Temperature dependence of the magnetic susceptibility of almost-ferromagnetic materials

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We present a microscopic calculation of the magnetic susceptibility of exchange-enhanced materials based on the paramagnon model for the interaction of electrons with spin fluctuations. By explicitly calculating the vertex corrections, we find that, in a spin-conserving approximation, the low-temperature magnetic susceptibility contains anomalous logarithmic corrections that account for the maximum in $\chi(T)$ observed in many materials. This is the result expected on the basis of phenomenological Landau theory. Our results, which are not limited to low temperatures, show that at temperatures above the spin-fluctuation temperature the inverse susceptibility increases almost linearly with T, in agreement with spin-fluctuation theories.

I. INTRODUCTION

The static magnetic susceptibility of nearly ferromagnetic Fermi systems, such as ^3He, Pd, and Ni_3Ga, is large and depends strongly on temperature. Since the natural temperature scale for Fermi systems is the Fermi temperature T_F and, under ordinary circumstances $T/T_F \ll 1$ this behavior is at first sight surprising. In many Fermi systems (e.g., ^3He, Ni_3Ga) the quantity $\alpha(T)=\chi^{-1}(T)\chi_p$ first rises as T^2 at very low temperatures ($T \ll \Theta$) and then changes over to a nearly linear behavior for $T > \Theta$, where Θ is a characteristic temperature of order T_F/S, and S is the enhancement of the zero-temperature susceptibility over its free-electron value. This behavior has been explained[1] by a dynamical model that represents almost-ferromagnetic systems of fermions as a collection of interacting spin and density fluctuations. The results of such a theory have been applied successfully[1] to itinerant systems, like ^3He and some intermetallic compounds (e.g., $HfZn_2$, $Ni_{1-x}Rh_x$).

There is yet another class of materials[2] (e.g., Pd, α-Mn, U_2C_3, $NpCo_2$, YCo_2, and $LuCo_2$), in which the susceptibility exhibits a maximum at low temperature before falling off as the temperature increases. The magnitude of this effect varies from material to material ranging from a maximum of about 6–10% above $\chi(0)$ in the case of Pd, to cases like YCo_2 and $LuCo_2$, where the maximum value of χ is almost twice the value at $T=0$ K. Experimentally it is well established[2] that this maximum can be described by adding a logarithmic correction to the susceptibility of the form $\delta\chi \sim T^2 \ln T$.

The origin of this anomalous feature has been the subject of much controversy.[3,4] In a number of papers the existence of a $T^2 \ln T$ term in the susceptibility has been claimed, both on the basis of general Fermi-liquid arguments,[5] and also as the result of the solution of particular microscopic models,[2,6] the two approaches giving different estimates of the size of the effect.

In the microscopic approach one tries to go beyond the standard random-phase approximation by dressing the fermion propagators with the self-energy corrections that result from their interaction with the spin-density fluctuations. It is well known,[7] however, that if such dressing is done, appropriate vertex corrections must be included in order for the response functions to obey the conservation laws. The main effect of the self-energy correction is to introduce nonanalytic terms in the expansion of the quasiparticle energies. These singular terms give a nonanalytic correction to the quasiparticle density of states near the Fermi level of the form $\delta N(\epsilon) \sim \epsilon^2 \ln\epsilon$. It is obvious that the convolution of this with the derivative of the Fermi factor will give a $T^2 \ln T$ term in the magnetic susceptibility. However, the vertex corrections may have compensating contributions and the anomalous terms can be canceled. Indeed, it has been suggested[3] that this is the case if one uses the ladder-bubble approximation for the self-energy. This is contradiction with some previous work.[2]

In this paper we adopt the paramagnon model[8] for the interaction of the d-band electrons with spin fluctuations. As is usual, no account is taken of the s-band electrons beyond including explicitly their screening of the Coulomb interaction between the d electrons. By direct calculation of the vertex corrections, we show the existence of a nonvanishing logarithmic contribution to the low-temperature susceptibility. Our results are valid throughout the degeneracy region and are not restricted to $T \ll \Theta$. Indeed, for $T \geq \Theta$ they agree with those of spin-fluctuation theory.[1] Our approach is related to the one used previously,[2] but is different in detail and avoids the approximations used in the latter work that lead to a spurious logarithmic anomaly.

