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Low-temperature renormalization-group study of the random-field model

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The continuous-spin random-field model is investigated by means of the low-temperature renormalization-group technique with the use of the replica trick. The Wilson-Kogut recursion method is applied. For short-range exchange, the results are in exact agreement with those of the random-axis model studied by Pelcovits. For long-range exchange varying with distance R as $R^{-(d+\sigma)}$, critical exponents are calculated to first order in $d-2\sigma$. They are identical to those in a $d-\sigma$ expansion of the nonrandom model. However, the hyperscaling law becomes $(d+\lambda_T)\nu=2-\alpha$ (λ_T is the eigenvalue associated with the dangerous irrelevant operator T), and, for m-component spins, $-\lambda_T=\sigma+(d-2\sigma)/m$.

I. INTRODUCTION

Imry and Ma[1] first discussed the effect of quenched random magnetic fields on the ordered phase of ferromagnets with short-range exchange interactions. They used renormalization-group arguments to show that the classical mean-field-like behavior is found at dimension d above $d_c=6$, instead of $d_c=4$ for pure systems. Arguing heuristically, they concluded that these systems with $m\geq 2$ (m is the number of spin components) have no long-range order for $d<d_d=4$ as compared with $d_d=2$ for pure systems. Shuster[2] reached the same conclusion using the replica method. Grinstein[3] has shown the hyperscaling relation $d\nu=2-\alpha$ (for ordered systems) becomes $(d+\lambda_u)\nu=2-\alpha$, where λ_u is negative and is related to the range of the ferromagnetic exchange interactions. For a short-range exchange interaction, $\lambda_u=-2+O(\epsilon^3)$ ($\epsilon=6-d$). If the exchange forces are long range, varying with distance R as $R^{-(d+\sigma)}$, then $\lambda_u=-\sigma\ +O(\epsilon^3)$ ($\epsilon=3\sigma-d$). Earlier, Lacour-Gayet and Toulouse[4] studied the ideal Bose gas in the presence of a random-source term and computed exact critical exponents for Bose condensation. They also found violations of the familiar scaling laws relating critical exponents. Aharony *et al.*[5] and Young[6] used direct perturbation methods to verify that the critical exponents of a phase transition in a d-dimensional ($4<d<6$) system with short-range exchange and a random quenched field are the same as those of a $(d-2)$-dimensional pure system. Parisi and Sourlas[7] reached the same conclusion by using a geometrical interpretation which stems from a hidden supersymmetry of the associated stochastic equation.

All of the calculations mentioned above are around the upper critical dimensionalities $d_c=6$ and 3σ for the short-range and long-range exchange interactions, respectively. Since the lower critical dimensionality is $d_d=4$ (short-range case) or $d_d=2\sigma$ (long-range case), it is expected that similar analyses can be made in $d-4$ or $d-2\sigma$ expansion.

Recently Pelcovits *et al.*[8,9] have studied the random-axis model (RAM) using the low-temperature renormalization-group method.[10–13] The critical exponents are calculated to first order in $\epsilon=d-4$; they are in exact agreement with those of the pure case where $\epsilon=d-2$. However, hyperscaling does not hold for the RAM unless one incorporates the dimensionality shift by two; it becomes $(d-2)\nu=2-\alpha$ up to $O(\epsilon^2)$. This agrees with Grinstein's predictions which were based on arguments at d near the upper critical dimensionality.

In this paper, we present details of low-temperature renormalization-group calculations for the random-field model (RFM). The replica method is used. We first discuss the case of short-range exchange near $d=4$. The recursion relations are derived; they and therefore their exponents are exactly the same as those in Ref. 9 which were derived by a different method for the RAM. We then discuss long-range exchange near $d-2\sigma$. The critical exponents ν and η are calculated to be $1/(d-2\sigma)+O(1)$ and $2-\sigma+O((d-2\sigma)^2)$, respectively. They are identical to $(d-\sigma)$ expansions in the pure system.[11,13] The hyperscaling law is checked to be $(d+\lambda_T)\nu=2-\alpha$, where λ_T is the eigenvalue associated with the "dangerous irrelevant variable" T.[14] We find $\lambda_T=-\sigma+(2\sigma-d)/m$.

The paper is organized as follows. In Sec. II the replica method is briefly described. The low-temperature "effective" Hamiltonian is obtained. In Sec. III we apply the Wilson-Kogut[15] renormalization-group method to this effective Hamiltonian. The critical behavior for long-range exchange is discussed in Sec. IV.

