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Experimental study of the flow of liquid ^{4}He through very small channels: Finite-size effects near T_λ

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The flow of liquid ^{4}He through the very small cylindrical channels present in Nuclepore filters has been studied. Filters with channel diameters in the range 100–3300 Å have been utilized, with emphasis on the behavior in the vicinity of the superfluid transition. As observed by many previous workers, we find that the onset of superfluid flow of the ^{4}He in the channels occurs at a temperature, $T_{\lambda s}$, which is below the transition temperature of bulk ^{4}He. Our results for the variation of $T_{\lambda s}$ with channel size are in good quantitative agreement with the current theory.

I. INTRODUCTION

For many years liquid ^{4}He has served as a "model" system with regards to testing the predictions of the general theory of phase transitions and critical phenomena.[1] Indeed, it seems fair to say that the behavior of ^{4}He near the superfluid transition T_λ has been studied in greater detail than has that of any other experimental system. In nearly all cases which have been examined, the behavior of ^{4}He near T_λ has been found to be in excellent quantitative agreement with the theory. One notable exception, however, is the behavior of ^{4}He in so-called "confined" geometries, i.e., ^{4}He confined to a region which is of finite extent in one or more dimensions. The geometries which are most convenient to consider theoretically are thin films of uniform thickness, and cylindrical channels with uniform diameters. Experimental study of ^{4}He in these geometries is made difficult by the fact that the confining dimensions—the thickness of the film or the diameter of the cylinder—must be fairly small, typically 1000 Å or less, in order for the effect of the finite size to be measureable. As a result, most of the early work in this area involved ^{4}He confined in packed powders, Vycor glass, or similar structures.[2] While the results of this work confirmed the qualitative predictions of the theory of finite-size effects near phase transitions,[3–6] quantitative comparisons were not possible because of the ill-defined geometries which were involved. In recent years experimental techniques have been developed which permit the study of ^{4}He in both uniform films and channels. The work on thin films is not of primary interest to us here, although we note that in this case the experimental results appear to be in good agreement with the relevant theory.[7] As for the work on ^{4}He in uniform channels, a very important development in this area has been the availability of Nuclepore filters.[8] These are thin pieces of plastic, which contain cylindrical holes of uniform size produced by etching the damage tracks left by heavy charged particles which have passed through the material.[9] Filters are available with holes (we will also refer to them as pores or channels) as small as 100 Å in diameter, and with a large number of holes ($\gtrsim 10^8$ per cm^2), and this makes them ideal as a confining container for liquid ^{4}He.

A number of workers[10–16] have studied the properties of ^{4}He in Nuclepore filters or similar materials. It has been found that the liquid in the pores becomes superfluid at a temperature $T_{\lambda s}$ which is below the transition temperature for bulk ^{4}He, T_λ. The first results[10–12] indicated that ΔT_λ $(=T_\lambda - T_{\lambda s})$ varied with the pore diameter in a manner which was in good agreement with the theory. However, recent experiments[13–16] report what appear to be significant deviations from the theoretical predictions. As discussed by Gasparini *et al.*,[14] one possible interpretation of the new experiments is that the confining geometry gives rise to a new "critical-length scale," which is distinct from the ordinary bulk correlation length. Such a result is very difficult to reconcile with current theoretical ideas. This state of affairs has stimulated us to perform a new study of ^{4}He in Nuclepore filters. The experiment consisted of simple flow measurements which, while in some ways quite crude in comparison with previous work, yielded a fairly accurate determination of ΔT_λ. Our results for the variation of ΔT_λ with pore size yield a value for the critical exponent Λ [see (1) below] of

1.55 ± 0.09, in good agreement with the theoretical prediction[3] $\Lambda = 1/\nu \cong 1.48$. However, as we will see, when possible experimental uncertainties are considered, most and possibly all previous experiments are consistent with our results.

This paper is organized as follows. In the next section (II) we review the theoretical predictions relevant to this work. In Sec. III we describe the experimental set up and samples used in our measurements. Section IV contains our results, and in Sec. V we compare our results with those of previous workers. In Sec. VI we give our conclusions.

II. THEORY

The first theoretical treatment of finite-size effects in liquid ^{4}He was given by Ginsburg and Pitaevskii,[17] who employed what is now known as Landau or mean-field theory.[18,19] They considered the case of a thin film, and were able to show that when the correlation length is a certain fraction of the film thickness, or larger, the mean-field differential equation for the superfluid wave function (i.e., the order parameter) no longer has a nonzero solution. They associated this with the suppression of the superfluid transition ΔT_λ and their result was

$$\Delta T_\lambda = A d^{-\Lambda} , \tag{1}$$

where d is the thickness of the film, A is a constant, and Λ is a critical exponent which according to the Ginzburg-Pitaevskii theory[17] is equal to 2. The case of a cylindrical pore was first considered by Mamaladze[20] who found a result of the same form as that given above (1), where d is the diameter of the pore, with $\Lambda = 2$, but with a different value for the constant A.

The shortcomings of mean-field theory, particularly with regard to the values of critical exponents, are well known.[3,19,21] This stimulated Mamaladze[20] to modify the Ginzburg-Pitaevskii theory. By choosing the coefficients in the expansion of the thermodynamic potential to be singular (and somewhat *ad hoc*) functions, Mamaladze was able to obtain exponents which agree with those found experimentally. These modifications resulted in the prediction $\Lambda = \frac{3}{2}$.

A more rigorous approach to the problem of finite-size effects has been developed by Fisher and co-workers,[3,4] and by a number of other workers.[5,6] This approach is based on general scaling ideas together with detailed model calculations. In simple (perhaps oversimplified) terms, the scaling arguments amount to the statement that "something" must happen when the bulk correlation length becomes comparable to the size of the system (in our case, the pore diameter). This leads directly[3] to the prediction $\Lambda = 1/\nu$, where ν is the usual correlation length exponent.[19,21,22] This prediction has been confirmed by a number of detailed calculations for a variety of models,[3-6] and it therefore appears to be on very firm theoretical footing. For ^{4}He, the best available estimates[23] give $\nu \simeq 0.675$, and hence yield the prediction[24] $\Lambda = 1.48$.

It is well known that superfluidity, or more precisely the presence of conventional long-range order in a superfluid, cannot exist in one dimension.[25] Thus, strictly speaking, ^{4}He confined to a pore with a finite diameter will not be a superfluid of the usual sort. Nevertheless, one can argue[3,25] that this effect will not be significant so long as the correlation length is much less than the pore diameter. However, near the transition the reduced dimensionality must be important, with the main result then being a "rounding off" of the transition.[3] Thus (1) should describe the shift of the "rounded" transition relative to that of the bulk.

One problem which arises in dealing with ^{4}He in confined geometries is that of boundary conditions. In the Ginzburg-Pitaevskii theory[17] the boundary conditions pertain to the superfluid wave function, and are, of course, needed in order to solve the mean-field differential equation. However, the actual conditions at the boundary between liquid ^{4}He and a solid are much more complicated than those usually considered by these theories. Here we have in mind, for example, the effect of the van der Waals force and the resultant solid layer of ^{4}He on the surfaces, as well as what are believed to be liquid layers at an effective pressure higher than ambient which are in contact with the solid layer. In this paper we will be concerned only with pores whose diameters are much (~ 20 to 600 times) greater than the thickness of the ^{4}He appreciably affected by the van der Waals force, so one might expect that these effects will not be important in our case. However, we should note that Brooks *et al.*[15] have estimated that for these pore diameters and at temperatures quite accessible to experiment, the "van der Waals effects" will depress the average superfluid density by $\sim 10\%$ or more. The upshot of all this is that these effects may well be important for our experiments. While a general scaling theory which describes the behavior in the limit of large d can presumably safely ignore these effects, they may be important in any microscopic calculations which, for example, attempt to estimate the correction terms to (1) for small d.

