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Spectrum of longitudinal fluctuations in an isotropic ferromagnet below the Curie point

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The spectrum for longitudinal fluctuations of an isotropic ferromagnet in the hydrodynamic critical region below the Curie point is calculated. The spectrum has a hole in the center in the absence of an external magnetic field. An applied field fills the hole and when it exceeds a critical value a central peak develops in the spectrum. This can be verified by neutron scattering experiments.

The fluctuation spectrum of an isotropic ferromagnet below the Curie point T_C is supposed to consist of spin waves and a central peak coming from the longitudinal fluctuations. In the hydrodynamic critical region, where $\xi^{-1}(\kappa) >> k$, and the spin-wave peaks are well separated, the central peak would be easiest to see. However, to date, the neutron scattering experiments[1] have been unable to detect the central peak. Vaks, Larkin, and Pikin[2] calculated the shape of the spectrum of the longitudinal fluctuations at very low temperature where the damping of the spin waves was negligible and showed that the spectrum does not have a central peak, but has sidebands peaked at the spin-wave frequencies. In the hydrodynamic critical region, where the damping of spin waves is important, we have repeated the Vaks, Larkin, and Pikin[2] calculations under these different conditions. We find that at zero magnetic field there is a hole in the center rather than a peak. Application of a finite magnetic field removes the dip gradually and for fields greater than a critical field the central peak should appear.

The order parameter ψ for the ferromagnet is the magnetization vector with components ψ_1, ψ_2, and ψ_3. They satisfy the Landau-Lifshitz equation

$$\dot{\vec{\psi}} = g\,\vec{\psi}\times\nabla^2\vec{\psi} \ , \tag{1}$$

where g is a mode-coupling constant. In the ordered phase $T < T_C$, one of the ψ's (say ψ_1) acquires an expectation value m. The longitudinal fluctuations are then the fluctuations in the variable $\psi_1 - m$. The static correlation of these fluctuations has the well-known long-wavelength problem because of the Goldstone modes for $D<4$. In the present work, as in Hohenberg *et al.*[3] for the study of light scattering near the λ transition in helium, we will overlook this problem and assume that the longitudinal susceptibility can be approximated by a χ_L which has an effective Ornstein-Zernike form

$$\chi_L^{-1} = k^2 + \kappa^2 \ . \tag{2}$$

Inserting the expectation value of ψ_1 in Eq. (1) and linearizing leads to the spin-wave spectrum for the transverse fluctuations. Dynamic scaling[4] arguments by Halperin and Hohenberg[5] yield the spin-wave frequency as

$$\omega = A\,\kappa^{2-\epsilon/2}k^2 \ , \tag{3}$$

and the damping can be written as

$$\Gamma_s = \Gamma_0 k^4 (k^2+\kappa^2)^{-\epsilon/4} \ , \tag{4}$$

where k is the wave number, A and Γ_0 are some amplitudes, and

$$\epsilon = 6 - D \ . \tag{5}$$

The spectrum $G_L(k,\kappa,\omega)$ of the longitudinal fluctuations can be written as

$$G_L(k,\kappa,\omega) = \mathrm{Re}\frac{1}{-i\omega + \Sigma(k,\kappa,\omega)} \ , \tag{6}$$

where $\Sigma(k,\kappa,\omega)$ is the self-energy. To the lowest order $\Sigma(k,\kappa,\omega)$ is obtained from the convolution of two spin-wave lines with the appropriate vertex factors. The spin waves can be treated as double Lorentzians, which allows the frequency convolution to be easily performed and leaves us with[6–8]

$$\Sigma(k,\kappa,\omega) = g^2\chi_L^{-1}\int\frac{d^Dp}{(2\pi)^D}\frac{p^{-2}p'^{-2}(p^2-p'^2)^2}{[-i\omega+\Gamma_0 p^4(p^2+\kappa^2)^{-\epsilon/4}+\Gamma_0 p'^4(p^2+\kappa^2)^{-\epsilon/4}+iA\kappa^{2-\epsilon/2}(p^2-p'^2)]} \ . \tag{7}$$

Working in units where $g^2=(2\pi)^D/C_D$, C_D being the area of the unit sphere in D dimensions, and making approximations appropriate to the hydrodynamic critical region, i.e., $k << \kappa$, we can rewrite Eq. (7) as

$$\Sigma(k,\kappa,\omega) = \chi_L^{-1}\frac{k^2\kappa^{(D/2)-3}}{\Gamma_0 C_D}\;2\int\frac{d^Dx}{x^4}\frac{x^2\cos^2\theta}{-i\Omega+x^4(1+x^2)^{-\epsilon/4}} \ , \tag{8}$$

where

$$\Omega = \frac{\omega}{2\Gamma_0 \kappa} - 1 + \frac{D}{2} \ . \tag{9}$$

Note that the small-k approximation leads to setting $p^2 - p'^2 \sim 2kp\cos\theta$ in the numerator and $p^2 = p'^2$ in the denominator of Eq. (7). This is what picks out the lowest-order contribution in k. In three dimensions, we get

$$\Sigma(k,\kappa,\omega) = \chi_L^{-1} k^2 \frac{\kappa^{-3/2}}{\Gamma_0} \frac{2}{3} \int \frac{dx}{-i\Omega + x^4(1+x^2)^{-3/4}}$$

$$= \chi_L^{-1} k^2 \frac{2}{3} \frac{\kappa^{-3/2}}{\Gamma_0} I(z) \ , \tag{10}$$

where $z = -i\Omega$.

