Theory of superconductivity in reentrant superconductors: Tunneling in paramagnetic phase

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(Received 27 September 1982)

We compute the temperature- (T-) dependent order parameter $\Delta(T)$, condensation energy $H_c^2(T)$, and frequency-dependent density of states $N_s(\omega)$, for a reentrant superconductor in the paramagnetic phase. Our parameters are chosen to be appropriate to $ErRh_4B_4$. To characterize microscopically the superconductivity, we numerically solve the analog of the Eliashberg equations using a simple diffusion model for the spin-fluctuation propagator. We find that, although the coupling between the local spins and the superconducting electrons is small by some measures, it nevertheless leads to results which strongly deviate from those observed in both the BCS and Abrikosov-Gorkov theory. Pair breaking increases overall as the temperature is lowered; this effect is partially compensated by the accompanying softening of the spin-fluctuation modes and the presence of the crystal-field splitting. All of these factors contribute to make the transition out of the superconducting state first order. Our results for $N_s(\omega)$ compare reasonably well with recent point-contact tunneling experiments. The calculated form for $\Delta(T)$ is consistent with that derived from experimental measurements of a Fraunhoffer diffraction pattern using a Josephson-junction configuration.

I. INTRODUCTION

Ternary compounds such as $ErRh_4B_4$ and $HoMo_6S_8$ exhibit reentrant superconductivity; superconductivity exists in the temperature interval $T_{c1} \geq T \geq T_{c2}$. Below T_{c2} the system becomes ferromagnetic. In these systems, there is a small (~ 0.3 K) temperature region ($\overline{T}_{c2} > T > T_{c2}$) in which domains of superconducting and magnetic order appear to coexist.[1] Scattering from local spins (associated with the rare-earth atoms) can have a profoundly negative effect on superconductivity. Therefore, it is believed that the superconducting state exists only because there is relatively weak exchange coupling between the Cooper pairs (associated with the Rh or Mo atoms) and the local spins.

In this paper we study the effect of this (dynamical) exchange coupling for temperatures $T > \overline{T}_{c2}$. For definiteness we consider the $ErRh_4B_4$ system. We demonstrate that, although the coupling is small by some measures, it nevertheless leads to results which strongly deviate from those obtained in both the BCS (Ref. 2) and Abrikosov-Gorkov (AG) theories[2] (of magnetically dirty superconductors). This is seen clearly in the temperature dependences of the order parameter $\Delta(T)$ and condensation energy $H_c^2(T)$ and in the frequency dependence of the superconducting density of states $N_s(\omega)$. While the size of the exchange interaction is small compared to that found in the rare-earth metals, it is not this fact alone which allows superconductivity to exist in the presence of a high concentration of rare-earth atoms. It is important to note that in, for example. $ErRh_4B_4$, there is also a large electron-phonon coupling constant $N(0)V \sim 0.34$, which is necessary to give the observed large $T_{c1} \approx 8.7$ K.

Furthermore, we find that the fact that the spin-fluctuation frequencies ω are spread over a wide range of low ω/ω_D (where ω_D is the Debye frequency) makes them less destructive to superconductivity than previous calculations[3] suggest. Pair breaking increases overall as the temperature is lowered; this effect is partially compensated by the accompanying softening of the spin-fluctuation modes and the presence of the crystal-field splitting. All of these factors contribute to leave the superconductivity relatively "intact" at the magnetic ordering temperature, so that the transition out of the superconducting state is necessarily first order.

Most previous studies[3] of the effects of spin fluctuations on superconductivity, in reentrant superconductors, have been based on the weak coupling approximation. This approximation, which includes the effects of the local spins through an effective (negative) coupling constant, is appropriate only when the spin-fluctuation frequencies are high compared to the Debye frequency. However, since a substantial proportion of the magnetic excitation frequencies are comparable to the size of the gap parameter, this necessitates the use of strong coupling theory. A weak coupling approach over emphasizes the negative effect of spin fluctuations on superconductivity; it predicts a second-order transition out of the superconducting state somewhat above T_M, which is not observed experimentally.[1,4] These remarks will be discussed in more detail in Sec. III.