The general formalism is presented in Sec. II. Our results for the paramagnon model are described in Sec. III. Comparison with other work and some concluding remarks follow in Sec. IV.

II. FORMALISM

In this section we derive an expression for the static susceptibility of a system of interacting fermions in terms of the real spin-antisymmetric scattering amplitude. This expression is a finite-temperature generalization of well-known ground-state results.[9] We use the temperature Green's-function formalism throughout.[10]

The magnetization of a system of fermions in the presence of a magnetic field $\vec{H}$ is given by

$$\vec{M} = -\mu_0^2 \frac{1}{(\beta V)^2} \sum_{pp'} D^k_{\alpha\beta,\nu\mu}(p,p') \vec{\sigma}_{\beta\alpha}(\vec{\sigma}_{\mu\nu}\cdot\vec{H}), \tag{1}$$

where the summations are over four-momenta $p \equiv (\vec{p}, i\nu)$, $i\nu$ are the fermion Matsubara frequencies, μ_0 is the Bohr magneton, and $D_{\alpha\beta,\nu\mu}$ is the so-called k limit of the Fourier transform of the two-particle Green's function, taken in the particle-hole channel,[10] i.e.,

$$D^k_{\alpha\beta,\nu\mu}(p,p') = \lim_{\substack{|\vec{k}|\to 0,\\ k_0/|\vec{k}|\to 0}} D_{\alpha\beta,\nu\mu}(k;p,p'), \tag{2}$$

where

$$D_{\alpha\beta,\nu\mu}(k;p,p') = \int d^4(1-2) \int d^4(1-1') \int d^4(2'-2) D_{\alpha\beta,\nu\mu}(11',22') \times \exp\{-i[k(1-2)+p(1-1')+p'(2'-2)]\}. \tag{3}$$

The two-particle Green's function is defined as

$$D_{\alpha\beta,\nu\mu}(11',22') = -\langle T\{\psi_\alpha(1)\psi^\dagger_\beta(1')\psi_\nu(2)\psi^\dagger_\mu(2')\}\rangle. \tag{4}$$

For spin-independent forces, $D_{\alpha\beta,\nu\mu}$ has only two independent components, namely those corresponding to the singlet- and triplet-spin states. In terms of these components

$$D_{\alpha\beta,\nu\mu}(k;p,p') = \tfrac{1}{2}[D_1(k;p,p')\delta_{\alpha\beta}\delta_{\nu\mu} + D_2(k;p,p')\vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\nu\mu}]. \tag{5}$$

Substituting (5) into (1) and contracting spin indices, one obtains the static susceptibility

$$\chi = \vec{M}/\vec{H} = -2\mu_0^2 \frac{1}{(\beta V)^2} \sum_{pp'} D^k_2(p,p'). \tag{6}$$

The two-particle Green's function is related to the vertex part[10] via the definition

$$D_2(k;p,p') = G(p)G(p+k)[\beta V \delta_{pp'} + \Gamma_2(k;p,p')G(p')G(p'+k)]. \tag{7}$$

Substituting (7) into (6) one gets an alternative and more useful expression for χ:

$$\chi = -2\mu_0^2 \frac{1}{\beta V} \sum_p R^k_p [1+\gamma^k_2(p)], \tag{8}$$

where $\gamma^k_2(p)$ is the k limit of $\gamma_2(k,p)$

$$\gamma_2(k,p) = \frac{1}{\beta V} \sum_{p'} \Gamma_2(k;p,p')G(p')G(p'+k), \tag{9}$$

and R^k_p is the k limit of the product of two Green's functions,

$$R_p(k) = G(p)G(p+k). \tag{10}$$

The vertex part is the solution of the Bethe-Salpeter equation which, in the low—momentum-transfer limit, may be written as

$$\Gamma_2(k;p,p') = \Gamma^{(0)}_2(p,p') + \frac{1}{\beta V} \sum_q \Gamma^{(0)}_2(p,q) R_q(k) \times \Gamma_2(k;q,p'), \tag{11}$$

where $\Gamma^{(0)}_2(p,p')$ is the irreducible vertex part.

