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Brief Reports

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Nonlinear electrodynamics in microwave-stimulated superconductivity

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In practical experiments on microwave-stimulated superconductivity the current source character of the microwave coupling leads to a strong dependence of the field strength on the value of the gap. Various consequences are pointed out, in particular for a quantitative comparison between critical current and gap or order-parameter enhancement.

INTRODUCTION

In the field of microwave-stimulated superconductivity reasonable qualitative agreement has been obtained between theory and experiment.[1] As the absolute microwave power level in the sample is unknown, a quantitative analysis so far has always relied on fitting the power and assuming that the rf level in the sample is proportional to the externally applied power. No specific attention has been paid to the high-frequency impedance of the sample that may change during the experiment because the gap is enhanced or depressed.

In this paper we point out that with a quantitative experiment it is essential to determine the rf coupling mode and the high-frequency impedance of the sample before confronting the data with theoretical results. On this basis, an explanation is offered for the fact that critical current enhancement often seems to be larger than theory predicts, compared with gap and order-parameter enhancement.

We will not perform detailed calculations or try to discuss all of the various coupling schemes and sample geometries. For each specific experiment a separate detailed analysis has to be performed, to which we hope the general points raised here may contribute.

HIGH-FREQUENCY PROPERTIES

In the theoretical calculations the microwave power is introduced by a vector potential $A_\omega \exp(i\omega t)$. A quantity α_ω is defined as $\alpha_\omega = 2e^2\hbar^{-1}DA_\omega^2$, where D is the diffusion constant. If the complex conductivity is $\sigma = \sigma_1 - i\sigma_2$, the amplitude of the associated microwave current density $j_\omega = |\sigma| E_\omega$ is equal to

$$j_\omega = |\sigma| \omega \left(\frac{\hbar}{2e^2 D} \alpha_\omega \right)^{1/2} . \tag{1}$$

In equilibrium the Mattis-Bardeen theory[2] gives values for σ_1 and σ_2 as functions of ω and Δ in the form of integrals over the excitation energies. For $\omega \ll \Delta$ and near the critical temperature T_c simpler expressions are obtained:

$$\frac{\sigma_1}{\sigma_n} = 1 + \frac{\Delta}{2kT} \left[\ln\left(\frac{8\Delta}{\hbar\omega} \right) - 1 \right] , \tag{2a}$$

$$\frac{\sigma_2}{\sigma_n} = \frac{\pi}{2} \frac{\Delta^2}{\hbar\omega kT} , \tag{2b}$$

where σ_n is the normal-state conductivity. In this article we will use these expressions although in microwave stimulation frequencies are sometimes comparable to Δ and also in the nonequilibrium state corrections may be required for accurate comparison. For our discussion we can make use of previous papers on the high-frequency impedance of superconducting thin films.[3]

Experimental investigations are typically performed on thin-film samples of 0.1-mm size or smaller. The actual sample is connected to larger thin-film areas onto which leads are attached. The wavelength is much longer than the sample size. The equivalent circuit is given in Fig. 1. The impedance of source, lines, and connecting thin-film areas is represented

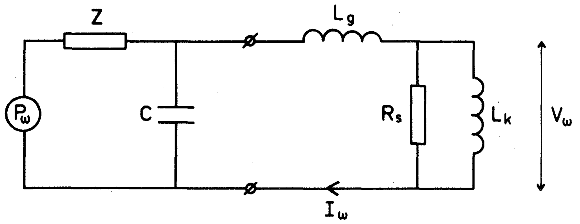

FIG. 1. Equivalent circuit for thin-film superconducting sample in microwave field.

by Z. Its character is mainly inductive; the absolute value is large. The capacitor shunting the sample is a simplified representation of the distributed capacity between the thin-film connecting areas. It is of order 10^{-14}–10^{-12} F. The sample itself contains three contributions. The geometrical inductance L_g is rather insensitive to the cross-sectional dimensions, its value is about 1 pH for each μm of length. The other two contributions are proportional to the normal-state resistance of the sample R_n. One is the normal-like resistance R_s, equal to $(\sigma_n/\sigma_1)R_n$, while the kinetic inductance L_k is given by $\omega L_k = (\sigma_n/\sigma_2)R_n$. Taking as a typical intermediate example an aluminum strip 100 μm long, 5 μm wide, and 0.1 μm thick at a frequency of 3 GHz, values are found as follows: $\omega L_g \approx 2\ \Omega$, $R_s \approx 2\ \Omega$, and $(\omega C)^{-1} \approx 50-5000\ \Omega$. $|Z|$ is of order 100 Ω. The value of L_k depends strongly on Δ. In equilibrium expression (2b) yields