III. APPARATUS AND SAMPLES

The apparatus used in our experiments is shown schematically in Fig. 1. The Nuclepore sample

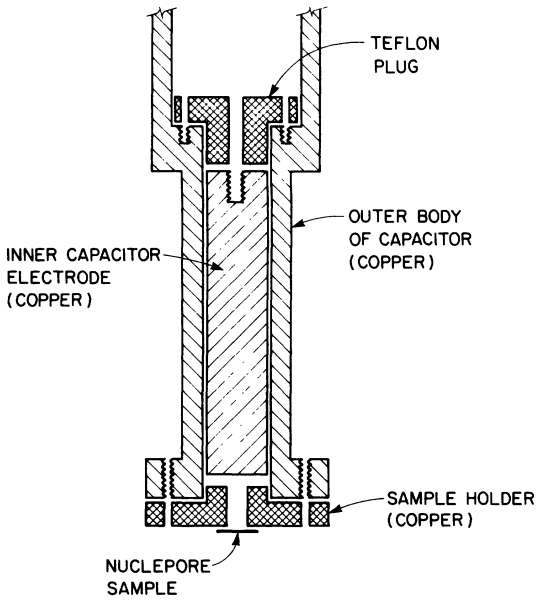

FIG. 1. Schematic diagram of the bottom portion of the coaxial capacitor container. This diagram is approximately to scale; important dimensions are given in the text. The Mylar strips which separate the capacitor electrodes, the indentations for the In O-ring seal between the container and the sample holder, and the screws which hold the various pieces together, are not shown. The upper part of the container attaches to a $\frac{1}{4}$-in. stainless-steel tube which is not shown.

separated two regions; one region was the ^{4}He bath used for cooling the apparatus, while the other region was enclosed by a copper container. The ^{4}He "sample" thus came from the ^{4}He bath. A cylindrical coaxial capacitor of standard design[26] was located in the bottom of the copper container. The capacitor was 5 cm tall and had an open gap of 100 μm. This spacing was maintained by three Mylar strips which were 3 mm wide and which ran the length of the capacitor. The capacitor container was mounted on the end of a stainless-steel tube which left the cryostat via an O-ring seal. This allowed the capacitor to be raised or lowered so as to keep the level of the ^{4}He bath in the "active" region of the capacitor.

The level of the ^{4}He in the capacitor was sensed by using the capacitor as part of the tank circuit of a tunnel diode oscillator. The oscillator components (except for this capacitor) were attached to the outside of the capacitor chamber, and were connected with the level sensing capacitor with leads which ran through a (leak-tight) epoxy feed through. The oscillator was connected to the room-temperature electronics via a coaxial cable[27] which ran down the inside of the stainless-steel support tube. At the bottom of the tube the coaxial leads were heat sunk to the chamber wall, then soldered to copper leads which were also heat sunk, and then ran out through the epoxy feed through to the oscillator. The oscillator circuitry was similar to that described by Boghosian *et al.*[28] The oscillator signal was amplified and counted directly with a frequency counter.[29] With the use of a digital to analog converter, the frequency could be recorded in analog form on a chart recorder.

The oscillator frequency with the capacitor empty was near 1.77 MHz, which is very close to the value expected from the estimated inductance of the tank coil (50 μH) and capacitance of the level sensing capacitor (140 pF). The resulting sensitivity to changes in the ^{4}He level was 0.6 Hz/μm. The intrinsic stability of the oscillator was better than ± 0.3Hz, but the frequency "noise" was usually due to ^{4}He-level fluctuations, and was typically ± 5 Hz.

The temperature was measured with two Ge thermometers,[30] both of which had been calibrated previously. One was measured with a standard four lead dc arrangement, while the other was part of a Wheatstone bridge operated at 110 Hz and which had "dummy" leads to compensate for changes of the lead resistance. The leads to both thermometers ran from room temperature directly into the ^{4}He bath. The thermometers were attached to the outside of the capacitor container about 1 cm above the sample, and their leads were heat sunk to the chamber with GE-7031 varnish. The thermometer measured with the Wheatstone bridge was used for high-resolution work. Its resistance was measured with an rms current of 0.25 μA (dissipated power $\sim 6\times 10^{10}$ W). The noise in the bridge was about a factor of 2 larger than the Johnson noise of the bridge components, and corresponded to typically ± 10 μK (peak-to-peak variation) when measured with a 0.1 Hz bandwidth.

An important part of this experiment was the accurate determination of T_λ for bulk ^{4}He. This was accomplished in the usual way,[31] by observing the temperature as a function of time as the ^{4}He bath was allowed to warm slowly. The point at which the temperature was observed to remain approximately constant for an extended period of time (typically to within ± 10 μK for more than 10 min) was identified as T_λ. Note that in this measurement, the ^{4}He bath acts as the "sample." Since the bath was fairly tall, typically 30 cm, T_λ will be ~ 40 μK higher at the top of the bath than at the bottom. However, since the thermometers were located only 1 cm above the sample and the ^{4}He level was always kept in the active region of the capacitor, the errors due to the pressure gradient were negligible for these experiments. The resistance of the thermometer at T_λ was determined during each cool down of the

apparatus, and varied by an amount equivalent to $\pm 100\ \mu$K during the course of these experiments.

Nuclepore filters[8] with nominal pore diameters ranging from 150 to 4000 Å were studied. They were glued over a 3.2-mm-diam hole in a copper disc using Stycast 2850FT epoxy with catalyst no. 9. This disc was fastened to the bottom of the capacitor container with an indium O-ring seal. The above epoxy was chosen because it adheres well to copper—in fact, the epoxy to copper seal was never observed to fail in these experiments. However, this epoxy is not completely ideal, for the following reason. It was found that as the epoxy hardened, a small amount of a clear substance flowed out of the main body of epoxy and onto the sample. Because it was transparent, this substance was presumably mostly catalyst, although it did appear to harden. If the only effect of this was a slightly reduced open sample area, there would be no problem. However, examination with a scanning electron microscope (SEM) revealed that this clear substance formed a very thin layer. In many areas it appeared not to fill the pores, but to flow into them , reducing the effective pore diameter. Fortunately, the area affected in this way was a ring less than 50-μm wide around the edge of the sample. Since the exposed region of the sample was typically 3 mm or more in diameter, the region affected by the epoxy "flow" was negligible for our experiments. Moreover, as will be seen in the next section, our measurements were sensitive only to the pores with the largest diameters, and hence should not be affected by the presence of a few pores with smaller than average size. It is interesting to note that Schubert and Zimmermann[16] used a similar epoxy along with very small open areas (as small as $40\times 40\ \mu m^2$). Their results for Nuclepore samples with 4000-Å-diam pores were very much out of line with those of other workers, and these authors speculated that some of the epoxy might have flowed into and partially filled the pores in their samples. In light of our findings, this seems to be a very likely explanation.