For a given approximation for the self-energy, $\Gamma^0_2(p,p')$ must be chosen as

$$\Gamma^{(0)}_2(p,p') = \frac{\delta\Sigma\uparrow(p)}{\delta G\downarrow(p')} \tag{12}$$

in order for the response function to be consistent with the conservation laws.[7]

Now we start a series of formal manipulations to rewrite (8) in terms of real scattering amplitudes. The following relations[10] will be used:

$$R_p^k = R_p^\omega - \beta Z_{\vec{p}}^2 \left[-\frac{\partial f}{\partial E_{\vec{p}}} \right] \delta_{p_0, E_{\vec{p}}}, \tag{13}$$

$$\frac{\partial \Sigma_{\vec{p}}(p_0)}{\partial p_0} = -\gamma_i^\omega(p), \quad i=1,2 \tag{14}$$

$$\sum_p R_p^\omega [1+\gamma_i^\omega(p)] = 0, \quad i=1,2. \tag{15}$$

Equation (14) is a Ward identity which can be derived from the conservation of particle number, Eq. (15) is a result of the structure of the perturbation series for the self-energy, and $\gamma_i^\omega(p)$ is the ω limit of $\gamma_2(k,p)$, i.e.,

$$\gamma_i^\omega(p) = \lim_{\omega\to 0} [\lim_{k\to 0} \gamma_i(p,k)].$$

The relation (13) between the k and ω limits of $R_p(k)$ (i.e., R_p^k and R_p^ω, respectively) involves the renormalization factor on the energy shell,

$$Z_{\vec{p}}^{-1} = 1 - \frac{\partial}{\partial \omega} \mathrm{Re}\Sigma_{\vec{p}}(\omega) \,|_{\omega=E_{\vec{p}}}, \tag{16}$$

where $E_{\vec{p}}$ are the quasiparticle energies

$$E_{\vec{p}} = e_{\vec{p}} + \mathrm{Re}\Sigma_{\vec{p}}(E_{\vec{p}}), \tag{17}$$

and $e_{\vec{p}}$ are the bare energies measured from the chemical potential.

The Bethe-Salpeter equation (11) may be combined with (13) to derive the finite-temperature version of the Landau equation[10] relating the k and ω limits of the vertex part:

$$\Gamma_2^k(p,p') = \Gamma_2^\omega(p,p') - \frac{1}{V} \sum_{\vec{q}} \Gamma_2^k(p,\vec{q}) Z_{\vec{q}}^2 \left[-\frac{\partial f}{\partial E_{\vec{q}}} \right] \times \Gamma_2^\omega(\vec{q},p'), \tag{18}$$

where the presence of $\vec{q}$ as an argument indicates that the momentum is taken on the energy shell, i.e., $q \equiv (\vec{q}, q_0) = (\vec{q}, E_{\vec{q}})$.

