Brief Reports

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Pulsed nuclear magnetic resonance and soliton lattice in ^{3}He-A

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When the magnetization of ^{3}He-A is tipped by a finite angle, the magnetization precesses with definite angular velocity reflecting the underlying texture. We analyze theoretically the precession frequency in the presence of composite solitons in ^{3}He-A and compare the results with recent experiments of Giannetta *et al.* and of Gould *et al.*

Recently, Kharadze and Maki (KM)[1] have formulated the precession frequency of the tilted magnetization in ^{3}He-A in the presence of textures as the eigenvalues problem and applied the formalism to the case of a twist composite soliton.[2] The object of this report is twofold: First, we shall extend the formalism to cover more general situations and apply to the general composite soliton[3] and the splay soliton lattice.[4] Second, we shall compare these results with recent tipping experiments of Giannetta *et al.*[5] and of Gould *et al.*[6] We shall show that the precession frequency observed by Giannetta *et al.* can be interpreted in terms of the one in the presence of isolated splay solitons, while the precession frequency seen by Gould *et al.* is consistent with the one in the presence of a slowly expanding splay soliton lattice.

Limiting ourselves to the case where the texture depends only on a single space variable $s = z\cos\sigma + y\sin\sigma$, we obtain an equation of motion for β, one of the Euler angles describing the spin motion[7] of ^{3}He-A:

$$\begin{aligned}\beta_{tt} - C^2\partial_s[(1-\tfrac{1}{2}\sin^2\alpha\cos^2\chi)\sin\gamma(\sin\gamma\,\beta_s - \cos\gamma\sin\beta\,\alpha_s)] \\ +\sin\beta[(\alpha_t-\omega_0)\gamma_t - C^2(1-\tfrac{1}{2}\sin^2\gamma\cos^2\chi)\alpha_s(\cos\beta\sin^2\gamma\,\alpha_s+\gamma_s+\cot\beta\sin\gamma\cos\gamma\,\beta_s)] \\ -\Omega^2\sin\beta\sin(\alpha-\chi)\sin\gamma[\cos(\alpha-\chi)\cos\gamma - \sin(\alpha-\chi)\cos\beta\sin\gamma] = 0 \ , \quad (1)\end{aligned}$$

which generalizes Eq. (15) of KM.

Here C is the spin-wave velocity; Ω is the Leggett frequency; ω_0 is the Larmor frequency; α, β, and γ are Euler angles describing the spin configuration; and χ describes the equilibrium $\hat{l}$ configuration given by

$$\hat{l} = -\sin\chi\,\hat{x} + \cos\chi\,\hat{y} \ . \quad (2)$$

We assumed that a strong magnetic field is applied along the z axis so that both $\hat{l}$ and $\hat{d}$ lie in the x-y plane in the equilibrium configuration. Finally, the subscripts t and s on α, β, and γ are used to denote the partial derivatives.

Now Eq. (1) has to be solved with the initial conditions[1,8]

$$\alpha_t = -\gamma_t = \omega_0, \quad \beta = \theta \quad (3)$$

and

$$\alpha + \gamma = \phi_0(s) \ , \quad (4)$$

where θ is the tipping angle of the magnetization and $\phi_0(s)$ describes the equilibrium $\hat{d}$ configuration given by

$$\hat{d}_0 = -\sin[\phi_0(s)]\hat{x} + \cos[\phi_0(s)]\hat{y} \quad (5)$$

in the presence of the composite soliton.

Substituting

$$\alpha + \gamma = \phi_0(s) + \phi' \ ,$$

$$\alpha - \gamma = 2\tilde{\omega}t + \psi' \ ,$$

and

$$\beta = \theta = \Psi \quad (6)$$

into Eq. (1) and linearizing in ϕ', ψ', and Ψ, we obtain

$$\omega_0\,\delta\omega\,\Psi = -\tfrac{1}{2}(\cos\theta)^{-1}\frac{d}{dx}[(1-\tfrac{1}{2}\sin^2\sigma\cos^2\chi)\Psi_s] + \tfrac{1}{8}C^2(2+\cos\theta)(-\tfrac{1}{2}\sin^2\sigma\cos^2\chi)(\phi_{0s})^2\Psi + \tfrac{1}{8}\Omega^2[1+3\cos\theta-2(1+\cos\theta)\sin^2 v]\Psi \;, \quad (7)$$

where

$$\delta\omega = \tilde{\omega}-\omega_0, \quad v=\chi-\phi_0 \;, \quad (8)$$

and $\tilde{\omega}$ is the precession frequency of the magnetization. Equation (7) generalizes a similar equation found for the twist soliton in KM. We shall apply Eq. (7) to two cases of particular interest.

(a) Isolated composite soliton. Since the equilibrium configuration of $\hat{d}$ and $\hat{l}$ (ϕ_0 and χ) are known in the general composite soliton,[3] the eigenvalue is determined variationally. We have chosen $\Psi \propto \mathrm{sech}^{v}(\eta s)$, where v is a variational parameter and η is the parameter characterizing the size of the composite soliton [viz., $\sin v \equiv \mathrm{sech}(\eta s)$] and has been determined in Ref. 3. In Fig. 1 the shifts in the precession frequency for $\sigma=0°$ (twist soliton), $\sigma=45°$, and $\sigma=90°$ (splay soliton) are shown as functions of θ the tipping angle. In particular, the precession frequency for the splay soliton ($\sigma=90°$) describes accurately that observed by Giannetta *et al.*[5] in the presence of the solitons. Therefore this provides another proof for existence of these solitons after a large angle tipping. As already pointed out in KM, these solitons become unstable for $\theta \gtrsim 90°$; a tipping experiment with $\theta > 90°$ in the presence of solitons appears to destroy these solitons in ^{3}He-*A*. This feature has already been noted in the above experiment.