The pore diameter of the Nuclepore filters is clearly a very important parameter in these experiments. The manufacturer quotes an uncertainty in this diameter of $+0$—20%, and this is quite consistent with the detailed study of Nuclepore filters by Chen *et al.*[32] However, it does not appear to be widely known that the manufacturer checks the pore sizes of each batch of filters using N_2-gas—flow measurements, and also with SEM examination for the larger pore sizes. In Table I we list the nominal, i.e., catalog, values of the pore sizes of our samples, together with the values obtained by the manufacturer using gas flow and/or SEM examination. Also given are values we have obtained via SEM examination. The different measurements of the pore size are seen to be in good agreement. The gas-flow measurements are probably the most reliable, particularly for the smaller pores. While in most cases, the measured values agree very well with the nominal size, there are significant differences in several cases. The nominal 4000-Å sample had a measured size of $\sim$3300 Å, barely within the speci-

TABLE I. Information on the pore diameters of the Nuclepore samples studied in this work. The sizes measured by N_2 gas flow and SEM examination were provided by S. Rohrbacker of the Nuclepore Corporation. The fourth column contains values measured via SEM examination in this laboratory. Our attempts to measure the size of the nominal 150-Å sample, either with an SEM, or using thinned samples and a transmission electron microscope, were unsuccessful. The last column gives our results for $\Delta T_\lambda = T_\lambda - T_{\lambda s}$. Note that the nominal 4000- and 2000-Å samples were 10 μm thick, and the others were all 5 μm thick.

Nominal size (Å)	Size measured with N_2 flow[a] (Å)	Size measured with SEM by Nuclepore[b] (Å)	Size measured with SEM (this work) (Å)	T_λ-$T_{\lambda s}$ (mK)
4000	3300	3300	3400±200	0.109±0.019
2000	1900	1900	1900±200	0.192±0.025
1000	800	800	850±120	0.91 ±0.13
800	800	800	750±100	0.906±0.094
500	500	450	500±100	1.94 ±0.19
300	300		350±70	4.00 ±0.08
150	100			10.54 ±0.24

[a]The uncertainties of these values are not available, but ±10% is probably a reasonable, and conservative, estimate.

[b]The uncertainties of these values are not available, but are probably comparable to those in the following column.

fied limits. The nominal 1000-Å sample had a measured size of ~800 Å, again barely within specifications. The nominal 150-Å sample had a measured size of ~100 Å, which is actually outside the specifications. Indeed, this was subsequently (after our purchase of the material) "reclassified" by the manufacturer as nominally 100-Å material. One conclusion which can be drawn from this discussion, is that when precise values of the pore size are important, one should *not* rely solely on the nominal values quoted by the manufacturer.

IV. RESULTS

The objective of this work was to simply determine the temperature at which the ^{4}He in the pores became superfluid, and this was accomplished by monitoring the flow of ^{4}He through the pores under various conditions. We did not attempt to perform quantitative measurements of the flow rate, and thus extremely precise regulation of the temperature for extended periods of time was not necessary. Instead, the temperature was allowed to drift slowly, via appropriate adjustment of the valve which controlled the pressure of the ^{4}He bath, while the level of the ^{4}He inside the capacitor was recorded. It was found that adjusting the pumping valve caused sudden changes in the ^{4}He level in the capacitor, presumably because this produced temporary differences in the pressure and/or temperature of the ^{4}He inside and outside the capacitor.[33] Therefore, most data were taken by setting up the desired temperature drift rate, and thereafter not adjusting the pumping valve.

Some typical results for an 800-Å sample are shown in Fig. 2(a) (in this section we will refer to samples according to their pore diameters as determined by N_2-gas–flow measurements). Here the temperature was allowed to drift slowly downwards at a rate of approximately 0.15 mK/min, and both the level of the ^{4}He in the capacitor and the temperature were monitored as a function of time. As can be seen from Fig. 2(a), the level change, and hence also the flow rate, is initially exponential in form. This corresponds simply to viscous flow. However, at a certain point, indicated by the arrow in Fig. 2(a), the level change ceases to be exponential, and the curve becomes concave downwards. The change in behavior is illustrated more clearly in Fig. 2(b) where we plot the flow rate, which is just the derivative of the curve in Fig. 2(a). Here the inflection point in Fig. 2(a) is evident as a minimum in the flow rate. This is precisely the behavior expected when one passes through the transition temperature for the ^{4}He in the pores, $T_{\lambda s}$. For $T > T_{\lambda s}$ the fluid in the pores will be normal and hence there will simply be viscous flow,[34] while for $T < T_{\lambda s}$ there will be superfluid flow in parallel with that of the normal fluid. For $T < T_{\lambda s}$, the curve in Fig. 2(a) is concave down because as the temperature decreases, the volume flow rate for the superfluid will increase due to an increasing superfluid density and an increasing critical velocity. The vertical arrows in Fig. 2 indicate the approximate location of the inflection in the curve based on this and other measurments for this sample,[35] and therefore also give the location of $T_{\lambda s}$. The horizontal bars indicate the estimated uncertainty[35] in $T_{\lambda s}$.

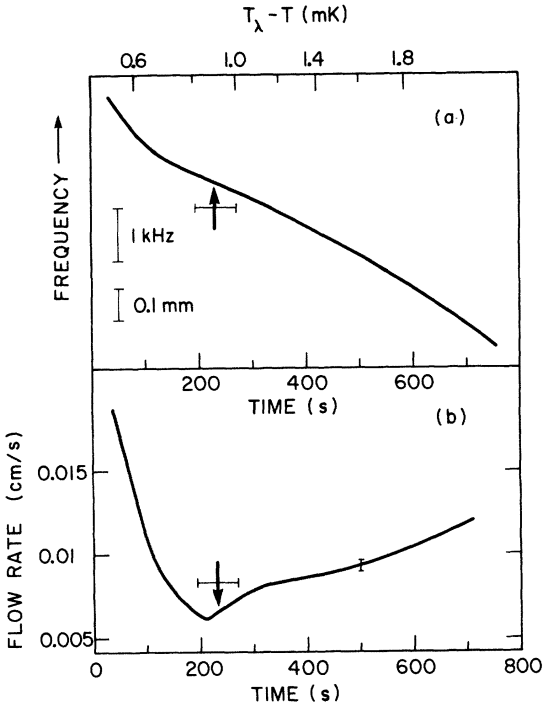

FIG. 2. (a) Oscillator frequency as a function of time for a measurement with an 800-Å sample. The temperature was allowed to drift slowly downwards as indicated on the upper scale. The decrease of the frequency with time corresponds to ^{4}He flowing into the capacitor container. (b) Flow velocity as a function of time, derived by differentiating the results in (a). The scale for the flow rate is computed assuming a uniform velocity across the pore diameter, and that all available pores are open (i.e., not blocked by condensed gases, etc.). For both (a) and (b) the vertical arrow indicates the "best" value of $T_{\lambda s}$ determined from this and other measurements with this sample (Ref. 35). The horizontal bar gives the estimated uncertainty in $T_{\lambda s}$. The estimated uncertainty in the flow rate is indicated by the vertical bar.

To ensure that the results in Fig. 2 do in fact locate $T_{\lambda s}$, the measurements were repeated with the

^{4}He at different levels, both inside and outside the capacitor, and with different temperature drift rates. Except for the cases to be discussed below, the results were always of the form shown in Fig. 2. Furthermore, these measurements along with others described below, always gave the *same* value for $T_{\lambda s}$.

Figure 3 shows similar results obtained for a 100-Å sample. Here again a clear inflection is seen in the plot of the ^{4}He level as a function of time [Fig. 3(a)]. The transition, i.e., the inflection in Fig. 3(a), is especially evident in Fig. 3(b) which shows the derivative curve.