From (14), (16), and (18) one can derive a relation between the k and ω limits of $\gamma_2(k,p)$, Eq. (9):

$$\gamma_2^k(p) = \gamma_2^\omega(p) - \frac{1}{V} \sum_{\vec{q}} \Gamma_2^k(p,\vec{q}) Z_{\vec{q}} \left[-\frac{\partial f}{\partial E_{\vec{q}}} \right]. \tag{19}$$

Evaluation of (19) on the energy shell and use of (13) and (16) gives

$$\gamma_2^k(\vec{p}) = Z_{\vec{p}}^{-1} - 1 \frac{-1}{V} \sum_{\vec{q}} \Gamma_2^k(\vec{p},\vec{q}) \left[-\frac{\partial f}{\partial E_{\vec{q}}} \right]. \tag{20}$$

Substituting (19) into (8) and using Eqs. (13)–(16) one obtains

$$\chi = 2\mu_0^2 \frac{1}{V} \sum_{\vec{p}} Z_{\vec{p}} \left[-\frac{\partial f}{\partial E_{\vec{p}}} \right] [1+\gamma_2^k(\vec{p})]. \tag{21}$$

Finally, by combining (20) and (21) one gets the susceptibility in terms of the vertex part, evaluated on the energy shell:

$$\chi = 2\mu_0^2 \left[\frac{1}{V} \sum_{\vec{p}} \left[-\frac{\partial f}{\partial E_{\vec{p}}} \right] - \frac{1}{V^2} \sum_{\vec{p},\vec{p}'} Z_{\vec{p}} Z_{\vec{p}'} \left[-\frac{\partial f}{\partial E_{\vec{p}}} \right] \left[-\frac{\partial f}{\partial E_{\vec{p}'}} \right] \Gamma_2^k(\vec{p},\vec{p}') \right]. \tag{22}$$

At zero temperature the momentum integrations are restricted to the Fermi surface. For a parabolic band (22) yields the well-known result[9]:

$$\chi(0) = 2\mu_0^2 N^*(0)[1-A_2(0)], \tag{23}$$

where

$$N^*(0) = \frac{1}{V} \sum_{\vec{p}} \delta(E_{\vec{p}}) \tag{24}$$

is the quasiparticle density of states at the Fermi level and

$$A_2(0) = Z_0^2 N^*(0) \int \frac{d\hat{p}}{4\pi} \Gamma_2^k(\hat{p},\hat{p}') \tag{25}$$

is the angular average of the scattering amplitude on the Fermi surface.

From Eq. (22) we obtain the finite-temperature analog of (25), i.e.,

$$\chi(T) = 2\mu_0^2 \int_{-\infty}^{\infty} dE \left[-\frac{\partial f}{\partial E} \right] N^*(E)\,[1-A_2(E)], \tag{26}$$

where the finite-temperature analog of the scattering amplitude is

$$A_2(E) = Z(E) \int_{-\infty}^{\infty} dE' \left[-\frac{\partial f}{\partial E'} \right] N^*(E') Z(E') \times \int \frac{d\hat{p}'}{4\pi} \Gamma_2^k(\hat{p},E;\hat{p}',E'). \tag{27}$$

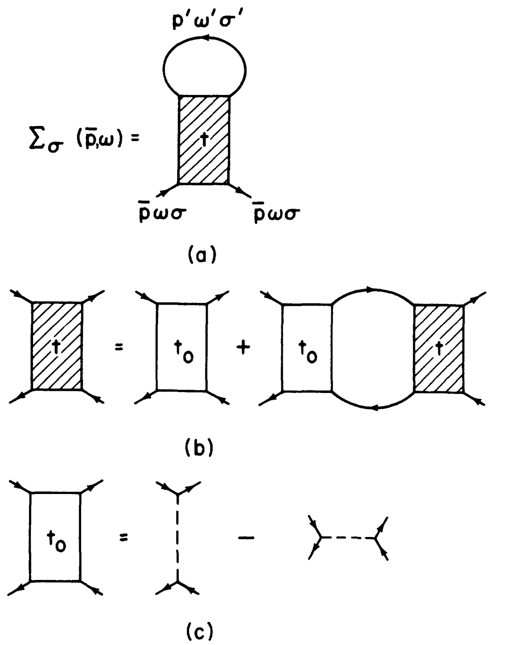

FIG. 1. (a)—(c) Diagrammatic representation of the paramagnon approximation to the self-energy. See text for the meaning of the symbols.

