given by $T/(q^2+r)+\Delta/(q^2+r)^2$, where $r=h/M$ is the solution of the equation

$$r = (T-T_c^0) + AM^2 + BTr^{(d-2)/2} + C\Delta r^{(d-4)/2} \;.$$

It is easy to check that these expressions obey all our scaling relations. For example, an explicit calculation[19] of χ verifies Eq. (4) with $\bar{\chi}(0,y)=e^{-y^2}$.

It is interesting to note that if one fits S by[11]

$$S = A/(\kappa^2+q^2)^2 + B/(\kappa^2+q^2) \;,$$

then Eq. (5) implies that $A/B = \kappa^2 s(T\xi^{2-d})$. If this $s(x)$ is finite as $x\to 0$ then A/B is proportional to κ^2 as observed experimentally.[11] We hope this paper will stimulate detailed checks of these results.

At low temperatures, the random anisotropy coefficient $\Delta = (D/J)^2$ obeys exactly the same scaling as the random field, $\Delta' = b^{4-d}\Delta$.[16] We therefore predict the same scaling results for the two problems for $T < T_c^0$. In particular we expect the susceptibility of the random anisotropy problem to obey the scaling form (4). If the function $\bar{\chi}$ is finite at T, $h\to 0$, this implies a finite value of χ for $T<T_c^0$, $\chi \to \Delta^{-2/(4-d)}$. In $d=3$ the χ^{-1} intercept of the experimental Arrot plots should therefore be proportional to $(D/J)^4$. Such a finite value could result from terms of high order in Δ, neglected in Ref. 9. It is, of course, possible that $\bar{\chi}(x,y)$ diverges when $x\to 0$ or $y\to 0$, in which case one could retain the infinite susceptibility phase. However, experimental results[20] seem to favor a finite value of χ. It would be useful to have an explicit (experimental or theoretical) determination of the function $\bar{\chi}$ for this case. A linear specific heat has been observed in Dy-Cu at low temperatures.[21] From Eq. (3) it follows that if this term comes from the singular free energy F_s, then the coefficient of the linear term is proportional to $\xi^{2(1-d)}$ or in three dimensions to (D/J).[8]

We now turn to the Ising case $n=1$ in a random field. Both the interface model[22] and many real-space renormalization-group calculations[23] give $T'=b^{1-d}T$ for $T \ll T_c^0$, consistent with $d_c^0=1$. As for $n\geq 2$ we assume that both M and Q are discontinuous for $d_c^0 < d < d_c$ such that $(h/T)' = b^d(h/T)$ and $(\Delta/T^2) = b^d(\Delta/T^2)$. Repeating the same steps as above we obtain

$$f(T,\Delta,h) = \xi^{-d}\bar{f}(T\xi^{1-d}, h\xi) \;, \tag{6}$$

where $\xi = \Delta^{-\nu_\Delta}$ with $\nu_\Delta = 1/(d-2)$ for $d<2$, and $\xi = e^{1/\Delta}$ at $d=2$ as obtained from $\Delta' = b^{2-d}\Delta$. Given all the stated assumptions this yields $d_c=2$.

The corresponding form of the susceptibility is

$$\chi = \xi\,\bar{\chi}(T\xi^{1-d}, h\xi) \tag{7}$$

with $\bar{\chi}(0,0)$ expected to be finite and nonzero.

The result $\Delta' = b^{2-d}\Delta$ can be rigorously proven for $d<1$. In this case there is no long-range order for any $T>0$ and we may assume that the susceptibility is analytic in Δ. Writing

$$T\chi = b^d\chi(b^{1-d}T, b^{\lambda_\Delta}\Delta) \;,$$

we find that

$$\partial(T\chi)/\partial(\Delta/T^2)\,|_{\Delta=0} \propto b^{\lambda_\Delta+3d-2} \;.$$

On the other hand, one can show rigorously[24] that

$$\partial(T\chi)/\partial(\Delta/T^2)\,|_{\Delta=0} = -(T\chi)^2 \propto b^{2d} \;,$$

and thus $\lambda_\Delta = 2-d$. This proof probably breaks down for $d>1$, when one is probably not allowed to expand in Δ for $\Delta\to 0$.

The result $d_c=2$ is consistent with the domain arguments.[2,4] If one believes that $d_c=3$, i.e., that $\lambda_\Delta = 3-d$, then *some of the above assumptions* (e.g., that Q has a discontinuity, or that one may use T as the appropriate temperature scaling field) *must be in-*

valid and there must be no discontinuity in Q for $n=1$, in contrast to $n>1$. This question is left for future study. We exphasize, however, that the result (1) must still hold.

