Critical fields of liquid superconducting metallic hydrogen

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Liquid metallic hydrogen, in a fully dissociated state, is predicted at certain densities to pass from dirty to clean and from type-II to type-I superconducting behavior as the temperature is lowered.

Hydrogen is likely to become a metal[1–5] at a density corresponding to $r_s \simeq 1.6$ [where for N hydrogen atoms in a volume V, $r_s = a_0^{-1}\,(3V/4\pi N)^{1/3}$]. The pressure required to achieve this state is in the megabar range.

It has been predicted[6–9] that crystalline monatomic forms of metallic hydrogen will be superconductors with high transition temperatures T_c ($\sim 10^2$ K). On the other hand, the recent work of Mon *et al.*[10] suggests that the melting points of monatomic phases can be quite low; for example, for $r_s \approx 1.6$, $T_m \sim 10^2$ K. On the basis of these observations, it is apparent that metallic hydrogen might become a liquid superconductor under suitable conditions, a possibility that has been pursued in some depth (Ref. 11, hereafter referred to as I). In particular, T_c has been computed as a function of density, the maximum value (142 K) occurring at $r_s = 1.36$.

In liquid metallic hydrogen, for $T \sim 10^2$ K, the prevailing physical conditions are such that the quantum statistics of the ions (in this case protons) are important. If off-diagonal long-range order is present in the electron gas, then the system under consideration becomes a superconducting metallic Fermi liquid. This material is likely to have unusual properties, particularly magnetic properties. In what follows we discuss the critical magnetic fields of superconducting liquid metallic hydrogen by means of a generalized Ginzburg-Landau (GL) theory.[12] This system appears to be a strong-coupling superconductor and near T_c is close to what is known as the dirty limit.[13]

In its early form, GL theory was limited to weak-coupling superconductors for temperatures close to T_c. The theory predicts the thermodynamic critical field $H_c(T)$ to be given by

$$H_c(T) = [4\pi N(\epsilon_F)]^{1/2}(1.76 k_B T_c)2(1-T/T_c) \quad , \tag{1}$$

and the magnetic field penetration depth to be given by

$$\xi^{-1}(T) = \frac{4\pi n_e e^2}{mc^2}2\left(1-\frac{T}{T_c}\right)\chi[\tilde{\tau}^{-1}] \quad , \tag{2}$$

where $\tilde{\tau}^{-1} = \tau_{\mathrm{Tr}}^{-1}\hbar/2k_B T_c$. In these expressions $N(\epsilon_F)$ is the density of states at the Fermi energy. The quantity τ_{Tr} is a characteristic relaxation time for transport processes (such as normal conductivity); the function $\chi[\ ]$ is discussed in some detail by Werthamer.[13] The Ginzburg-Landau parameter κ follows from Eqs. (1) and (2):

$$\kappa = \frac{e}{hc}\frac{1}{\sqrt{2}}H_c(T)\xi^2(T) \quad . \tag{3}$$

As long as τ_{Tr} is essentially independent of temperature, then κ is also. This is usually the case with ordinary superconductors where the scattering is mostly attributed to impurities or other common defects (at the temperatures important for superconductors). When $\kappa < 1/\sqrt{2}$ the material is classified as type I. If $\kappa > 1/\sqrt{2}$, it is type II; and in this case the upper and lower critical fields are given by

$$H_{c2}(T) = \sqrt{2}\kappa H_c(T) \quad , \tag{4}$$

and, for $\kappa >> 1$,

$$H_{c1}(T) = \frac{\ln\kappa}{\sqrt{2}\kappa}H_c(T) \quad . \tag{5}$$

The essential physical point we will make here is that the transport time in normal liquid metallic hydrogen has a strong temperature dependence, and, as will be seen below, this has an important bearing on the corresponding magnetic behavior of the superconducting state.

Various efforts have been made to extend the GL theory to temperatures well below T_c; these are reviewed in Ref. 12. Generally, it is possible to continue using relations similar to (1)–(5) provided we replace κ by $\kappa_1(T)$ in (4) and $\kappa_3(T)$ in (5). Here κ_1 and κ_3 are smooth functions which equal κ at $T = T_c$ and increase as T decreases. For example, for a small τ_{Tr} material in the dirty limit we have $\kappa_1/\kappa \simeq \kappa_3/\kappa \simeq 1.2$ at $T=0$.