$$\omega L_k = 0.016/(1-T/T_c)\ \Omega \ .$$

In the usual practice Z is large while C is too small to effectively shunt the source. Consequently the sample is current biased. We will call I_ω the microwave current and V_ω the voltage across R_s and L_k. The theoretical quantity α_ω is proportional to V_ω^2. As I_ω^2 is proportional to the externally applied power P_ω, α_ω is only proportional to P_ω at each frequency if the impedance of the sample remains constant when the gap changes due to temperature variations or enhancement. Whether this is true is largely determined by the relative values of R_s and L_k. For each frequency we can define Δ_c as the value of Δ where $\sigma_1 = \sigma_2$ and crossover occurs between inductive and resistive behavior. A rough approximation for Δ_c is given by $\Delta_c^2 \sim \hbar\omega k T_c$. For large enough ω or small enough Δ, the current I_ω is drawn by R_s. In this case

$$\alpha_\omega \propto \left(\frac{\sigma_n}{\sigma_1}\right)^2 P_\omega, \quad \Delta << \Delta_c \ . \tag{3}$$

As σ_n/σ_1 depends only weakly on Δ, here α_ω is roughly proportional to the externally applied power. This is certainly not true at lower frequencies and for higher values of Δ where

$$\alpha_\omega \propto \frac{(\hbar\omega k T)^2}{\Delta^4} P_\omega, \quad \Delta >> \Delta_c \ . \tag{4}$$

Clearly small changes in Δ lead to strong variations in the proportionality factor between α_ω and P_ω. Near equilibrium, at fixed P_ω, we see that α_ω is proportional to $(T_c-T)^2$. At fixed temperature turning on P_ω, the gap will increase because of stimulation. As a consequence α_ω will decrease.

CRITICAL CURRENT ENHANCEMENT

The critical current is measured in long narrow strips. The previous discussion and the equivalent circuit of Fig. 1 can be applied directly. At fixed P_ω, the microwave field inside the sample and hence the enhancement of I_c will be much smaller at low temperatures than near T_c. This effect is more pronounced at low frequency. In our group this has been observed; a quantitative analysis will be published.

It is important to realize that in order to measure I_c, the gap is depressed to a factor $(\frac{2}{3})^{1/2}$ by the dc current. At lower temperatures where α_ω is proportional to Δ^{-4}, this means that at fixed P_ω, α_ω increases by a factor $\frac{9}{4}$ when the dc current is increased from zero to I_c.

The critical current of a strip is determined by the weakest spot. In the enhanced state, a different weak spot may determine I_c. Weak spots are connected with local depressions of T_c, smaller values of the mean free path, or a smaller cross section. For all three mechanisms, the current source character of the microwave coupling leads to higher values of α_ω at such a weak spot. It will depend on the relative importance of the enhancement with respect to the original critical current if I_c will remain to be determined at the same position. The transition from strip to banks constitutes a particular example of cross-section variation that deserves special consideration.

GAP ENHANCEMENT

The energy gap is measured with a tunnel junction, usually between two crossing strips. The equivalent circuit for this geometry is represented in Fig. 2. The upper and lower parts are the two strips. All four ends will pick up an rf signal of its own amplitude and phase. The tunnel junction connects the two strips. R_T is the tunneling resistance (high-frequency differential resistance at its bias voltage), C_T the junction capacitance. We will assume the junction is small. It will sense gap values determined by the quasiparticle distributions in the four strip areas con-

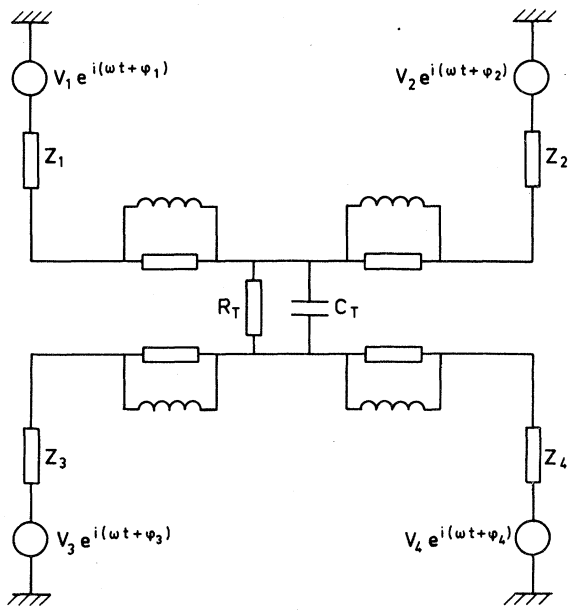

FIG. 2. Equivalent circuit for tunnel junction experiment. One electrode is between "leg" 1 and "leg" 2, the other between legs 3 and 4. The electrodes are connected by the tunneling conductance indicated with the resistor R_T and the capacitance C_T.