All the above results are expected to hold for $d_c^0 < d < d_c$ only for low temperatures, $T < T_c^0$. As T approaches T_c^0, we expect a crossover to the scaling behavior associated with T_c, e.g.,

$$\chi(t,\Delta) = |t|^{-\gamma}\tilde{\chi}(\Delta|t|^{-\phi}) \quad , \tag{8}$$

where $t=(T-T_c^0)/T_c^0$. In the random field case $\phi=\gamma$,[24] and thus $\chi \sim \Delta^{-1}$, $\xi \sim \Delta^{-\nu/\gamma}$, etc. In the random anisotropy case[25] $\phi = 2\phi_a - d_\nu$ ($\simeq 0.35$ at $d=3$), where ϕ_a is the spin anisotropy crossover exponent. The crossover from $\xi \sim \Delta^{-\nu/\phi}$ near T_c^0 to $\xi \sim \Delta^{-\nu_\Delta}$ for $T \ll T_c^0$ may complicate the analysis of the experiments.

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[1]P. Lacour-Gayet and G. Toulouse, J. Phys. (Paris) 35, 425 (1974).

[2]Y. Imry and S.-k. Ma, Phys. Rev. Lett. 35, 1399 (1975).

[3]G. Grinstein, Phys. Rev. Lett. 37, 944 (1976); A. Aharony, Y. Imry, and S.-k. Ma, *ibid.* 37, 1367 (1976); A. P. Young, J. Phys. C 10, L257 (1977); G. Parisi and N. Sourlas, Phys. Rev. Lett. 43, 744 (1979).

[4]G. Grinstein and S.-k. Ma, Phys. Rev. Lett. 49, 685 (1982); J. Villain, J. Phys. (Paris) Lett. 43, L551 (1982).

[5]E. Pytte, Y. Imry, and D. Mukamel, Phys. Rev. Lett. 46, 1173 (1981); D. Mukamel and E. Pytte, Phys. Rev. B 25, 4779 (1982); K. Binder, Y. Imry, and E. Pytte, *ibid.* 24, 6736 (1981); H. S. Kogon and D. J. Wallace, J. Phys. A 14, L527 (1981).

[6]R. A. Pelcovits, E. Pytte, and J. Rudnick, Phys. Rev. Lett. 40, 476 (1978).

[7]A. Aharony, J. Phys. C 14, L841 (1981).

[8]A. Aharony, Solid State Commun. 28, 667 (1978).

[9]A. Aharony and E. Pytte, Phys. Rev. Lett. 45, 1583 (1980).

[10]S. Fishman and A. Aharony, J. Phys. C 12, L729 (1979).

[11]H. Yoshizawa, R. A. Cowley, G. Shirane, R. J. Birgeneau, H. J. Guggenheim, and H. Ikeda, Phys. Rev. Lett. 48, 438 (1982).

[12]B. Nienhuis and M. Nauenberg, Phys. Rev. Lett. 35, 477 (1975); M. E. Fisher and A. N. Berker, Phys. Rev. B 26, 2507 (1982).

[13]Note that in this definition of the correlation function the long-range part $\langle S_i^\mu \rangle \langle S_j^\mu \rangle$ has not been subtracted off.

[14]Scaling of a structure factor near a first-order transition with the dth power of a characteristic length was recently suggested, in a dynamic context, by P. A. Rikvold and J. D. Gunton, Phys. Rev. Lett. 49, 286 (1982).

[15]For example, S. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975).

[16]For example, E. Brezin and J. Zinn-Justin, Phys. Rev. B 14, 3110 (1976).

[17]R. A. Pelcovits, Phys. Rev. B 19, 465 (1979).

[18]Young, Ref. 3.

[19]R. M. Hornreich and H. G. Schuster, Phys. Rev. B 26, 3929 (1982).

[20]S. von Molnar, B. Barbara, T. R. McGuire, and R. J. Gambino, J. Appl. Phys. 53, 1350 (1982); J. Appl. Phys. (in press); M. J. O'Shea, S. C. Cornelison, and D. J. Sellmyer, *ibid.* (in press).

[21]J. M. D. Coey, J. Phys. (Paris) Lett. 39, L327 (1978).

[22]D. J. Wallace and R. K. P. Zia, Phys. Rev. Lett. 43, 803 (1979).

[23]W. Klein, D. J. Wallace, and R. K. P. Zia, Phys. Rev. Lett. 37, 639 (1976).

[24]Y. Shapir and A. Aharony, J. Phys. C 14, L905 (1981).

[25]A. Aharony, Phys. Rev. B 12, 1038 (1975).