In deriving (26) and (27) we assumed that the explicit momentum dependence of the renormalization factor and the vertex part is smooth and, for a degenerate system ($T/T_F \ll 1$), the momenta can be placed on the Fermi surface. Their implicit momentum dependence, through the momentum dependence of the quasiparticle energies, has, however, been maintained.

III. RESULTS FOR THE PARAMAGNON MODEL

The approximation[8] used for the self-energy in the paramagnon model is shown in Figs. 1(a)—1(c). The solid lines represent fermion propagators for the d-band electrons, and the dotted lines represent their mutual Coloumb interaction as screened by the s electrons. We take this screened interaction to be of zero range and strength U. In this approximation the electron is coupled to particle-hole excitations only, and no scattering in the particle-particle channel is taken into account. As will be discussed later, the inclusion of particle-particle scattering would enormously complicate the calculation without changing the nature of the results. The analytic expression associated with Fig. 1(a) is

$$\Sigma(p)=\frac{1}{2\beta V}\sum_{p'}[t_1(p,p')+3t_2(p,p')]G(p'), \tag{28}$$

where the spin-symmetric and antisymmetric parts of the t matrix are

$$t_1(k)=-\frac{U}{1+U\pi(k)}, \tag{29}$$

$$t_2(k)=\frac{U}{1-U\pi(k)}, \tag{30}$$

and

$$\pi(k)=-\frac{1}{\beta V}\sum_p G(p+k)G(p) \tag{31}$$

is the normalized polarizability. Near the ferromagnetic instability $t_2 \gg t_1$, and so t_1 can be neglected and the self-energy becomes

$$\Sigma(\vec{p},i\nu)=\frac{3U}{2V}\sum_{\vec{p}}\int_{-\alpha}^{\infty}\frac{d\omega}{\pi}\int_{-\infty}^{\infty}\frac{dE'}{\pi}D''(\vec{q},\omega)\rho(\vec{p}+\vec{q},E')\left[\frac{f(E')+g(\omega)}{E'-\omega-i\nu}\right] \tag{32}$$

$$=\Sigma_F(\vec{p},i\nu)+\Sigma_B(\vec{p},i\nu), \tag{33}$$

where the last equality defines the Fermi and Bose parts of the self-energy. In Eq. (33) $f(\omega)$ and $g(\omega)$ are the Fermi and Bose distribution functions and $D''(\vec{q}\omega)$ and $\rho(\vec{q},E)$ the spectral functions for spin fluctuations and fermions, respectively.

Within the quasiparticle approximation, for the fermion's Green's function we have

$$\rho(\vec{p},\omega)=Z(\omega)\pi\delta(\omega-E_{\vec{p}}) \tag{34}$$

and

$$D''(\vec{q},\omega)=-\frac{\gamma\omega/qv_F^*}{\alpha^2+(\gamma\omega/qv_F^*)^2}, \tag{35}$$

where

$$\alpha=1-UN^*(0)Z^2(0), \tag{36}$$

γ depends on the band structure ($\gamma=\pi/2$ for a parabolic band), and v_F^* is an average over the Fermi surface of the quasiparticle velocity. Equation (35) is valid for $\omega \lesssim qv_F^*$ and $q \lesssim \Lambda$. The cutoff momentum Λ depends on the band structure and will be written as $\Lambda=\eta k_F$, where η is a constant of order 1 ($\eta=2$ for a parabolic band).