While, as mentioned above, most of the results resembled those shown in Figs. 2 and 3, there were some notable exceptions. One example of this is illustrated in Fig. 4 which shows results obtained with the same 100-Å sample as in Fig. 3. Here the normal fluid flow rate above $T_{\lambda s}$ is much lower than in Fig. 3. At a certain temperature, the flow rate (i.e., the derivative of the curve) increases in an unusual, and nearly discontinuous fashion, and as the temperature is lowered further, the flow rate remains approximately constant. To within experimental error,[36] the temperature at which the flow rate increases is the same as the transition temperature determined from measurements like those shown in Fig. 3. Intuitively this is not surprising, since one would expect the flow rate to increase at $T_{\lambda s}$, i.e., when the ^{4}He in the pores becomes superfluid. However, the manner in which the increase takes place in Fig. 4 is difficult to understand. First, it is surprising that the normal fluid flow above $T_{\lambda s}$ is so low, since we would expect the flow to be governed by the viscosity and the pressure difference, both of which are approximately the same as in Fig. 3. Second, it is also surprising that the flow rate changes so abruptly at $T_{\lambda s}$, as one would expect the superfluid flow rate to increase continuously from zero as the temperature is lowered below $T_{\lambda s}$. Finally, the slight increase in the ^{4}He level very near $T_{\lambda s}$ was also unexpected. Results of the type shown in Figs. 3 and 4 were both reproducible. although the behavior shown in Fig. 3 was far more common. The behavior shown in Fig. 4 tended to occur when the temperature drift rate was relatively high and/or when the temperature was varied rapidly just prior to making the measurement. This suggests that some sort of "turbulence" and/or vorticity (recall that the ^{4}He outside the pores is superfluid above $T_{\lambda s}$) might have acted to block the pores,[37] but this "explanation" is extremely tentative at best. In any event, the change in the flow rate is quite pronounced, and it occurs at the *same* temperature regardless of the behavior at higher temperatures. It therefore seems quite certain that this temperature is $T_{\lambda s}$.

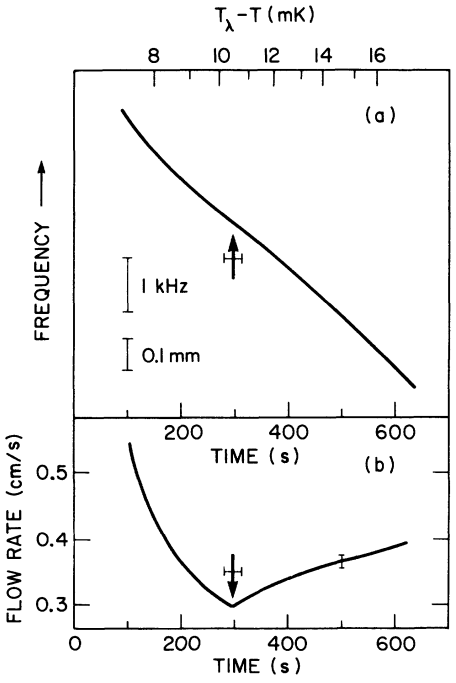

FIG. 3. (a) Oscillator frequency as a function of time for a measurement with a 100-Å sample as the temperature is allowed to drift slowly downwards as indicated on the upper scale. (b) Flow velocity as a function of time, derived from (a). The notation, etc., is the same as in Fig. 2.

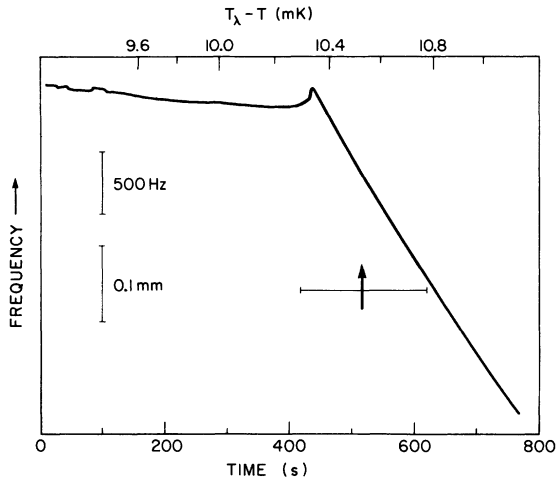

FIG. 4. Oscillator frequency as a function of time for a measurement with a 100-Å sample as the temperature is allowed to drift slowly downwards. The corresponding temperatures are indicated on the upper scale. Note that a decrease in frequency corresponds to ^{4}He entering the capacitor container. The arrow and horizontal bar have the same meaning (Ref. 36) as in Fig. 2.

The measurements described above were made with liquid in the capacitor at all times. It was also possible to perform measurements in which this was not the case. If the temperature was allowed to increase relatively rapidly, usually to values well above T_λ, it was found that the liquid in the capacitor would evaporate, leaving only low pressure gas in the capacitor volume. This presumably occurs because as the temperature is increased, some of the liquid in the capacitor must evaporate to maintain the proper vapor pressure.[38] Since above T_λ no new liquid can enter through the pores,[39] increasing the temperature sufficiently will cause all of the liquid to evaporate. This explanation is supported by the observation that the amount of heating required to evaporate the liquid increased as the initial amount of liquid in the capacitor was increased.

After heating to remove the liquid from the capacitor volume, it was then possible to cool down below T_λ and again monitor the ^{4}He "level" as a function of time as the temperature was allowed to drift slowly downwards. Some typical results for a 100-Å sample are shown in Fig. 5. Here it is seen that that capacitor remains essentially empty, with ^{4}He flowing in only very slowly, until the point indicated approximately by the arrow, at which time the flow rate increases rapidly.[40] This temperature coincides, within experimental error,[36] with the value of $T_{\lambda s}$ found in Figs. 3 and 4. Moreover, in repeated measurements of the type shown in Fig. 5, with the ^{4}He bath level at different positions relative to the capacitor, the temperature at which the flow rate increased was, to within the errors, always the same. It therefore seems quite certain that this increase in flow rate locates $T_{\lambda s}$. However, it is not clear why the normal fluid flow rate is so small (i.e., negligible) above $T_{\lambda s}$. The suppression of the normal flow is similar to that illustrated in Fig. 4, and may have the same origin.

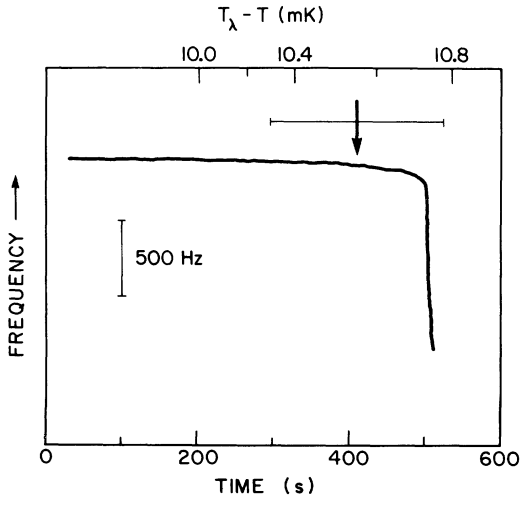

FIG. 5. Same as Fig. 4 for a 100-Å sample, but in this case there is very little liquid ^{4}He in the capacitor container at time equal to zero.