II. REPLICA METHOD

Generally there are two methods to treat quenched random systems. The Hamiltonian is

$$H=-\sum_{\langle ij\rangle}J_{ij}\vec{S}_i\cdot\vec{S}_j-\sum_i\vec{H}_i\cdot\vec{S}_i\ . \tag{2.1}$$

Here $\vec{S}_i$ is an m-component vector spin of unit length and $\vec{H}_i$ is the random field.

The first method is to perform renormalization-group transformations directly on the probability distribution. Recursion relations are obtained. Then the critical exponents can be calculated. Pelcovits has applied this method to the RAM in a low-temperature renormalization-group study. The Hamiltonian is

$$H=-\sum_{\langle ij\rangle}J_{ij}\vec{S}_i\cdot\vec{S}_j-D\sum_i(\hat{x}_i\cdot\vec{S}_i)^2\ , \tag{2.2}$$

where $\hat{x}_i$ is a random direction at site i.

The second method is to transform the quenched random problem to a translationally invariant one using the $n\to0$ replica trick.[16] Renormalization-group procedures are carried out on the replica Hamiltonian and the $n\to0$ limit taken at the end. For the RFM we replicate the Hamiltonian of Eq. (2.1) and write the free energy as

$$-\beta F=\frac{1}{n}\sum_{\{\vec{S}\}}e^{-\beta\sum_\alpha H^\alpha}\Big|_{n=0}\ . \tag{2.3}$$

Here $\alpha=1,\ldots,n$ is the replica index and $\vec{S}$ denotes the set $\{\vec{S}^\alpha\}$. The next step is to average F over the random field $\vec{H}_i$ which occurs in each Hamiltonian H^α. We assume the H_i to be independent and Gaussian-distributed with $\overline{H_i}=0$, $\overline{H_i^2}=\Delta$. We obtain

$$-\beta\bar{F}=\frac{1}{n}\int d\{\vec{S}\}e^{-\beta H}\Big|_{n=0}\ , \tag{2.4}$$

where

$$H=-\sum_\alpha\sum_{\langle ij\rangle}J_{ij}\vec{S}_i^\alpha\cdot\vec{S}_j^\alpha-\beta\Delta/2\sum_{\alpha,\beta}\sum_i\vec{S}_i^\alpha\cdot\vec{S}_i^\beta\ , \tag{2.5}$$

which is translationally invariant. Here $J_{ij}\propto|R_{ij}|^{-(d+\sigma)}$, $\sigma<2$ if it is long ranged.

We turn now to the low-temperature expansion. From Eq. (2.5) we can see that the effective Hamiltonian is exactly the same as the fixed length spin $O(nm)$ (nonlinear σ model) model[10] in the pure case, except for the presence of the off-diagonal local scalar product of the second term. Thus we may use the standard techniques already developed for the pure case for both short-range[10] and long-range[11,13] J_{ij}: We write $\vec{S}_i^\alpha=(\vec{\sigma}_i^\alpha,\vec{\pi}_i^\alpha)$ where $\vec{\pi}_i^\alpha$ has $m-1$ components and $\vec{\sigma}_i^\alpha$ is in the direction of spontaneous magnetization. Since $(\sigma_i^\alpha)^2+\vec{\pi}_i^\alpha\cdot\vec{\pi}_i^\alpha=1$ we may integrate the σ_i^α in Eq. (2.4) and then have to analyze the following functional integral:

$$-\beta\bar{F}=\int d\{\vec{\pi}_i^\alpha\}e^{-\beta\mathscr{H}}\ , \tag{2.6}$$

$$\begin{aligned}\mathscr{H}=\sum_\mu\sum_{\alpha,\beta}\int\frac{d^dk}{(2\pi)^d}\Bigg[&\frac{k^\sigma}{2T}\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})+\frac{k^\sigma}{8T}\pi^{\alpha^2}(\vec{k})\pi^{\alpha^2}(-\vec{k})+\cdots\\&-\frac{\Delta}{2T^2}[\pi_\mu^\alpha(\vec{k})\pi_\mu^\beta(-\vec{k})+\tfrac{1}{4}\pi^{\alpha^2}(\vec{k})\pi^{\beta^2}(-\vec{k})+\cdots]+\rho T[\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})+\cdots]\Bigg]\ ,\end{aligned} \tag{2.7}$$