The self-energy enters the calculation of the susceptibility only through the frequency and temperature-dependent renormalization factor. The latter can be calculated from Eqs. (32)–(36). After some tedious but straightforward algebra one obtains the following results, valid in the limit of large enhancement ($\alpha\to 0$):

$$Z^{-1}(\omega T)=\frac{m^*}{m}[1-L_F(\omega,T)-L_B(T)] \tag{37}$$

with

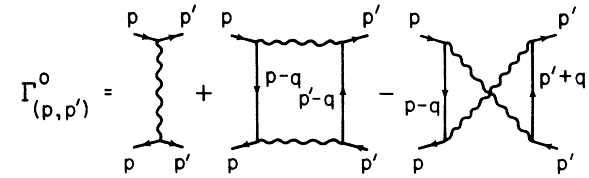

FIG. 2. Irreducible vertex part obtained from Fig. 1 by functional differentiation. The solid wavy lines stand for the total renormalized electron-hole interaction given in Eqs. (29) and (30) of the text.

$$L_F(\omega,T)=-\frac{3}{8}\left[\frac{\tilde{U}}{\alpha}\right]\frac{m}{m^*}\int_{-\infty}^{\infty}d\omega'\left[-\frac{\partial f}{\partial\omega'}\right]\left[\frac{\omega-\omega'}{k_B\Theta}\right]^2\ln\left[\frac{(\omega-\omega')^2+(k_B\Theta)^2}{(\omega-\omega')^2}\right], \tag{38}$$

$$L_B^{-1}(T)=1+\left[\frac{6}{\pi^2}\left[\frac{T}{\Theta}\right]^2\int_0^{\eta\Theta/T}yF(y)dy\right]^{-1}, \tag{39}$$

where

$$F(y)=\frac{1}{2}\left[\ln\left[\frac{y}{2\pi}\right]-\frac{\pi}{y}-\psi\left[\frac{y}{2\pi}\right]\right], \tag{40}$$

and $\psi(x)$ is the digamma function. In Eqs. (38) and (39) Θ is the spin-fluctuation temperature, $\Theta=2T_F/\gamma S$, and m^* is the zero-temperature effective mass.[8] The normalization factor can be evaluated in closed form both for $T/\Theta \ll 1$ and $T/\Theta \gg 1$ with the following results:

$$Z^{-1}(\omega,T)=\begin{cases}\dfrac{m^*}{m}\left\{1+\dfrac{3}{4}\left[\dfrac{\tilde{U}}{\alpha}\right]\dfrac{m}{m^*}\left[\left[\dfrac{\omega}{k_B\Theta}\right]^2+\dfrac{\pi^2}{3}\left[\dfrac{T}{\Theta}\right]^2\right]\ln\left|\dfrac{\omega}{\eta k_B\Theta}\right|\right\}, & T\ll\Theta\\[2ex] \dfrac{m_*}{m}\left[1+\dfrac{3\eta}{\pi}\left[\dfrac{T}{\Theta}\right]\right]^{-1}, & T\gg\Theta\end{cases} \tag{41}$$

where, again, only the leading terms in the enhancement are kept. The low-temperature result, which comes from the Fermi contribution, has been discussed by several authors[11] in the context of the low-temperature anomaly in the specific heat of Fermi systems. The high-temperature result, on the other hand, comes from the Bose term in the self-energy and is essential for the determination of the temperature dependence of χ for $T \gtrsim \Theta$.

The irreducible vertex associated with this choice of self-energy is shown in Fig. 2. In this figure the solid wavy lines represent the total renormalized particle-hole interaction whose spin decomposition is given in Eqs. (29) and (30) above. Had particle-particle interactions been kept in the self-energy, the expression for the vertex part would have included the whole series of parquet diagrams. The evaluation of these would have made the problem intractable. Since the physical origin of the effects of interest here is the energy transfer between electrons and a long-lived soft mode (the paramagnon), they should not be qualitatively affected by particle-particle scattering, a process that does not lead to the appearance of soft modes. Figure 2, evaluated in the ω limit, represents $\Gamma^{\omega}(p,p')$. In the limit of large enhancement the paramagnon pole dominates the integrals contributing to Fig. 2. Because of this "paramagnon dominance," Γ^{ω} can be calculated in closed form. The expressions are, however, complicated and not very illuminating, and we only quote here the high- and low-temperature results:

$$A^{\omega}(\omega,\omega')=\begin{cases}-\left[\dfrac{\tilde{U}}{\alpha}\right]\left[1-\left[\dfrac{\omega-\omega'}{k_B\Theta}\right]^2\ln\left|\dfrac{k_B\Theta^*}{\omega-\omega'}\right|\right], & T\ll\Theta\\[2ex] -\left[\dfrac{\tilde{U}}{\alpha}\dfrac{m}{m^*}\right]^2 2\pi\alpha\left[\dfrac{T}{\Theta}\right], & T\gg\Theta\end{cases} \tag{42}$$

where we have defined a dimensionless angular average

$$A^{\omega}(\omega,\omega')=N^*(0)Z_0\int\frac{d\hat{p}'}{4\pi}\Gamma^{\omega}(\hat{p},\omega;\hat{p}'\omega'). \tag{43}$$

The integral equation for $\Gamma^k(p,p')$, Eq. (18), can now readily be solved in the degenerate case, $T/T_F \ll 1$, with no conditions imposed on the ratio T/Θ. Once $\Gamma^k(p,p')$ is known, the susceptibility can be evaluated from (26) and (27). The algebra is tedious and the results rather complicated. However, they again simplify for both high and low temperature in the limit of large enhancement. Explicitly, as $\alpha \to 0$,

$$\frac{\chi(T)}{\chi(0)}=\begin{cases}1+S\dfrac{\pi^2}{6}\left[\dfrac{m}{m^*}\right]\left[\dfrac{T}{\Theta}\right]^2\ln\left[\dfrac{T^*}{T}\right], & T\ll\Theta\\[2ex] \dfrac{m^*}{m}\dfrac{6}{S\pi}\left[\dfrac{\Theta}{T}\right], & T\gg\Theta\end{cases} \tag{44}$$

where S is the enhancement, $S=\chi(0)\chi_p^{-1}$. The temperature T^* cannot be determined reliably from the low-temperature expansion, but is of the order of Θ. In any case, it can be estimated numerically.

The low-temperature result exhibits the triply enhanced contribution to the logarithmic term discussed by Barnea[2] (recall that $\Theta^{-1}\propto S$), although we find a different coefficient. The actual dependence on the enhancement is model dependent. It is believed[12] that the paramagnon model overemphasizes the role of spin fluctuations and hence this coefficient is probably too large.

The high-temperature result, on the other hand, is the classical spin-fluctuation behavior first pointed out for itinerant ferromagnets by Murata and Doniach[13] from a very different point of view, and discussed in detail for these systems by Mishra and Ramakrishnan.[1]

For intermediate temperatures χ must be calculated numerically. As an illustration, Fig. 3 shows the temperature dependence of the inverse susceptibility for $S=20$, with such a large value chosen to exhibit clearly the minimum at low temperature. Notice that the curve reaches its asymptotic linear dependence quite rapidly. This behavior is qualitatively similar to that observed in many strongly enhanced paramagnetic metals, such as Pd, U_2C_3, $NpCo_2$, and $LuCo_2$.[2]

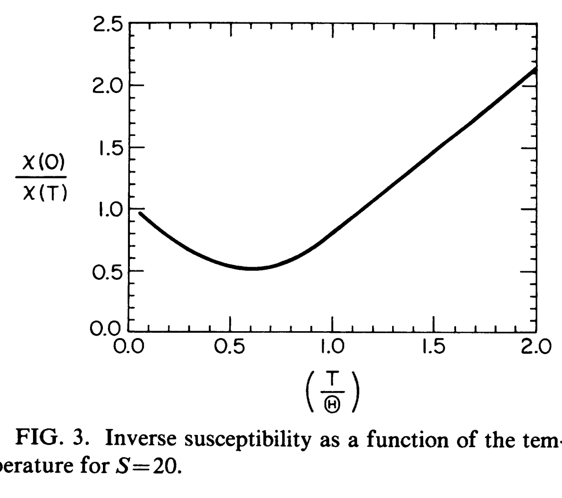

FIG. 3. Inverse susceptibility as a function of the temperature for $S=20$.