The results shown in Figs. 2–5 were obtained with relatively small pores, and hence correspond to relatively large values of $\Delta T_\lambda = T_\lambda - T_{\lambda s}$. For the larger pores which were studied, ΔT_λ was accordingly smaller, and it was difficult to set up a slow yet uniform temperature drift of the sort required for measurements like those shown in Figs. 2 and 3. However, it was still possible to perform measurements like those shown in Figs. 4 and 5. Some typical results for a 3300-Å sample are shown in Fig. 6. Here we show the liquid level as a function of tem-

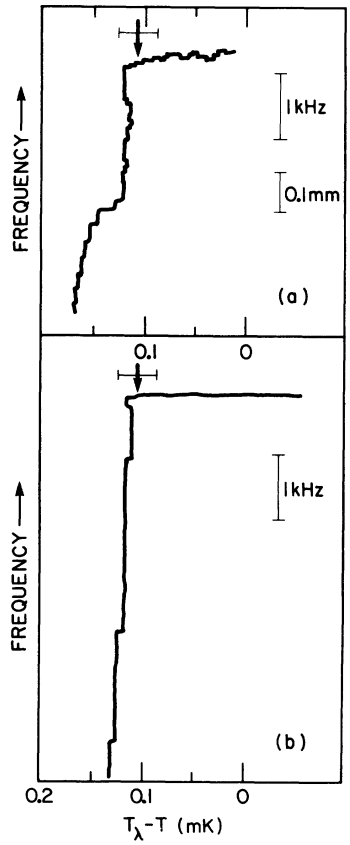

FIG. 6. (a) Oscillator frequency as a function of temperature for a measurement with a 3300-Å sample. At T_λ there was a significant amount of liquid ^{4}He in the capacitor container. Note that the temperature scale is reversed relative to Figs. 2–4. (b) Same as (a) but in this measurement there was very little liquid in the capacitor container at T_λ. The discrete "jumps" in the curves are due to the sampling time of the frequency counter (1 s). The arrows and horizontal bars have the same meaning as in Fig. 2.

perature, as the temperature is swept slowly downwards from near T_λ. In Fig. 6(a) there was initially liquid in the capacitor, while in Fig. 6(b) there was initially no liquid in the capacitor. In both cases the liquid level is seen to remain fairly constant until a certain temperature at which ^{4}He rapidly enters the container. These results are very similar to those shown in Figs. 4 and 5. It can be seen from Fig. 6 that the temperature at which the flow rate increases, indicated approximately by the arrows, is the same for the two types of measurements—that is, it does not depend on whether liquid is or is not present initially in the capacitor. Repeated measurements with different amounts of liquid in the capacitor and different temperature drift rates showed that the increase in flow rate always occurred at the same temperature, which we again identify as $T_{\lambda s}$.

It is worthwhile at this point to summarize the results presented above. We have observed three different "types" of ^{4}He-flow behavior near $T_{\lambda s}$ in our apparatus. Examples of these three types of flow are shown in Figs. 3–5. The type of flow which is observed depends upon the conditions of the measurement, such as whether liquid ^{4}He is initially present in the capacitor or not. Certain features of the flow, such as the very small normal-fluid flow rates observed in Figs. 4 and 5, are not understood. However, the three different types of flow curves *all* show that at a certain temperature the flow rate increases significantly. This temperature is *independent* of the particulars of the measurement, such as temperature drift rates, and is the *same* for the three different types of flow behavior. It therefore seems quite certain that this temperature is the temperature at which the ^{4}He in the pores can first undergo superfluid flow, $T_{\lambda s}$.

Measurements of the sort illustrated in Figs. 2–6 have been performed with the Nuclepore samples described in Table I. Our results for $\Delta T_\lambda = T_\lambda - T_{\lambda s}$ are also given in Table I. The uncertainties are due to the errors in both T_λ and $T_{\lambda s}$. Errors in $T_{\lambda s}$ dominate for all but the largest samples. In several cases a given sample was "measured" more than once (i.e., was used for a measurement, then stored at room temperature for several weeks or months, and then measured again), and the results were the same for each measurement. Also, in a few cases different samples from the same Nuclepore batch were measured, and again the results were found to be the same.

The results for ΔT_λ as a function of pore diameter d are shown in Fig. 7. Note that here we use the pore diameters determined by the manufacturer from gas-flow measurements, since as mentioned in Sec. III these are presumably the most accurate values.[41] It is difficult to estimate the uncertainties

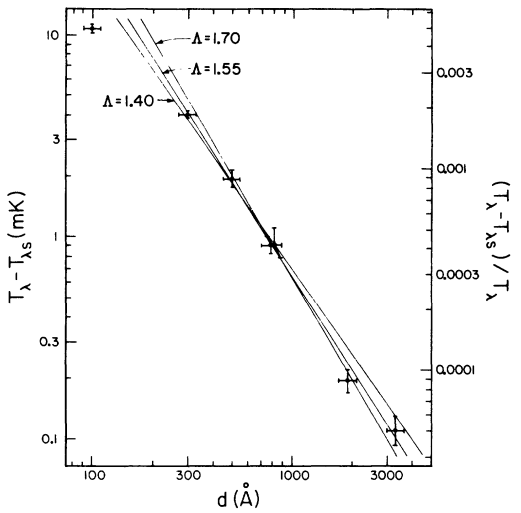

FIG. 7. $\Delta T_\lambda = T_\lambda - T_{\lambda s}$ as a function of pore diameter, d. These data are tabulated in Table I. The horizontal error bars correspond to the $\pm 10\%$ uncertainty in d as discussed in the text. The solid lines correspond to (1) with the indicated values of Λ.

in the pore sizes, but $\pm 10\%$ is probably a reasonable value, and the corresponding error bars are shown in Fig. 7. In Fig. 7 we also show curves corresponding to the relation

$$\Delta T_\lambda = A d^{-\Lambda} , \tag{2}$$

for three different values of Λ. It can be seen that for no values of the parameters A and Λ is a curve of the form (2) consistent with our results for the entire range of d; in particular, the result for $d = 100$ Å always falls well below the curve (2). It is interesting to note that if the nominal value of d is used for this sample, namely 150 Å, then this data point would fall on the curve for $\Lambda \simeq 1.5$. According to the manufacturer, it is unlikely that the measured value of d could be in error by this amount. However, given the nature of the fabrication process, inhomogeneities in the pore size will be largest for the smaller pore sizes. Such an inhomogeneity could account for the observed behavior. Another possible explanation of this result is that the behavior for $d = 100$ Å is outside of the asymptotic region, so that (2) would no longer be expected to apply. An independent measure of the pore size for this sample would resolve this question.[42] In any case, for pore sizes of 300 Å and greater, our results can be described by a power law (2). A least-squares fit to the data for $d \geq 300$ Å gives $\Lambda = 1.55 \pm 0.09$, where we quote the standard error.[43] This is in good agreement with the theoretical prediction, $\Lambda = 1.48$. A curve corresponding to $\Lambda = 1.55$ is shown in Fig. 7; it clearly provides a good "fit" to the data. To give

a feeling for how other values of Λ compare with our results, we also show curves corresponding to $\Lambda=1.40$ and 1.70, approximately two standard errors above and below the "best fit" value. While these values are roughly consistent with our results, the deviations of the data from these curves are clearly systematic. We therefore conclude that our results are not consistent with values of Λ much outside the range 1.40—1.70.