$$\pi^{\alpha^2}(\vec{k})=\int[\pi^\alpha(\vec{x})]^2e^{-i\vec{k}\cdot\vec{x}}d^dx=\sum_\mu^{m-1}\int\frac{d^dp}{(2\pi)^d}\pi_\mu^\alpha(\vec{p})\pi_\mu^\alpha(\vec{k}-\vec{p})\ , \tag{2.8}$$

where μ are the $m-1$ (transverse) spin components, α,β are replica indices, and

$$\rho=\frac{1}{(2\pi)^d}\int_0^1d^dq=K_d\int_0^1q^{d-1}dq=\frac{K_d}{d}\ ,$$
$$K_d=\frac{2^{-d+1}\pi^{-d/2}}{\Gamma(d/2)} \tag{2.9}$$

We can make a standard loopwise expansion of Eq. (2.5). The corresponding Feynman diagrams involve the propagators[11,13]

$$G_{\mu\mu'}^{\alpha\beta}=\delta_{\alpha\beta}\delta_{\mu\mu'}T/q^\sigma\ , \tag{2.10}$$

where $\sigma=2$ for short-range J_{ij}.[10]

III. RECURSION RELATIONS

In this section only the short-range interaction is considered. The long-range interaction will be discussed in the next section. In the actual calculation, all the terms except the first one in Eq. (2.7) are treated as perturbations. The vertices relevant to the perturbation expansions are shown in Fig. 1. It is noted that the wavy line contributes a factor k^2/T, while the solid line denotes T/k^2.

The standard Wilson-Kogut recursion method is used here. We decompose the Fourier-transformed spin field $\vec{\pi}(\vec{k})$,

$$\vec{\pi}(\vec{k})=\begin{cases}\vec{\pi}_<(\vec{k}), & 0<|\vec{k}|<1/b\\ \vec{\pi}_>(\vec{k}), & 1/b<|\vec{k}|<1\end{cases} \tag{3.1}$$

and integrate out $\vec{\pi}_>(\vec{k})$. Upon rescaling the momenta by b and the spins $\vec{\pi}_<(\vec{k})$ by ζ, we obtain a Hamiltonian of the form (2.7) with a new temperature prefactor. We need three recursion relations involving T and Δ to identify the fixed points and calculate the critical exponents. We first consider $(1/T)k^2\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})$. It is easy to see that there are three Feynman diagrams in Fig. 2 that might contribute. The first diagram does not contribute in the $n\to 0$ limit because of the presence of the loop which carries a factor of n. The expression for the second is

$$\int_{1/b<|p|<1} d^dp\,\frac{(\vec{k}-\vec{p})^2}{p^2}\ . \tag{3.2}$$

The coefficient of k^2 is

$$K_d\frac{1}{d-2}[1-(1/b)^{d-2}]$$
$$=K_d[\ln b-\tfrac{1}{2}(d-2)\ln^2 b+\cdots]\ .$$

In calculating the differential recursion relations

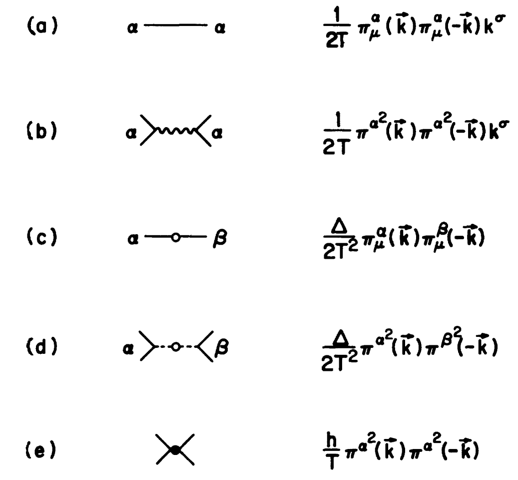

FIG. 1. Vertices relevant to the perturbation expansions in Eq. (2.6).