Since the absence of a maximum in χ in the case of Ni_3Ga has been explained in terms of impurities or imperfect crystalline order,[2] it is of interest to discuss their effects. Impurities and imperfect ordering introduce damping in the motion of the quasiparticles near the Fermi surface and this, in turn, causes the appearance of a diffusion pole in the paramagnon propagator. The $T^2\ln T$ term in the low-temperature susceptibility is replaced by $T^2\ln(T+T_{\rm imp})$ where $T_{\rm imp}=\Theta/(k_F l)$, and l is the mean free path. Thus the maximum in $\chi(T)$ is reduced in size and is shifted towards lower temperatures as $T_{\rm imp}$ increases. In fact, for $T_{\rm imp}\gtrsim\Theta$ the maximum disappears completely and χ^{-1} increases approximately as T^2. In contrast, the behavior for $T\gtrsim\Theta$ remains unchanged, and one still obtains a linear increase of χ^{-1} with temperature. This behavior, quadratic at low temperatures and linear at higher temperatures, describes qualitatively the observations.

IV. DISCUSSION AND COMPARISON WITH OTHER WORK

We calculated from a microscopic model the magnetic susceptibility of almost ferromagnetic materials. We find that at low temperatures the susceptibility exhibits an anomalous maximum that can be described by a $T^2\ln T$ term, whereas, at high temperatures, our results agree with those of spin-fluctuation theories.[1] It is of interest to discuss the origin of this discrepancy between these two theories at low temperatures. In the spin-fluctuation model[1] the spin-fluctuation self-energy was calculated by considering a set of diagrams that are essentially identical to the ones in Fig. 2, with their external legs restored. However, in the evaluation of the diagrams, the effects of four-momentum transfer between spin fluctuations and electrons was neglected. Now, typical paramagnon momenta are restricted to the range $|\vec{q}| \ll k_F$ whereas, in the degenerate case, typical electron momenta are of order k_F. Hence, it is a good approximation to neglect the momentum transfer. However, for given $|\vec{q}|$, the paramagnon energy ($\sim qv_F/S$) and the electronic excitation energy which lies in the interval $0<E<qv_F$ may become of the same order depending on the direction of $\vec{q}$. Under those circumstances it is not permissible to neglect the energy transfer. Although the phase space for such processes is very small in the case of large enhancement (it is essentially limited to 90° electron-paramagnon scattering) it is large enough to introduce the low-temperature logarithmic singularities.

In previous work[2] based on the microscopic approach, the susceptibility was evaluated from an expression that can be shown to be equivalent to (22), namely

$$\chi=\frac{2\mu_0^2}{V}\sum_{\vec{p}} Z_{\vec{p}} \left[-\frac{\partial f}{\partial E_{\vec{p}}}\right] \left[1-\frac{1}{\mu_0}\frac{\partial}{\partial H}\text{Re}\Sigma_{\vec{p}}(E_{\vec{p}})\right].$$

In the evaluation of this formula the explicit temperature dependences of the renormalization factor and of the field derivative of the self-energy were neglected. Unfortunately, it can readily be shown that the temperature-dependent part of $Z_{\vec{p}}$ exactly cancels the logarithmic contributions obtained from the anomalous terms in the quasiparticle spectrum, and all the interesting temperature dependence must come from the neglected thermal part of $(\partial/\partial H)\Sigma(\vec{p},\omega)$. What we have shown in this paper is that a consistent evaluation of the latter quantity is possible, at least for large enhancement and in the degenerate case, and produces the expected results throughout all the temperature range of interest. In this work band structure and other effects of relevance for real materials were neglected. However, their introduction is not expected to produce any change in the qualitative results although, of course, they have to be included if detailed comparison of the theoretical results with experiments is desired.

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