We, therefore, adopt a strong coupling or dynamical treatment of the spin fluctuations in order to study reentrant superconductors over a wide range of temperatures. Keller[5] has applied strong coupling theory to compute the phase diagram as a function of x in $(Er_xHo_{1-x})Rh_4B_4$ alloys. Our general formalism is similar to his. However, we will focus on calculating various properties of the superconducting state in the entire paramagnetic phase, so that unlike Ref. 5, we do not deal exclusively with equations linearized in the gap parameter. Our work is, in large part, motivated by recent tunneling experiments[6,7] on $ErRh_4B_4$. These suggest that the behavior of $N_s(\omega)$ in reentrant superconductors is characteristically distinct from that of BCS and magnetically dirty superconductors. We find, as observed experimentally,[6,7] a filling in or rounding of the gap in the density of states. In contrast with the usual gapless superconductors, this is accompanied by a rather pronounced peak in $N_s(\omega)$ at ω approximately equal to the order parameter Δ. From several different types of tunneling data[6,7] it appears that the order parameter Δ tends to saturate as T approaches T_{c2}. We have shown that the growth of spin-fluctuation scattering leads, as might be expected, to a decrease in Δ as T is decreased near T_{c2}. However crystal-field splitting inhibits spin-flip scattering and when this is included we find that Δ saturates at low T, as is observed.

II. GENERAL THEORY

In this section we summarize results previously obtained for the order parameter $\Delta(T)$ and superconducting density of states. Our formalism for treating the reentrant superconductors in the paramagnetic phase is equivalent to that we previously applied[8] to spin-glass superconductors.

The Hamiltonian for a system of d electrons interacting with localized f spins is

$$\mathscr{H}=\mathscr{H}^0+\mathscr{H}^{fd}\ , \tag{2.1}$$

where

$$\mathscr{H}^0=\sum_{\vec{k},\sigma}\epsilon(k)C^\dagger_{\vec{k}\sigma}C_{\vec{k}\sigma}+\sum_{\vec{k},\vec{k}'}V_{\vec{k},\vec{k}'}C^\dagger_{\vec{k}\uparrow}C^\dagger_{-\vec{k}\downarrow}C_{-\vec{k}'\downarrow}C_{\vec{k}'\uparrow} \tag{2.2}$$

and

$$\mathscr{H}^{fd}=J^{fd}\sum_{\substack{i,\vec{k},\vec{k}'\\ \sigma,\sigma'}}\{\exp[i(\vec{k}-\vec{k}')\cdot\vec{R}_i]\}\times(\vec{S}_i\cdot C^\dagger_{\vec{k}\sigma}\vec{\sigma}_{\sigma\sigma'}C_{\vec{k}'\sigma'})\ . \tag{2.3}$$

Here $\vec{\sigma}$ is the vector Pauli matrix, $\vec{S}_i$ the localized spin operator at the site i, $\vec{S}_i\equiv(g-1)\vec{J}$, where g is the Landé g factor and $\vec{J}$ the total angular momentum. Here J^{fd} is the exchange interaction between the localized spin and superconducting electrons. The interaction Hamiltonian $\mathscr{H}^{fd}$ gives rise to a self-energy correction

$$\Sigma(k,i\omega_n)=\frac{-(J^{fd})^2}{\beta}\sum_m\int\frac{d^3k'}{(2\pi)^3}\vec{\alpha}G(k',i\omega_m)\vec{\alpha}\chi(\vec{k}-\vec{k}',i(\omega_n-\omega_m))\ , \tag{2.4a}$$