We now need to evaluate the integral $I(z)$. This is best done in the two limits, $z \ll 1$ and $z \gg 1$. In the former case

$$I(z) \approx \int \frac{dx}{x^4 + z} = \frac{\sqrt{2}}{4} \frac{\pi}{z^{3/4}} \ , \tag{11}$$

for $z \gg 1$,

$$I(z) \approx \int \frac{dx}{z + x^{5/2}} = \frac{2}{5} \frac{1}{z^{3/5}} \frac{\pi}{\sin(2\pi/5)} \ . \tag{12}$$

This suggests the interpolation formula

$$I(z) = \frac{\sqrt{2}}{4} \pi z^{-3/4} (1 + az)^{7/20} \ , \tag{13}$$

where a is given by the equation

$$\frac{\sqrt{2}}{4} \pi a^{7/20} = \frac{2}{5} \frac{\pi}{\sin(2\pi/5)} \ . \tag{14}$$

Equation (13) gives a fit to the numerically evaluated $I(z)$ to within 10% for all values of z.

By using Eqs. (2), (6), (10), and (13),

$$\Gamma_0 \kappa^{1+D/2} G = \mathrm{Re}\left[-i\Omega + \frac{2}{3} \frac{x^2}{\Gamma_0^2} \frac{\sqrt{2}}{4} \pi (-i\Omega)^{-3/4} [1 + a(-i\Omega)]^{7/20}\right]^{-1}$$

$$= C \ \mathrm{Re}\{-i\Omega' + (-i\Omega')^{3/4} [1 + b(-i\Omega')]^{7/20}\}^{-1} \ . \tag{15}$$

The dimensionless ratio k/κ is denoted by x. Ω' is a rescaled frequency which allows the first line to be expressed in the cleaner form of the second. This spectrum clearly vanishes at $\Omega = 0$ and hence the absence of a central peak.

We now study the effect of an external field h. The spin waves, which are the Goldstone modes of our system, acquire a mass, and the static susceptibility χ_T can be written as

$$\chi_T^{-1} = k^2 + \kappa_T^2 \ , \tag{16}$$

where

$$\kappa_T^2 \propto h \ . \tag{17}$$

The self-energy of Eq. (8) now becomes

$$\Sigma(k,\kappa,\omega,h) = \chi_L^{-1} \frac{k^2 \kappa^{(D/2)-3}}{\Gamma_0 C_D} 2 \int \frac{d^D x}{(x^2+y^2)^2} \frac{x^2 \cos^2\theta}{z + x^2(x^2+y^2)(1+x^2)^{-\epsilon/4}} \ , \tag{18}$$

where $y = \kappa_T/\kappa$. In three dimensions,

$$\Sigma(k,\kappa,\omega,h) = \chi_L^{-1} \frac{k^2 \kappa^{-3/2}}{\Gamma_0} \frac{2}{3} \tilde{I}(z,y) \ , \tag{19}$$

with

$$I(z,y) = \int \frac{x^4 \, dx}{(x^2+y^2)^2 [z + x^2(x^2+y^2)(1+x^2)^{-3/4}]} \ . \tag{20}$$

The zero-frequency value is

$$\tilde{I}(0,y) = \int \frac{x^2(1+x^2)^{3/4}}{(x^2+y^2)^3} dx \ . \tag{21}$$

For $y \ll 1$,

$$\tilde{I}(0,y) \approx \frac{\pi}{16} \frac{1}{y^3} \ , \tag{22}$$

and for $y \gg 1$,

$$\tilde{I}(0,y) \approx \frac{5\sqrt{2}}{64 y^{3/2}} \pi \ , \tag{23}$$

leading to the interpolation

$$\tilde{I}(0,y) = \frac{\pi}{16 y^3} [1 + 2(14 y^2)]^{3/4} \ . \tag{24}$$

The high-frequency limit of $\tilde{I}(z,y)$ is the same as that of $I(z)$, and we can consequently represent $\tilde{I}(z,y)$ as

$$\tilde{I}(z,y) = \tilde{I}(0,y)(1 + Cz)^{-3/5} \ , \tag{25}$$

where

$$C^{-3/5} \tilde{I}(0,y) = \frac{2}{5} \frac{\pi}{\sin(2\pi/5)} \ . \tag{26}$$

The spectrum is now given by

$$\Gamma_0\kappa^{1+D/2}G(k,\kappa,\omega,h) = \mathrm{Re}\left\{z + \chi_L \frac{x^2}{\Gamma_0^2}\frac{2}{3}\tilde{I}(0,y)(1+Cz)^{-3/5}\right\}^{-1} . \quad (27)$$

This spectrum has a similar mathematical structure to the spectrum above T_C.[9,10] Whether such a spectrum will have a central peak or a dip in the center can be settled by expanding Eq. (27) in powers of z and examining the coefficient of z^2. As can be seen from Eqs. (26) and (27) this depends on $\tilde{I}(0,y)$, i.e., on the strength of the applied field. For a small field, $\tilde{I}(0,y)$ is large and there will be a dip in the center, but for fields exceeding a critical field (where the coefficient of z^2 changes sign) there will be a central peak in the spectrum obtained by neutron scattering. Note that the smaller the value of $x = k/\kappa$, the smaller is the magnetic field required to see the central peak.

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