V. COMPARISON WITH PREVIOUS WORK

There have been many previous studies of ^{4}He in confined geometries. However, in only a relatively few cases has the geometry been sufficiently well defined to allow a quantitative comparison with the theory of finite-size effects near T_λ. For ^{4}He in a one-dimensional geometry, such experiments have only been possible with etched nuclear tracks in either plastic (i.e., Nuclepore filters) or mica. In Fig. 8 we compare our results for ΔT_λ as a function of pore size with those of all previous workers who have studied the suppression of the superfluid transition in etched tracks. In general, the agreement is quite good, especially for the larger pore sizes. We now discuss the previous results in more detail.

The first results of this kind were obtained by Notarys,[10] who performed flow measurements using etched tracks in mica. The measurement of ΔT_λ was not the major goal of this work, and no estimates of the uncertainties, either in ΔT_λ or the pore diameter, were given. As a result, it is really not possible to quantitatively analyze the results of Notarys, although as seen from Fig. 8, they are quite consistent with later work.

Pobell and co-workers[11,12] have performed very-high-resolution studies of the depression of T_λ by studying the efficiency with which vibrating Nuclepore filters generate second sound. These workers were able to follow the appropriate signal very near to $T_{\lambda s}$, so that the uncertainty in ΔT_λ was quite small. A detailed analysis of their results[11] gave $\Lambda=1.54\pm0.05$. Our results are seen to be in very good agreement with those of Pobell *et al.*

Zimmerman and co-workers[15,16] have performed extensive measurements with Nuclepore filters using a Helmholtz resonator technique. With this method they were able to determine the superfluid density of the ^{4}He in the pores, ρ_{sp}. In order to locate $T_{\lambda s}$ it was then necessary to extrapolate ρ_{sp} to zero. This extrapolation was made difficult the fact that the functional form for ρ_{sp} is not known. This problem has been considered in detail by Brooks *et al.*,[15] who have shown that two different functional forms, $\rho_{sp}\sim(\Delta T_\lambda)^{0.67}$ and $\rho_{sp}\sim(\rho_s-\rho_0)$, where ρ_s is the bulk superfluid density, and ρ_0 is a constant, both

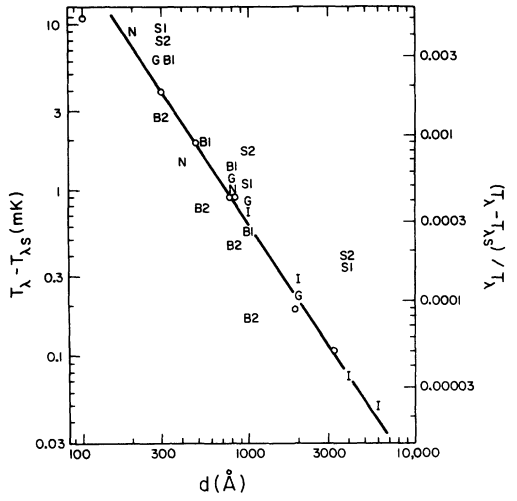

FIG. 8. $\Delta T_\lambda=T_\lambda-T_{\lambda s}$ as a function of pore diameter, d, for all reported measurements involving ^{4}He in etched tracks. The solid curve is the same as the "best fit" curve plotted in Fig. 7 and corresponds to $\Lambda=1.55$. The open circles are the present work and the letters corresponds to previous work as follows: *N*—Notarys (Ref. 10); *I*—Ihas and Pobell (Ref. 11; note that these values were obtained from Sec. IV B of this paper and *not* from their Table II); *G*—Gasparini *et al.* (Ref. 14); *B*1, *B*2—Brooks *et al.* (Ref. 15; *B*1 is the value of their t_{01} with ζ held fixed at 0.675 and ϵ_p/ϵ_b set equal to the calculated value, and *B*2 is the value of their t_{02} evaluated with ϵ_p/ϵ_b set equal to the calculated value and their parameter A_1 allowed to vary. As noted by Brooks *et al.*, these are presumably the most reliable values which can be derived from their data at this time); *S*1,*S*2—Schubert and Zimmermann (Ref. 16; *S*1 is the value obtained in a manner similar to *B*1 above, and *S*2 is the value obtained from mass flow measurements as opposed to measurements of the superfluid density). For clarity, the estimated uncertainties are not shown; they are typically the size of the symbols or smaller.

describe the data quite well. With the use of these two forms, different estimates for ΔT_λ were obtained. The values believed to be most reliable are shown in Fig. 8. As noted by these authors, their results seem to suggest a value of Λ somewhat higher than our "best" value of ~1.5. However, the possible errors,[44] due mainly to uncertainties in the extrapolation procedure, are sufficiently large that these results are also consistent with $\Lambda=1.5$, so there is no real discrepancy here. It would be interesting to obtain values of ρ_{sp} closer to $T_{\lambda s}$, but as discussed by Brooks *et al.*,[15] such measurements may not be possible with the Helmholtz resonator technique.

The only experiments with ^{4}He in etched tracks which did not involve flow are the heat-capacity studies of ^{4}He in Nuclepore filters by Gasparini and

co-workers.[13,14] In this work $T_{\lambda s}$ was identified as the temperature at which the heat capacity exhibited a maximum. As can be seen from Fig. 8, the results of Gasparini *et al.*[14] agree fairly well with our values for large pores, but there are systematic differences for the smaller pores. This difference led these authors to conclude that $\Lambda = 1.72 \pm 0.14$, which is slightly higher than our "best" value and does not agree with the theory. Such a large value of Λ was also found by Gasparini *et al.* in measurements on ^{4}He films. From these results along with a detailed analysis of their heat-capacity data,[45] Gasparini *et al.* concluded that either corrections to scaling [i.e., corrections to (1)] dominate the observed behavior, or that there may be a critical length scale which is not proportional to the bulk correlation length. These conclusions appear to be at odds with current theories of finite-size effects near phase transitions, and it is therefore very important to consider how, or if, the results of Gasparini *et al.* can be reconciled with our findings. First, it would be interesting to determine the actual pore sizes for the samples studied by Gasparini *et al.* In particular, if the pore size for their 300-Å sample were to fall at the low end of the nominal range, then the results for this sample would be in very good agreement with our results. Thus this *could* be a way out of the discrepancy.[46] However, this would not explain the high value of Λ obtained by Gasparini *et al.* from measurements on films. Here we can only note that, as discussed above, our nominally 150-Å sample may not lie in the asymptotic region,[47] and this could well be the case for the films studied by Gasparini *et al.*, since they all exhibited much larger values of ΔT_λ than did our 150-Å sample. Moreover, one would not necessarily expect the asymptotic region to be the same for different quantities. Our flow measurements suggest that the asymptotic region extends to values of d as small as 300 Å, but this may not be the case for heat-capacity measurements. This possibility was considered by Gasparini *et al.*[14] in that they also included corrections to scaling in their analysis. While one would expect that the inclusion of corrections to scaling would allow a successful analysis of data slightly outside the asymptotic region, it is not entirely clear that this will always be the case. Another possible reason for the difference between our results and those of Gasparini *et al.* should also be noted. Namely, that the onset of mass flow (which is the quantity we have measured) may simply not occur at the temperature at which the heat capacity has a maximum (which Gasparini and co-workers have measured). Scaling arguments predict that the variation these two temperatures with pore size must be the same, so, as pointed out by Gasparini *et al.*[14] the experimental results could conceivably indicate a serious breakdown of scaling. In any case, further effort to understand the reason(s) for the differences between our results and those of Gasparini and co-workers certainly seems to be called for.