where

$$\vec{\alpha}=(\rho_3\sigma_1,\rho_0\sigma_2,\rho_3\sigma_3) \tag{2.4b}$$

and $\chi(k,i\omega_n)$ is the time-ordered spin-fluctuation propagator. Here $\{\rho_i\}$ are the Pauli matrices on a four-dimensional vector space. It is convenient to introduce a notation for the matrix elements in Σ by writing $\Sigma(k,i\omega_n)\equiv G_0^{-1}(k,i\omega_n)-G^{-1}(k,i\omega_n)$ with

$$G_0^{-1}(k,i\omega_n)=i\omega_n-\xi\rho_3-\Delta\rho_2\sigma_2 \tag{2.5a}$$

and

$$G^{-1}(k,i\omega_n)=i\tilde{\omega}_n-\xi\rho_3-\tilde{\Delta}\rho_2\sigma_2\ , \tag{2.5b}$$

where ξ is the kinetic energy measured relative to E_F. Using Eqs. (2.4) and (2.5) we may deduce an integral expression for $u(\omega)=\tilde{\omega}/\tilde{\Delta}$ which is valid in the paramagnetic phase:

$$u(\omega)=\omega/\Delta+\frac{N(0)(J^{fd})^2}{\Delta} \times \int_0^{2k_F} d^3q \int_0^\infty d\Omega\,\phi(q)B(q,\Omega)\left[\pi n(\Omega)\left[\frac{u(\omega-\Omega)+u(\omega)}{[1-u^2(\omega-\Omega)]^{1/2}}+\frac{u(\omega+\Omega)+u(\omega)}{[1-u^2(\omega+\Omega)]^{1/2}}\right] + \int_0^\infty d\omega'\left[\mathrm{Im}\frac{u(\omega')}{[1-u^2(\omega')]^{1/2}}+u(\omega)\mathrm{Im}\frac{1}{[1-u^2(\omega')]^{1/2}}\right] \times\left[\frac{f(-\omega')}{\omega'-\omega+\Omega}\mp\frac{f(-\omega')}{\omega'+\omega+\Omega}+\frac{f(\omega')}{\omega'-\omega-\Omega}\mp\frac{f(\omega')}{\omega'+\omega-\Omega}\right]\right], \tag{2.6}$$

where the upper and lower signs correspond to the first and second term in the preceding large parentheses, and f and n are the Fermi and Bose statistical factors. Here $\phi(q)$ is the joint density of states of conduction electrons whose wave vectors are separated by q and $B(q,\omega)$, the Fourier transform of the commutator $i\langle\Theta(t)[S_i(t),S_j(0)]\rangle$, is the spectral weight function for the spin-fluctuation propagator. In the presence of long-range magnetic order Eqs. (2.5) must be generalized as was done previously, for example, for antiferromagnetic superconductors.[9]

Both the density of states and order parameter can[2] be written in terms of $u(\omega)$ as

$$N_s(\omega)=N(0)\mathrm{Im}\left[\frac{u(\omega)}{[1-u^2(\omega)]^{1/2}}\right] \tag{2.7}$$

and

$$\Delta=N(0)V\int_0^{\omega_D} d\omega'\mathrm{Im}\left[\frac{1}{[1-u^2(\omega')]^{1/2}}\right]\tanh\left[\frac{\beta\omega'}{2}\right]. \tag{2.8}$$

In this last equation we have assumed that the electron-phonon interaction can be treated in the usual BCS weak-coupling approximation. Finally, it is useful to determine the magnetic condensation energy,

$$\frac{H_c^2}{8\pi}=F_N-F_S .$$

An expression for the free-energy difference F_S-F_N is derived in Ref. 10,

$$F_S-F_N=-N(0)\int_0^{\omega_D} d\omega'\left[\frac{N_S(\omega')}{N(0)}-1\right]2\omega'\tanh\left[\frac{\beta\omega'}{2}\right] +\frac{\Delta^2}{V}-N(0)\Delta^2\frac{-4N(0)}{\beta}\int_0^{\omega_D} d\omega'\left[\frac{N_s(\omega')}{N(0)}-1\right]\left[\ln(1+e^{-\beta\omega'})+\frac{\beta\omega'}{1+e^{\beta\omega'}}\right].$$