The upper critical field of a dirty *strong-coupling* superconductor has been analyzed by Rainer and Bergmann.[14] Their analysis shows that strong-coupling effects enter in two ways. First, the electron mass m [and hence the density of states $N(\epsilon_F)$] is scaled by a renormalization factor $(1+\lambda)$ where λ is the effective phonon-mediated pairing attraction between electrons. Second, there is an additional overall factor multiplying H_{c2}, which turns out to be close to unity in, for example, amorphous superconductors. Since liquid metallic hydrogen possesses an Eliashberg function rather close to the corresponding function for an amorphous system,[11] it is reasonable to take this factor to be unity here as well. On the other hand, strong-coupling corrections to H_{c1} are not presently well understood; accordingly, we will continue to use (5) when $\kappa_3(T) \gg 1$ and a simple interpolation for smaller κ. It is important to note that as a consequence of these assumptions the results to be discussed below will be more reliable for $H_{c2}(T)$ than for $H_{c1}(T)$.

Using T_c and λ (as given by Ref. 11), we can compute κ and the critical fields provided, however, that τ_{Tr} is known. The scattering *rate* τ_{Tr}^{-1} can be calculated by a method similar to the one we used earlier to determine the Eliashberg function.[11] The idea is due to Baym[15]: All phonon-related quantities of interest are expressed in terms of the dynamic structure factor [or equivalently, by the fluctuation-dissipation theorem, the imaginary part of the dynamic response function, $\chi''(q,\omega)$]. The dynamic structure factor $S(q,\omega)$ is well defined for the liquid states of metallic hydrogen that we are considering, and we assume that the expressions which involve χ'' remain the same.[11] From the Born approximation, we find

$$\tau_{\mathrm{Tr}}^{-1}=2\pi N(\epsilon_F)\int_0^{2k_F}\frac{dq\,q}{2k_F^2}\left[\frac{q^2}{2k_F^2}\right]|v_{ei}(q)|^2\int_0^{\infty}d\omega\beta\hbar\omega\,\mathrm{sech}^2\left[\frac{\beta\hbar\omega}{2}\right]\chi''(q,\omega)\ , \tag{6}$$

where $v_{ei}(q)$ is the Fourier transform of the screened electron-proton interaction. At high temperatures it is easily seen that (6) reduces to the weak scattering result[15]

$$\tau_{\mathrm{Tr}}^{-1}=\frac{n_i m k_F}{\pi h^2}\int_0^{2k_F}\frac{dq\,q}{2k_F^2}\left[\frac{q^2}{2k_F^2}\right]|v_{ei}(q)|^2 S(q)\ ,$$

where $S(q)$ is the static structure factor.

Equation (6) is easily evaluated using the approximation for $v_{ei}(q)$ and $\chi''(q,\omega)$ discussed in I. As can be seen in Fig. 1, we find τ_{Tr}^{-1} to have a rather pronounced temperature dependence: it drops very rapidly as temperature decreases, an effect that can be understood as a direct consequence of the quantum statistics of the protons (which are manifested in χ''). Specifically, the Pauli principle reduces the phase space available for protons to absorb momentum from the electrons; some of the final states are blocked, the effect becoming most pronounced when $T \ll T_F$.

As a consequence of the temperature dependence of τ_{Tr}^{-1}, a remarkable temperature dependence of κ results, as also shown in Fig. 1. This is in addition to the much weaker temperature dependence ordinarily associated with the modified Ginzburg-Landau parameters $\kappa_1(T)$ and $\kappa_3(T)$. The temperature dependence of κ gives, in turn, a quite unusual tem-

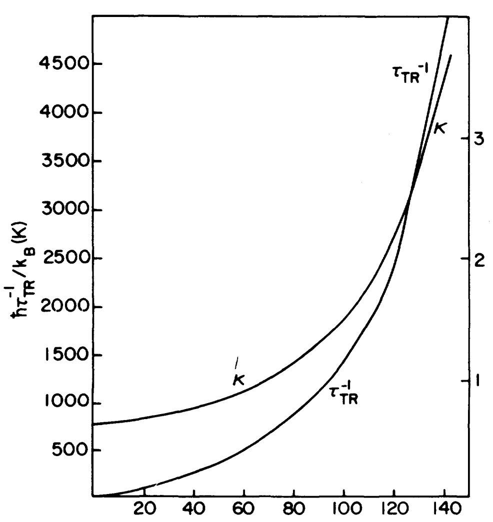

FIG. 1. Scattering rate and Ginzburg-Landau parameter of superconducting liquid metallic hydrogen when $r_s=1.36$.