($b\gtrsim 1$) we only need $K_d\ln b$. For the third diagram it is

$$K_d\frac{1}{d-4}[1-(1/b)^{d-4}]$$

because the denominator is p^4. Similarly only $K_d\ln b$ is necessary. Therefore we have the first recursion relation,

$$\frac{1}{T'}=\zeta^2 b^{-d-2}\left[\frac{1}{T}+K_d\ln b+\frac{\Delta}{T}K_d\ln b\right]\ . \tag{3.3}$$

To determine the spin-rescaling factor ζ, we can either consider $(1/T)k^2\vec{\pi}^{\alpha^2}(\vec{k})\vec{\pi}^{\alpha^2}(-\vec{k})$ or following Nelson and Pelcovits[12] (NP) add a magnetic field h to $H_{\rm eff}$,

$$H_{\rm eff}\to H_{\rm eff}+\frac{h}{T}\sum_\alpha\int\sigma^\alpha(\vec{x})d^dx=H_{\rm eff}+\sum_\alpha\frac{h}{T}\int d^dx\{1-\tfrac{1}{2}[\vec{\pi}^\alpha(\vec{x})]^2-\tfrac{1}{8}[\vec{\pi}^\alpha(\vec{x})]^4\cdots\}\ . \tag{3.4}$$

The quadratic term $(h/T)\int d^dx[\vec{\pi}^\alpha(\vec{x})]^2$ can be absorbed into the propagator. $(h/T)\int d^dx[\vec{\pi}^\alpha(\vec{x})]^4$ is treated as a perturbation. We consider the recursion relation for $(h/T)\vec{\pi}^2$. Nelson and Pelcovits[12] and Pelcovits[9] have determined it in the ordered system and the RAM, respectively. The Feynman diagrams involving randomness are shown in Fig. 3. Note that Fig. 3(c) does not contribute in the limit $n\to 0$ since it has a closed loop. The recursion relation for $(h/T)\vec{\pi}^{\alpha^2}$ is

$$-\frac{h'}{2T'}=\zeta^2 b^{-d}\left[-\frac{h}{2T}-\frac{h}{4}\frac{(m-1)K_d\ln b}{1+h}-\frac{h}{4}\frac{\Delta}{T}\frac{(m+1)K_d\ln b}{(1+h)^2}-\frac{\Delta}{2T}\frac{K_d b}{(1+h)^2}+\frac{\Delta}{2T}\frac{K_d\ln b}{(1+h)}\right]$$
$$=\zeta^2 b^{-d}\left[-\frac{h}{2T}-\frac{h}{4T}\frac{(m-1)[\Delta+T(1+h)]K_d\ln b}{(1+h)^2}\right]\ . \tag{3.5}$$

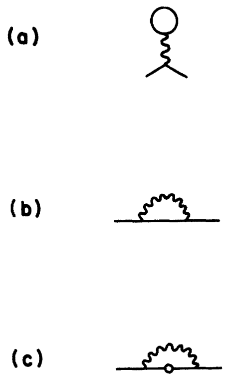

FIG. 2. Feynman diagrams contributing to the recursion relation for the two-point function $(1/2T)\pi_\mu^\alpha(\vec{k})\times\pi_\mu^\alpha(-\vec{k})k^\sigma$.

By rotational symmetry (which is preserved in the configurational averaged system) the magnetic field renormalizes trivially as

$$h'/T'=\zeta h/T\ . \tag{3.6}$$

From Eqs. (3.5) and (3.6), we obtain ($h\to 0$)

$$\zeta=b^d[1-\tfrac{1}{2}(m-1)(T+\Delta)(K_d\ln b)]\ . \tag{3.7}$$

The third recursion relation we need is for $(\Delta/2T^2)\pi^\alpha\pi^\beta(\alpha\neq\beta)$. The relevant Feynman diagrams are shown in Fig 4. Figure 4(a) goes to zero in the limit $n\to 0$. We therefore obtain

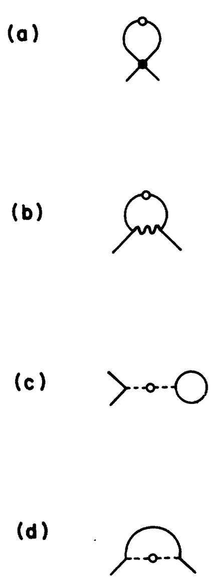

FIG. 3. Feynman diagrams contributing to the recursion relation for $(h/T)\vec{\pi}^{\alpha^2}$.

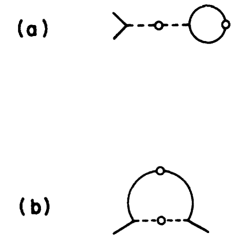

FIG. 4. Feynman diagrams contributing to the recursion relation for $(\Delta/2T^2)\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})$ $(\alpha\neq\beta)$.

$$\frac{\Delta'}{T'^2}=\zeta^2 b^{-d}\left[\frac{\Delta}{T^2}+\frac{\Delta^2}{T^2}K_d\ln b\right]\ . \tag{3.8}$$

It turns out the three recursion relations, Eqs. (3.3), (3.7), and (3.8) are in exact agreement with Eqs. (2.10), (2.15), and (2.17) in Ref. 9 for the RAM, where the critical behavior is discussed in detail. Therefore it is not repeated here.