Above the magnetic ordering temperature T_M, we adopt a hydrodynamical model for $B(q,\omega)$

$$B(q,\omega)=\frac{\omega D k^2\chi(k)}{\omega^2+[Dk^2-\omega^2\tau]^2}, \tag{2.9}$$

where

$$\chi(k)=\frac{c_1 S(S+1)}{(T-T_M)/T_M+\xi_0^2k^2} \tag{2.10}$$

and the spin-diffusion constant D is given by

$$D=c_2[\chi(k)]^{-1}. \tag{2.11}$$

D vanishes at $k=0$, $T=T_M$ as expected[11] from critical slowing down. The parameter τ in Eq. (2.9) is a phenomenological relaxation time which describes dissipative processes acting on the local magnetization.

The constants c_1 and c_2 are given by

$$c_1=g^2S(S+1)\hbar/(3k_BT_M) \tag{2.12a}$$

and

$$c_2 = a^2 J^{ff}[ZS(S+1)]^{1/2} c_1 T_M/(6\hbar T) \ . \tag{2.12b}$$

Here Z is the number of nearest neighbors of a localized spin and a the near-neighbor separation. J^{ff} is the local-spin—local-spin exchange constant (which is taken to be of order $k_B T_M$) and ξ_0 is the magnetic coherence length.

The spectral weight $B(q,\omega)$ obeys the sum rule [over the Brillouin zone (BZ)]

$$\int_{BZ} d^3q \int_{-\infty}^{\infty} d\omega \frac{2B(q,\omega)}{1-e^{-\beta\omega}} = 2\pi S(S+1) \int_{BZ} d^3q \ . \tag{2.13}$$

Since all parameters except τ in Eq. (2.9) are reasonably well known experimentally, we may view Eq. (2.13) as an equation which determines τ as a function of temperature. This yields a τ parameter which increases as the magnetic transition temperature T_M is approached from above.

In order to solve Eq. (2.6) it is useful to define the density of states associated with spin fluctuations by $\alpha^2F(\omega)$:

$$\alpha^2F(\omega) = N(0)(J^{fd})^2 \int_{BZ} d^3q \ \phi(q) B(q,\omega) \ . \tag{2.14}$$

For a spherical Fermi surface (which we assume, for simplicity in the remainder of this paper), $\phi(q)d^3q = q\,dq/2k_F^2$ and the integral is taken between 0 and $2k_F$.

III. NUMERICAL RESULTS

In this section we calculate $N_S(\omega)$, Δ, and the thermodynamic critical field H_c^2 using parameters appropriate to $ErRh_4B_4$. In this material electrons in the inner unfilled f band of Er^{3+} are ferromagnetically ordered below $T_{c2} = 0.93$ K. The conduction electrons, deriving primarily from the d band of the Rh_4B_4 clusters are superconducting below $T_{c1} = 8.7$ K and above T_{c2}. A type of coexistent phase appears to exist between $\bar{T}_{c2} = 1.2$ K and T_{c2}. Because this phase is not well understood we will consider only $T > \bar{T}_{c2}$. While the magnetic instability temperature T_M is not clearly known, in this work we will choose it to be 0.9 K which is roughly equal to T_{c2}. Free-energy arguments[12] suggest that once magnetic order can exist the ferromagnetic state will rapidly (as a function of decreasing T) reach a lower free energy than that of the superconducting state, so that T_{c2} and T_M cannot be far apart.

Our choice of parameters for $ErRh_4B_4$ is as follows. In order to obtain a T_{c1} of 8.7 K, in weak coupling (phonon) theory, we needed $N(0)V \approx 0.342$.