VI. SUMMARY AND CONCLUSIONS

We have performed a study of the flow of ^{4}He through the etched particle tracks in Nuclepore filters. Our results for the supression of the superfluid transition as a function of pore size yield $\Lambda = 1.55 \pm 0.09$, in good agreement with the current theory. We have also made a detailed comparison with previous results for ^{4}He in similar geometries, and find that on the whole, all of the data are in reasonable agreement. One possible exception is the recent result of Gasparini and co-workers. We have discussed several ways in which their findings could possibly be reconciled with ours, but a definitive conclusion on this point will have to await further experiments.

ACKNOWLEDGMENTS

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[1]See, for example, G. Ahlers, in *Physics of Liquid and Solid Helium, Part I,* edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1978), p. 85; P. C. Hohenberg, in Proceedings of the 16th International Conference on Low Temperature Physics [Physica 109&110B, 1436 (1982)].

[2]See, for example, E. Guyon and I. Rudnick, J. Phys. (Paris) 29, 1081 (1968); D. J. Bishop, J. E. Berthold, J.

M. Parpia, and J. D. Reppy, Phys. Rev. B 24, 5047 (1981); these papers also contain extensive references to earlier work in this area.

[3]M. E. Fisher, in *Critical Phenomena,* edited by M. S. Green, *Proceedings of the "Enrico Fermi" Summer School, Varenna, Italy, Course No. 51* (Academic, New York, 1971), p. 1.

[4]See, for example, M. E. Fisher and A. E. Ferdinand, Phys. Rev. Lett. 19, 169 (1967); A. E. Ferdinand and M. E. Fisher, Phys. Rev. 185, 832 (1969); M. E. Fisher and M. N. Barber, Phys. Rev. Lett. 28, 1516 (1972); M. E. Fisher, J. Vac. Sci. Technol. 10, 665 (1973).

[5]K. Binder and P. C. Hohenberg, Phys. Rev. B 6, 3461 (1972); 9, 2194 (1974).

[6]References 4 and 5 also contain extensive references to other work in this area.

[7]See, for example, D. J. Bishop and J. D. Reppy, Phys. Rev. B 22, 5171 (1980); I. Rudnick, Phys. Rev. Lett. 40, 1454 (1978).

[8]Nuclepore Corporation, 7035 Commerce Circle, Pleasanton, CA 94566.

[9]For an introduction to the field of etched particle tracks, see R. L. Fleischer, P. B. Price, and R. M. Walker, *Nuclear Tracks in Solids* (University of California Press, Berkeley, 1975).

[10]H. A. Notarys, Phys. Rev. Lett. 22, 1240 (1969).

[11]G. G. Ihas and F. Pobell, Phys. Rev. A 9, 1278 (1974).

[12]W. C. Thomlinson, G. G. Ihas, and F. Pobell, Phys. Rev. B 11, 4292 (1975).

[13]T. P. Chen and F. M. Gasparini, Phys. Rev. Lett. 40, 331 (1978).

[14]F. M. Gasparini, T. P. Chen, and B. Bhattacharyya, Phys. Rev. B 23, 5797 (1981).

[15]J. S. Brooks, B. B. Sabo, P. C. Schubert, and W. Zimmermann, Jr., Phys. Rev. B 19, 4524 (1979).

[16]P. C. Schubert and W. Zimmermann, Jr., J. Low Temp. Phys. 44, 177 (1981).

[17]V. L. Ginzburg and L. P. Pitaevskii, Zh. Eksp. Teor. Fiz. 34, 1240 (1958) [Sov. Phys.—JETP 7, 858 (1958)]. For a recent review of this theory and extensions see V. L. Ginzburg and A. A. Sobaynin, Usp. Fiz. Nauk 120, 153 (1976) [Sov. Phys.—Usp. 19, 773 (1976)].

[18]See, for example, L. D. Landau and E. M. Lifshitz, *Statistical Physics,* 2nd ed. (Pergamon, Oxford, 1969).

[19]For a discussion of the general philosophy of Landau theory see, for example, L. P. Kadanoff, W. Götze, D. Hamblen, R. Hecht, E. A. S. Lewis, V. V. Palciauskas, M. Rayl, J. Swift, D. Aspnes, and J. Kane, Rev. Mod. Phys. 39, 395 (1967).

[20]Yu. G. Mamaladze, Zh. Eksp. Teor. Fiz. 52, 729 (1967) [Sov. Phys.—JETP 25, 479 (1967)].

[21]See, for example, H. E. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Oxford University Press, New York, 1971).

[22]Here and in what follows we assume the validity of the scaling relations $\nu=\nu'$, $\alpha=\alpha'$, etc. Strictly speaking, the theory (Ref. 3) predicts $\Lambda=1/\nu'$, since the correlation length for $T<T_\lambda$ is governed by the exponent ν'.

[23]The value of ν can be estimated in several ways. The measured specific-heat exponent, $\alpha=-0.026\pm0.004$ [K. H. Mueller, G. Ahlers, and F. Pobell, Phys. Rev. B 14, 2096 (1976)], together with the scaling relation $d\nu=2-\alpha$, where d is the spacial dimensionality, yields $\nu=0.675\pm0.001$ [see also G. Ahlers, Physica 107B, 347 (1981)]. Recent theoretical estimates [J. C. Le Guillou and J. Zinn-Justin, Phys. Rev. B 21, 3976 (1980)] give $\nu=0.669\pm0.002$.

[24]Given the uncertainties described in Ref. 23, we would expect the uncertainty in the prediction for Λ to be less than ±0.01, which is negligible for our purposes.

[25]N. D. Mermin and H. Wagner, Phys. Rev. Lett. 17, 1133 (1966); P. C. Hohenberg, Phys. Rev. 158, 383 (1967); G. V. Chester, M. E. Fisher, and N. D. Mermin *ibid.* 185, 760 (1969).

[26]See, for example, M. E. Banton, J. Low Temp. Phys. 16, 211 (1974).

[27]Lakeshore Cryotronics type S-1.

[28]C. Boghosian, H. Meyer, and J. E. Rives, Phys. Rev. 146, 110 (1966).

[29]A Hewlett-Packard 461A amplifier and a Data Precision 5740 frequency counter were used.

[30]Both thermometers were Cryocal model 2500H.

[31]See, for example, M. Barmatz and I. Rudnick, Phys. Rev. 170, 224 (1968).

[32]T. P. Chen, M. J. DiPirro, B. Bhattacharyya, and F. M. Gasparini, Rev. Sci. Instrum. 51, 846 (1980).

[33]The fountain pressure almost certainly played a major role in this behavior. While the fountain effect has been shown to be very important in these kinds of flow experiments [see, for example, W. E. Keller and E. F. Hammel, Cryogenics 5, 245 (1965)], it should not affect our results for $T_{\lambda s}$ since as will be seen below, we are concerned only with determining the location of the "qualitative" onset of superfluid flow.

[34]The flow rates above $T_{\lambda s}$ in Fig. 2(a) correspond to $J\sim4\times10^{-5}$ cm^3/s for this sample. This is to be compared with the theoretical value for viscous flow, $J=(\pi r^4\Delta p/8\eta l)n$ [see, for example, K. R. Symon, *Mechanics,* 3rd ed. (Addison-Wesley, Reading, Mass., (1971)], where r is the pore radius, Δp is the pressure drop, η is the viscosity, l is the length of a pore, and n is the number of pores. Using $\eta=25$ μP, $r=400$ Å, $l=5$ μm, $n=11.7\times10^8$, all values appropriate for this sample, and assuming $\Delta p=1$ mm of ^{4}He, one finds $J\sim2\times10^{-5}$ cm^3/s, in very reasonable agreement with the observed value.