We make one remark about the replica method in the nonlinear σ model. In Eq. (2.7) there are an infinite number of relevant and marginal operators with the coefficients $1/T$ and Δ/T^2, respectively. A natural question arises whether they renormalize consistently.[17] We have checked $(\Delta/2T^2)\pi^\alpha\pi^\beta$. Indeed for $\alpha=\beta$ and $\alpha\neq\beta$ they have the same $\Delta'/2T'^2$. It is essentially impossible in the present formulation to check all orders in π^2. This point needs further investigation.[18]

IV. LONG-RANGE INTERACTION

In this section we discuss the critical exponents and scaling laws in the presence of a long-range exchange coupling which dies off as $R^{-d-\sigma}$ in position space. As mentioned earlier, $\vec{k}^2$ in $H_{\rm eff}$ for the short-range case is replaced by k^σ if the interaction is long ranged. Therefore in Fig. 1 the wavy line represents a factor $(1/T)k^\sigma$, while the solid line denotes the propagator T/k^σ.

Following exactly the same procedures as in Sec. III, we consider the recursion relations for

$$(1/T)k^\sigma\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})\ ,$$

$$(h/T)\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})\ ,$$

and

$$(\Delta/2T^2)\pi_\mu^\alpha(\vec{k})\pi_\mu^\beta(-\vec{k})\ .$$

The recursion relation for T is obtained as usual from propagator renormalization as in Fig. 2. Figure 2(a) gives zero again. The expression for Fig. 2(b) becomes

$$\int_{1/b<|p|<1} \frac{d^d p}{(2\pi)^d} \frac{|\vec{k}-\vec{p}|^\sigma}{p^\sigma} . \tag{4.1}$$

Since the expression for Fig. 2(b) depends on k analytically it has an expansion in k^2 and has no k^σ term. The same is true for Fig. 2(c). Thus the diagrams in Fig. 2 do not contribute. We have

$$\frac{1}{T'}=\zeta^2 b^{-d-\sigma}\frac{1}{T} . \tag{4.2}$$

The second and the third relations are exactly the same as those of the short-range case, since

$$(h/T)\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})$$

and

$$(\Delta/T^2)\pi_\mu^\alpha(\vec{k})\pi_\mu^\beta(-\vec{k})$$

do not contain any factors k^σ and only $K_d \ln b$ is needed for the Feynman integral evaluated at arbitrary dimensionality. From Eqs. (4.2), (3.7), and (3.8), we can obtain the following differential recursion relations ($e^l=b$):

$$\frac{d\Delta}{dl}=(-d+2\sigma)\Delta+K_d[m\Delta+(m-1)T]\Delta , \tag{4.3}$$

$$\frac{dT}{dl}=(-d+\sigma)T+K_d(m-1)T(T+\Delta) . \tag{4.4}$$

From these equations we can identify four sets of fixed points,

$$T^*=0,\ \ \Delta^*=-\frac{2\sigma-d}{mK_d} , \tag{4.5}$$

$$T^*=\frac{d-\sigma}{K_d(m-1)} ,\ \ \Delta^*=0 , \tag{4.6}$$

$$T^*=0,\ \ \Delta^*=0 , \tag{4.7}$$

$$T^*=\frac{\sigma(m-1)+d-\sigma}{K_d(m-1)},\ \ \Delta^*=-\frac{\sigma}{K_d} .$$

The third and fourth sets of fixed points are unphysical. The second set (for the pure case) has been discussed before.[11,13] Here we consider the first set of fixed points.

By linearizing about $T^*=0$,

$$\Delta^*=-(2\sigma-d)/mK_d ,$$

and we easily obtain ($d>2\sigma$)

$$\lambda_\Delta=d-2\sigma+O((d-2\sigma)^2)\ \ \text{(relevant)} \tag{4.8}$$

$$\lambda_T=-\sigma+(d-2\sigma)(-1/m)+O((d-2\sigma)^2)\ \ \text{(irrelevant)} . \tag{4.9}$$

Just as in the pure case[13] the line separating the domains of attraction of the long- and short-range fixed points is given by $\sigma=2-\eta_{SR}$. On this line, λ_T given here is equal to that of the short-range case [Eq. (3.4) of Ref. 9].