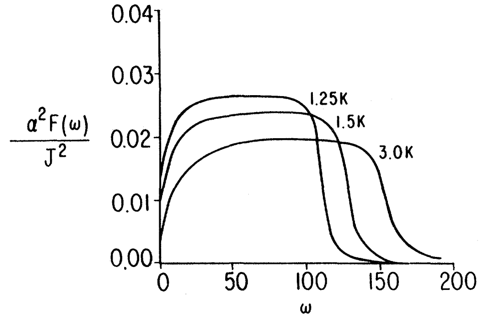

FIG. 1. Paramagnon density of states vs ω, measured in units of $\omega_D/200$.

This corresponds to a BCS transition temperature of 10.7 K (for $\hbar\omega_D/k_B = 200$ K) which must lie somewhat above T_{c1} due to spin-fluctuation effects. [Our exact results yield a T_{c1} higher than experiment, suggesting that $N(0)V \sim 0.335$ might actually be more appropriate.] It should be emphasized that $ErRh_4B_4$ has a surprisingly high critical temperature for a material with a ferromagnetic ground state. This can be explained by assuming a rather large electron-phonon coupling constant. The other parameters appearing in Eqs. (2.6) and (2.9) are the spin $\vec{S} = (g-1)\vec{J}$ which is $\frac{3}{2}$ for Er^{3+}; the two exchange constants J^{ff} and J^{fd} are, respectively, 0.1 and 10 meV. The former corresponds to $k_B T_M$ and the latter has been estimated from measurements of the depression of superconducting transition temperature as a function of impurity concentration in the rhodium boride series. We also take $Z = 8$, the magnetic coherence length $\xi_0 = 4.4 k_F^{-1}$ which corresponds to the nearest-neighbor separation $a = 5.5$ Å. Finally $g = 1.2$ and[13] $N(0) = 0.33$ eV^{-1}. Note that our choice of parameters differs significantly from that of Ref. 3. Our results are relatively insensitive to ξ_0, which is not well known. Using these parameters and the sum rule [Eq. (2.13)] to determine τ, we may compute $\alpha^2F(\omega)$ as a function of T. This is

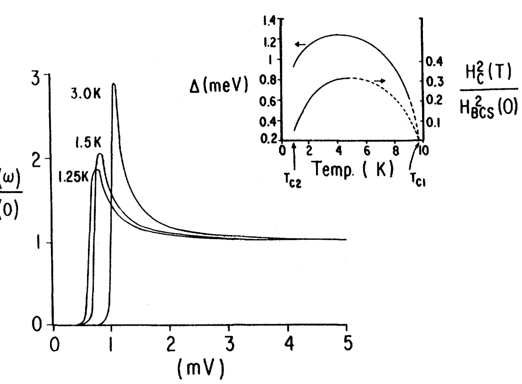

FIG. 2. $N_S(\omega)$ vs ω at several T (in the absence of crystal-field—splitting effects). Inset shows gap parameter Δ and free-energy difference H_c^2.

plotted for three different temperatures in Fig. 1. Note as T decreases the weight in $\alpha^2F(\omega)$ moves to lower frequencies, corresponding to the softening of the spin-fluctuation modes.

We numerically solve Eqs. (2.6) and (2.8) using an iterative approach. We began our iterations by making a guess for $u(\omega)$ using the AG equation

$$u(\omega)=\frac{\omega}{\Delta}+\frac{1}{\tau_2\Delta}u(\omega)/[1-u^2(\omega)]^{1/2} \tag{3.1}$$

with $(\tau_2\Delta)^{-1}$ chosen to be 0.125, which was a reasonable estimate. The final $u(\omega)$'s converged, normally, within 1% usually after four to six iterations.[14]

Our results for $N_s(\omega)$ for three low temperatures are shown in Fig. 2.[15] The inset plots Δ and H_c^2 as a function of T. The growth of spin fluctuations as T decreases toward T_M leads to a slight drop in Δ (and H_c^2) at lower T. Since Δ stays relatively large, in the vicinity of magnetic ordering, it follows that the transition out of the superconducting state must be first order.