(b) Soliton lattice. Now we shall consider the precession frequency in the presence of a tightly packed soliton lattice. This is motivated by the fact that, whereas the experiment of Giannetta *et al.*[5] is consistent with the picture of the isolated solitons, the experiment of Gould *et al.*[6] appears to require a dense array of solitons. This is most likely due to the difference in the waiting times in these experiments. In the experiment of Giannetta *et al.* the waiting time is about 10 sec; after a large-angle tipping of magnetization, which creates a large number of solitons, they waited about 10 sec to start the pulsed NMR experiment. On the other hand, in the experiment of Gould *et al.* the waiting time is of the order of 10 msec, so that there appears to be no time for the tightly packed soliton lattice to relax.[4,9] Making use of the variational solution for the equilibrium soliton lattice obtained by Bruinsma and Maki,[4] the precession frequency is calculated variationally as a function of θ the tipping angle for different values of L_0/ξ, where L_0 is the distance between two solitons (i.e., the soliton lattice constant) and $\xi = C/\Omega$ ($\sim 10\ \mu$m) is the dipole coherence distance. These results are shown in Fig. 2 for $L_0/\xi = 0$, 2, 5, and ∞. In the limit of $l = L_0/\xi \ll 1$, we have an asymptotic result:

$$\delta\omega = (8\omega_0)^{-1}\Omega^2\left\{1+3\cos\theta-\tfrac{7}{32}(1+\cos\theta)\times\left[1-\tfrac{10}{21}\left(\frac{l}{2\pi}\right)^2\right]\right\} \;, \quad (9)$$

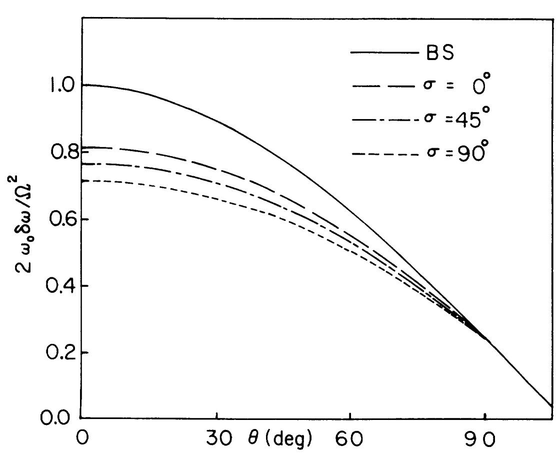

FIG. 1. Shift in the precession frequencies in the presence of the composite solitons with $\sigma=0°$ (twist case), $\sigma=45°$ and $\sigma=90°$ (splay case) are shown as functions of θ, the tipping angle. These curves terminate at $\theta=90°$, implying that solitons become unstable for $\theta > 90°$.

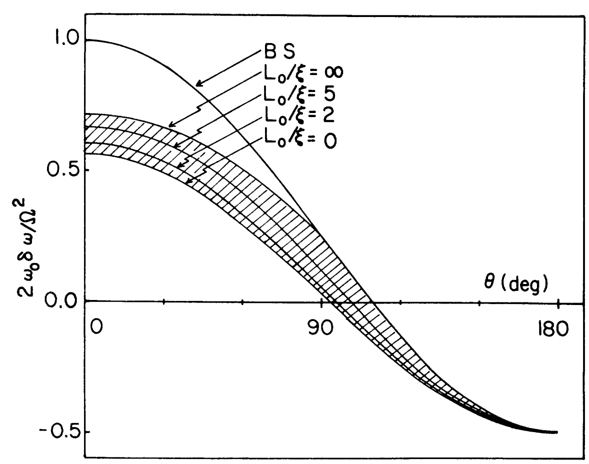

FIG. 2. Shift in the precession frequencies of the splay soliton lattice for several lattice constants L_0 are shown as functions of θ. The precession frequencies associated with soliton lattices appear only in the shaded area.

where the first term in the bracket is the well-known Brinkman-Smith (BS) result.[8]

In general, the precession frequency for given θ increases monotonically as $l\,(=L_0/\xi)$ increases. Therefore the precession frequency in the presence of the soliton lattice is limited from below by that corresponding to the soliton lattice with vanishing l ($l \rightarrow 0$), and from above by that corresponding to the isolated solitons for $0° \leq \theta \leq 90°$ and by that of the uniform texture (BS)[8] for $\theta > 90°$ (the shaded area in Fig. 2); the precession frequency should be always below the Brinkman-Smith curve. By comparing this result with the experiment of Gould *et al.*, it appears that for small θ the observed precession frequency is consistent with the theory with small l (i.e., $l < 2$). On the other hand, in order to account for the observed precession frequency for $\theta \simeq 90°-100°$ we need a soliton lattice with $l \sim 10$. It is, however, quite possible that in the course of applying the second tipping signal the soliton lattice is somewhat expanded, although this has to be verified.

Furthermore, according to the present theory, the precession frequency for the soliton lattice never exceeds that for the uniform texture. However, some of the experimental data[6] for $\theta \gtrsim 100°$ appear to be above the BS limit, which is difficult to interpret within the present theory.

In summary, we have analyzed the pulsed—nuclear-magnetic-resonance experiments[5,6] after a large-angle tipping of the magnetization. The observed precession frequency of the tilted magnetization is interpreted in terms of the solitons and the soliton lattice in ^{3}He-A, which are created by the large-angle tipping of the magnetization. It appears that the precession frequency will provide a new signature of soliton lattice, which reflects sensitively the soliton density.

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