From Eq. (4.8) we have

$$\nu=\frac{1}{\lambda_\Delta}=\frac{1}{d-2\sigma}+O(1) . \tag{4.10}$$

The determination of η requires some care because of the "dangerous irrelevant variable," namely the temperature.[9] The scaling law near the fixed points in Eq. (4.5) for the connected correlation function in momentum space is

$$\begin{aligned} G_c(k,T,\Delta^*)&=\zeta^2 b^{-d}G_c(bk,\delta T b^{\lambda_T},\Delta^*)=b^d(1-(m-1)(T^*+\Delta^*)K_d\ln b)G_c(bk,Tb^{\lambda_T},\Delta^*)\\ &=b^{[d+(m-1)(2\sigma-d)/m]}G_c(bk,Tb^{\lambda_T},\Delta^*) . \end{aligned} \tag{4.11}$$

Here $\delta T=T-T^*=T-0=T$. It is trivial to see $G_c(1,T',\Delta^*)\approx T'$ as $T'\to 0$. By setting $bk=1$ in Eq. (4.11) and using Eq. (4.9), we obtain the scaling behavior of $G_c(k,T,\Delta^*)$ as follows:

$$\begin{aligned} G_c(k,T,\Delta^*)&=k^{-d-(1-1/m)(2\sigma-d)}G_c(1,Tk^{-\lambda_T},\Delta^*)\\ &\sim k^{-d-(1-1/m)(2\sigma-d)-\lambda_T}\\ &\sim k^{-\sigma}\\ &\sim k^{-2+\eta} \end{aligned} \tag{4.12}$$

Therefore we have

$$\eta=2-\sigma+O((d-2\sigma)^2) . \tag{4.13}$$

Young[6] has shown in ϕ^4 theory that ϵ expansion ($\epsilon=3\sigma-d$) for the RFM is different from ϵ expansion ($\epsilon=2\sigma-d$) for the pure system at order ϵ^2. Here in the nonlinear σ model, we only show that to the lowest order in ϵ ($\epsilon=d-2\sigma$ for the RFM and $\epsilon=d-\sigma$ for the pure system) they are the same. However, there is no reason to expect them to remain identical at order ϵ^2.

Now we check the hyperscaling law by studying the singular part of the averaged free energy $F(T,\Delta_r)$ where $\Delta_r=\Delta-\Delta^*$. We follow exactly the steps in Refs. 3 and 9. We have

$$F(T,\Delta_r)=b^{-d}F(Tb^{\lambda_T},\Delta_r b^{1/\nu}) , \tag{4.14}$$

where λ_T and $1/\nu$ are given by Eqs. (4.9) and (4.10), respectively. Choosing $\Delta_r b^{1/\nu}=1$, we obtain

$$F(T,\Delta_r)=\Delta_r^{d\nu}F(T\Delta_r^{-\lambda_T\nu},1) .$$

We check the behavior of $F(T',1)$ as $T'\rightarrow 0$. It is not difficult to see that Fig. 5 contributes the factor

$$\left[\frac{1}{T'}\right]\left[\frac{1}{T'^2}\right]^2 T'^4=\frac{1}{T'} .$$

The first factor $1/T'$ comes from the four-point interaction in Fig. 2(b), $(1/T'^2)^2$ is due to the two vertices of Fig. 1(c) ($\Delta=1$), and four propagators contribute T'^4. Therefore

$$F(T,\Delta_r)\sim\Delta_r^{(d+\lambda_T)\nu}f(T) ,$$

where $f(0)$ is a finite constant. Thus the hyperscaling law becomes

$$(d+\lambda_T)\nu=2-\alpha ,$$

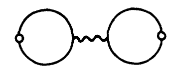

FIG. 5. Typical Feynman diagram for the free energy. Each circle contributes a factor Δ/T^2.

Grinstein[3] has shown $\lambda_T=-\sigma+O(\epsilon^3)$ near $d=3\sigma$ and it is speculated that $\lambda_T=-\sigma$ exactly. However, in the nonlinear σ model near $d=2\sigma$, we find

$$\lambda_T=-\sigma+(d-2\sigma)(-1/m)+O((d-2\sigma)^2) ,$$

which is not exactly equal to $-\sigma$. In the limit $m\rightarrow\infty$ the second term vanishes. It is probable that $\lambda_T=-\sigma$ exactly for $m\rightarrow\infty$, although we have not checked it.

ACKNOWLEDGMENTS

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