This first-order transition can be directly compared with the second-order transition obtained in Ref. 3 using weak coupling theory. The strong coupling equations (2.6)–(2.8) yield the results found by Maekawa and Tachiki when $B(q,\Omega)$ has all of its weight at high frequencies. For example, to reproduce the results in Ref. 3 one may take

$$B(q,\Omega)=\frac{-\Omega\chi(q)}{2}\delta(\Omega-\omega^{\rm sp})\ , \tag{3.2}$$

where $\omega^{\rm sp}$ is a characteristic magnetic frequency which is high compared to all other energies in the system. It follows from Eq. (2.8) that

$$T_c=(1.14\hbar\omega_D/k_B)\exp[-1/g^{\rm eff}N(0)]\ , \tag{3.3a}$$

where the effective coupling constant[3]

$$g^{\rm eff}N(0)=g^{\rm BCS}N(0)-\frac{3T_M(J^{fd})^2N(0)}{8k_F^2J^{ff}\xi_0^2}\times\ln\left[\frac{T-T_M+4k_F^2\xi_0^2}{T-T_M}\right]. \tag{3.3b}$$

It may be seen from Eqs. (3.3) at low temperatures with $T>T_M$, that $g^{\rm eff}$ vanishes and there is thus a second-order transition out of the superconducting state. This is in contrast to the results obtained in our model for $B(q,\Omega)$, which contains a spread of frequencies $0<\omega\lesssim\omega_D$. This distribution of ω is clearly crucial and weakens the negative effect of magnetic order on superconductivity. While we believe the spread in frequencies to be most important, to some extent the difference in the order of the transition is related to the softening of the magnetic excitation frequencies. It may be argued that most of the *negative* contribution to $\Delta(\omega)$ comes from ω smaller than the spin-fluctuation frequency. This is opposite to the phonon case in which most of the *positive* contribution to $\Delta(\omega)$ comes from frequencies below the Debye frequency. (The sign difference of these two effects follows from the fact that phonons strengthen and magnons weaken superconductivity.) Hence as the characteristic magnetic frequency is lowered, with decreasing temperature, we would expect that the magnetic excitations are less damaging to the superconducting order.

The behavior of $N_s(\omega)$ is clearly different both from that obtained for magnetically dirty superconductors and from BCS theory. We observe a small but finite density of states at $\omega<\Delta$ unlike the BCS results. However, by contrast with results obtained in AG theory for a gapless superconductor we see a much more well-defined and narrow peak at $\omega\sim\Delta$. This peak shifts to lower T as T decreases near T_M in the same way as $\Delta(T)$.

The T_{c1} obtained for $N(0)V=0.342$ was found to be about 9.5 K, which is slightly higher than observed experimentally. We estimate that $N(0)V=0.335$ might be more appropriate. Note that the ratio of twice the maximum order parameter $\Delta^{\rm max}$ to k_BT_{c1} is ~3.0 rather than 3.8 as observed experimentally. This arises from the fact that our T_{c1} is slightly larger and our $\Delta^{\rm max}$ is slightly smaller (by 5%) than found experimentally. However, we see no reason to argue for strong coupling phonon effects to explain the deviation of the observed ratio from BCS theory.[6] It presumably arises from the fact that spin-fluctuation effects decrease T_{c1} more than Δ, as is seen in AG theory.[2]

It is believed that crystal-field effects, which have been neglected up to this point, play an important role in the rhodium boride series. The lowest energy levels of Er^{3+} ($J=\frac{15}{2}$) consist[16] of a pair of doublets separated by 1 K; the next levels are a doublet at 12 K and four doublets at approximately 32 K above the ground-state pair. This implies that over much of the temperature region of interest the effective "spin" value of the Er atom is considerably less than the value of $S=\frac{3}{2}$. Furthermore, due to inelastic processes this "spin" must be viewed as temperature dependent. With regard to the first point we note that estimates[13] of J^{fd} are based on assuming that the spin-flip lifetime is given by the AG result for the full spin value: $\tau_s^{-1}\propto(J^{fd})^2S(S+1)$. However, to some extent we have underestimated (J^{fd}) by overestimating S. This effect makes it possible to argue that near T_{c1} our calculations of pair breaking effects and thus of the thermodynamic variables are reasonably reliable. However, as T decreases, some