[35]The "best" value for $T_{\lambda s}$ (the location of the arrow) and the uncertainty in $T_{\lambda s}$ (the horizontal bar) were arrived at as follows. For each measurement (e.g., Fig. 2) a value for $T_{\lambda s}$ and the associated uncertainty were obtained—this uncertainty arose from the scatter in the data [as evidenced, for example, by the vertical bar in Fig. 2(b)]. The measurement was then repeated a number of times (typically 5). The best value of $T_{\lambda s}$ (as indicated by the arrow and listed in Table I) was obtained by averaging the values obtained from the different measurements. The uncertainty in $T_{\lambda s}$ was then chosen such that it encompassed *every* individual measurement

of $T_{\lambda s}$ (given its uncertainty). We should also note that to within the individual experimental errors, the individual measurements were always consistent with each other. This explains why the arrow in Fig. 2 for example does not quite coincide with the best value of the inflection point for this measurement.

[36]In several of the measurements there is some ambiguity as to where the flow rate begins to increase. In Fig. 5 for example, the flow rate begins to increase significantly somewhat before the very sharp drop in frequency. In these cases the uncertainties in $T_{\lambda s}$ have been chosen to be large enough to encompass any reasonable choice of $T_{\lambda s}$. The meaning of "reasonable" is best illustrated by the size of the uncertainty shown in Fig. 5. For measurements like those shown in Fig. 4, we have chosen to assign uncertainties similar in size to those for the measurements shown in Figs. 2 and 3. This appears to be fairly conservative since the onset of increased flow occurs over a very small temperature range—i.e., this range is much smaller than the horizontal bar in Fig. 4. Nevertheless, since the detailed shape of the curve in Fig. 4 is not understood, we prefer to assign what we believe to be rather conservative uncertainties.

[37]The findings of P. Leiderer and F. Pobell, Phys. Rev. A 7, 1130 (1973), may be relevant to this problem.

[38]Moreover, since the capacitor chamber tube joined the pumping line for the ^{4}He bath outside the Dewar, the pressure in the empty capacitor may have been slightly lower than that in the surrounding ^{4}He bath.

[39]Actually, some liquid could enter through the pores via viscous flow of the normal fluid, but this was found to be negligible for the typical time scale of these experiments.

[40]In interpreting in detail the measurements in Fig. 5, it is necessary to consider the effect of the "dead volume" of the capacitor. This is the volume (see Fig. 1) which is above the sample but below the "active" region of the capacitor. If the data in Fig. 5 were followed to lower frequency, it would be seen that the frequency rapidly attains a value f_0, at which time it decreases more slowly until it reaches f_1. Thereafter, it decreases more rapidly. This behavior occurs for the following reason. When ^{4}He first begins to flow through the pores, the capacitor chamber rapidly fills with gas. This is the cause of the frequency decrease shown in Fig. 5. The frequency at which the capacitor is filled with gas is f_0. The liquid then flows into the dead volume of the capacitor causing only a relatively small change in frequency, i.e., $f_0 \rightarrow f_1$. When the liquid reaches the active region of the capacitor, the frequency change becomes more rapid. The type of behavior described above occurred in every measurement of this type. Moreover, independent measurements verified that the measured values of f_0 and f_1 did indeed correspond to the capacitor being filled with gas, and to the dead volume being filled with liquid, respectively.

[41]If any of the other measured estimates for the pore diameters were used, none of our results would be significantly altered.

[42]Attempts to measure the pore size of thinned samples using a transmission electron microscope were unsuccessful.

[43]The $\pm 10\%$ uncertainty in the pore diameters has been taken into account in arriving at this estimate for the uncertainty in Λ. The fit also yielded $A = 34 \pm 18$. The only theoretical prediction for A comes from the modified mean-field theory of Mamaladze (Ref. 20) which predicts $A = 47.4$. The units here are such that d is in Å and ΔT_λ in K.

[44]Note also the potential problems with epoxy partially filling the pores (see Ref. 16 and also our discussion in Sec. III). This effect would presumably be most pronounced for the larger pore sizes.

[45]The analysis we refer to here was a detailed comparison of the heat-capacity data to a model "equation of state." It was therefore possible to use *all* of the heat-capacity data in the analysis, instead of just using the data which determine the location of the maximum in the heat capacity. Thus, in a sense one might expect [F. M. Gasparini (private communication); see also Ref. 14] that the former analysis would be more accurate and/or reliable, since it involves many more data points than an analysis which involves only the location of the heat-capacity maximum. However, we do not find this argument to be convincing. Previous experience with magnetic systems in which one can compare a measured equation of state with theoretical predictions of "data collapsing," indicates that one must be very careful in estimating the uncertainties associated with such an analysis [see, for example, G. F. Tuthill, F. Harbus, and H. E. Stanley, Phys. Rev. Lett. 31, 527 (1973), together with the discussion by M. Blume, L. M. Corliss, J. M. Hastings and E. Schiller, Phys. Rev. Lett. 32, 544 (1974)]. The difficulties stem mainly from the lack of a quantitative theoretical prediction for the equation of state, as well as uncertainties in the range over which data collapsing should occur. This leads to very subjective "goodness of fit" criteria. These kinds of problems are also present in the analysis described in Ref. 14. While there certainly do appear to be good theoretical reasons for believing that Eq. (20) in Ref. 14 is an accurate prediction for the size-dependent heat capacity, the range of temperature and pore size over which this prediction should be accurate is very uncertain. For example, it is by no means obvious that the free energy can be neatly separated into bulk and surface contributions (a key assumption of the theory) when the correlation length is equal to the pore radius, but data in this range was used in the analysis. When these types of uncertainties are combined with experimental ones (such as the possible systematic uncertainties in d), determining the "goodness" of a fit, or the uncertainty in the value of an exponent becomes very subjective. Thus, while we believe that the analysis in Ref. 14 was performed as well as possible given the associated theoretical and experimental uncertainties, it is our opinion that because of potential pitfalls of the type discussed above, the

values of exponents derived from this analysis are no more (and perhaps even less) reliable than those obtained in the much more straightforward analysis of the heat-capacity maximum, etc.

[46]Note that the detailed SEM analysis of Ref. 32, which presumably utilized samples from the same batches as those studied in Refs. 13 and 14, yielded a pore size about 10—20% below the nominal value for a 2000-Å sample. One can also see from Fig. 8 that a slight change in the value of d for their nominal 2000-Å sample would make a line drawn through the data of Gasparini *et al.* have a slope (and hence exponent value) very close to what we find. Our point here is that small changes in the values of d, changes which are well within possible systematic uncertainties, could bring the results of Gasparini *et al.* into agreement with our results.

[47]It is interesting to note that heat-capacity measurements on bulk ^{4}He [see, for example, F. M. Gasparini and M. R. Moldover, Phys. Rev. B 12, 93 (1975)] indicate that the "asymptotic" region for this quantity extends to about $\Delta T \simeq 7$ mK. Of course, the size of this region depends upon the function used to fit the heat capacity, and the size of the deviations one is willing to accept. However, this value of ΔT does suggest that the asymptotic region in our experiments could well correspond to $d > 150$ Å.