necting to the junction. If the junction impedance is high, the sample consists of two separate strips, each conforming to the equivalent circuit of Fig. 1. For each strip the previous discussions regarding the proportionality between α_ω and P_ω apply.

The values C_T and R_T vary over a considerable range. A reasonable intermediate value of C_T is 10 pF; at 3 GHz, $(\omega C)^{-1}$ is then equal to 5 Ω. R_T will be an order of magnitude larger. The impedance of the junction connecting the two strips is low enough that significant currents will pass through it. The resultant rf voltage across the junction gives rise to photon-assisted tunneling or its classical equivalence. The accompanying structure in the I-V characteristic makes it very difficult to determine the gaps of the two films. This has already been experienced as a major obstacle in observing gap enhancement, in particular at high power levels.[4–6]

The high-frequency current through the junction will be carried partly by the capacitance. The real conductance is usually high enough to take a nonnegligible proportion. This Ohmic current leads to dissipation and heating. How strong this heating effect is depends on the cross sections of the strips and of the junction. To obtain a small, well-observable enhancement effect in aluminum an rf-current density of 10^8 A m^{-2} is required. If the two strips have a 10×0.1-μm cross section, I_ω is of order 100 μA. It is not unreasonable then to suppose that the rf current through R_T is 3 μA. With a junction resistance of 100 Ω, dissipation of 10^{-9} W occurs. This dissipation of about 10 W/m^2 will lead to a temperature rise of order 10 mK. Actual junction data should be used for a real analysis, in conjunction with an estimate of the rf current through the junction from the photon-assisted tunneling structure. However, the numbers given indicate that in many experiments designed to observe gap enhancement, this enhancement may be outdone by the Joule heating in the junction sensor.

In the experiments of Horstman and Wolter,[7] a tunnel junction consisting of two long narrow strips, almost entirely on top of each other, was used. In this geometry there is a strong capacitive coupling between the strips along their lengths. As a consequence there is no voltage across the junction. The problems experienced in the usual crossed-strips geometry were avoided in this way and enhancement was observed very convincingly.

CRITICAL CURRENT VERSUS GAP ENHANCEMENT

As a relative effect, critical current enhancement is intrinsically larger than gap or order-parameter enhancement, simply because I_c is proportional to Δ^3 or ψ^3 to a rough first approximation. Moreover, a correction term involving the quasiparticle occupation at the gap edge has a positive effect on I_c. Still, taking these known circumstances into account, very often Δ or ψ enhancement seems to be a smaller effect than I_c enhancement. We believe that these differences are due to microwave-coupling-associated effects as discussed in this article. In particular a dissipative tunneling current will have a strongly depressing effect on the gap as observed. Moreover, it should be realized that in measuring I_c the gap is depressed thereby increasing α_ω by a factor $\frac{9}{4}$ at lower temperatures.

Order-parameter enhancement has been demonstrated by Pals *et al.*[8,9] In their experiments changes are measured of the order parameter by means of fluxoid conservation in a closed superconducting loop. Very recently, Van Attekum and Ramekers[10] performed a comparison between I_c enhancement and ψ enhancement. The strip in which I_c was measured formed part of the closed loop in which the changes in ψ were determined. Their conclusion was that the order-parameter enhancement was smaller by a factor of about 2 with respect to the meausured enhancement of the critical current. This is exactly what our analysis predicts. For our analysis no convincing experimental proof is available as yet. Other factors may be important. However, any comparison between enhancement effects should certainly take microwave coupling into account very carefully.

CONCLUSIONS

In practical experiments on microwave-stimulated superconductivity the microwave field strength is not proportional to the square root of the externally applied power. The resultant nonlinear electrodynamics, together with possible Ohmic heating by microwave tunneling currents may offer an explanation for apparent differences between critical current enhancement on one side and gap or order-parameter enhancement on the other.

ACKNOWLEDGMENTS

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