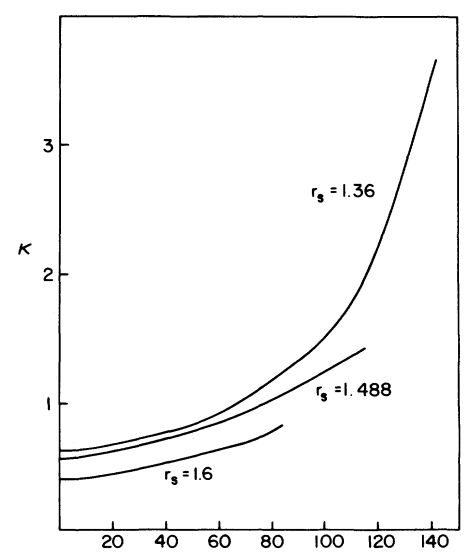

FIG. 2. Ginzburg-Landau coefficients of superconducting liquid metallic hydrogen at three densities.

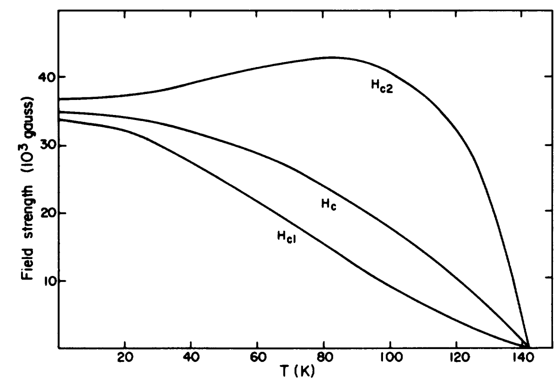

FIG. 3. Critical fields of superconducting liquid metallic hydrogen as functions of temperature for $r_s = 1.36$.

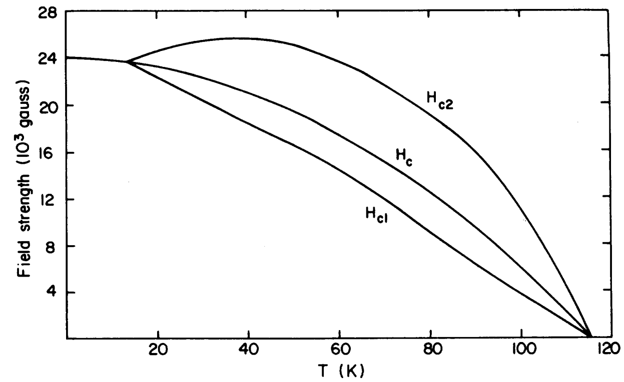

FIG. 4. Critical fields of superconducting liquid metallic hydrogen for $r_s = 1.488$.

perature dependence to the critical fields. The system can be qualitatively described as a dirty superconductor near T_c which gradually becomes "clean" as the temperature is reduced. Since the Ginzburg-Landau parameter then *decreases* with temperature, it is necessary that H_{c2} passes through a maximum value as a function of temperature. This leads to the interesting physical conclusion that the system may pass from type-II to type-I behavior as T decreases.

The temperature dependence of κ is shown in Fig. 2, and the corresponding critical fields are shown in Fig. 3, for density corresponding to $r_s = 1.36$ and in Fig. 4 for $r_s = 1.488$. At the higher density no transition to type-I behavior is observed even though κ drops below $1/\sqrt{2}$ (because κ_1 and κ_3 are both somewhat greater than this value). For $r_s = 1.488$ a transition between type-I and type-II behavior evidently is predicted and might provide a probe for discerning the existence of liquid superconductive states in closed geometries, where macroscopic fluid flow will be difficult to detect.

This kind of magnetic behavior can be contrasted with that of certain crystalline superconductors which make a transition[16,17] from type I to type II with decreasing T. In these materials the electron transport relaxation rate is dominated (at temperatures relevant to superconductivity) by impurity scattering, with the result that τ_{Tr}^{-1} is independent of temperature. However, in samples where κ is slightly less than $1/\sqrt{2}$ at $T = T_c$, the gradual increase in the ratio $\kappa_1(T)/\kappa$ with decreasing T may be sufficient to cause $H_1(T)$ to exceed the value $1/\sqrt{2}$ as T drops below some value T^*. In such cases the material is type I when $T^* < T < T_c$ and type II when $0 < T < T^*$. In liquid metallic hydrogen, however, we predict that the role of impurities is taken over by *excitations* of the proton system, and the number density of these drops sharply with decreasing T. This results in the strong intrinsic temperature dependence of κ itself (in addition to κ_1) and should lead to a type-II to type-I transition in the opposite direction (with respect to temperature change) from that found experimentally in some ordinary superconductors. The observations presented here should also apply, though possibly at different temperatures, to liquid metallic deuterium,[18] provided the deuterons are not themselves ordered.

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