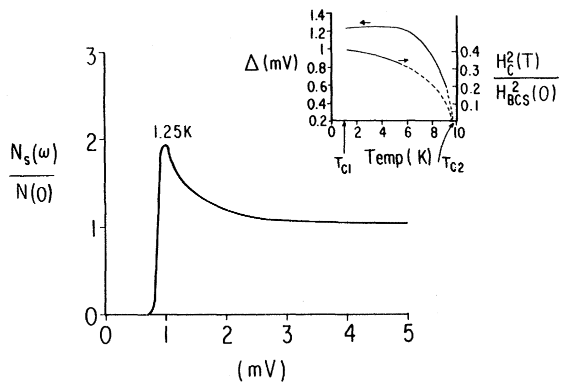

FIG. 3. $N_S(\omega)$ vs ω (when crystal-field splitting is included) at $T=1.25$ K. The gap parameter Δ and free-energy difference are shown in the inset.

inelastic contributions to the spin-flip lifetime must be frozen out; we now estimate these and their effect on $\Delta(T)$.

A crude approximation of an effective $1/\tau_s$ which includes inelastic processes is given by[17]

$$\frac{1}{\tau_s^{\rm eff}} \propto \left[\sum_{\omega} \beta\omega/(e^{\beta\omega}-1)\right]\left[\sum_{\omega} 1\right]^{-1}. \qquad (3.4)$$

Using the first three energy-level splittings (at 1, 12, and 32 K) we deduce that $(1/\tau^{\rm eff}_{s\,(5\,{\rm K})})/(1/\tau^{\rm eff}_{s\,(1.25\,{\rm K})})\sim 2$. Thus the neglect of crystal-field effects leads to an overestimate of $1/\tau_s^{\rm eff}$ by about a factor of 2 relative to a typical "high" temperature value. Using AG theory as a guideline we estimate[2] that this leads to an increase in Δ at $T=1.25$ K by about 20% over the value obtained by neglecting crystal-field effects. The net result is that Δ will tend to be rather T independent at low T, rather than decreasing slightly as was shown in the inset of Fig. 2.

To simulate this behavior we have chosen the cutoff parameter τ in our theory, to yield a constant Δ at low T. This is shown in Fig. 3, along with the resulting H_c^2. In the main portion of the figure is plotted $N_s(\omega)$ for the lowest temperature $T=1.25$ K. Note by comparison with Fig. 2, that the position of the peak has now moved up in frequency as expected from the change in Δ (from Fig. 2 to Fig. 3). We expect very little shift in the peak position as T is decreased (at low T) toward T_M when crystal-field effects are included. This appears consistent with what is experimentally observed,[6,7] as will be discussed in the next section.

IV. COMPARISON WITH EXPERIMENT

There have been three sets of experimental tunneling measurements on the $ErRh_4B_4$ system. The earliest work by Rowell, Dynes, and Schmidt[18] using proximity-effect tunneling is evidently suspect, for it gives unexpectedly low values ($\Delta\sim 0.7$ meV) for the gap parameter. Goldman and co-workers have fabricated superconductor-insulator-normal[6,7] and superconductor-insulator-superconductor[19] junctions using a variety of oxides and counterelectrodes with more success. Their dI/dV peaks appear to be less well defined than those found by Poppe[6] using a point-contact technique. Our theoretical results compare most favorably with those of Poppe, despite the fact that point-contact tunneling techniques are still in the earlier stages of development.

To make contact with experiment we compare our estimated values of $\Delta(T)$ and of $N_S(\omega)$. The former can be roughly determined experimentally using the period ΔH of the Fraunhofer diffraction pattern[7,19] in a Josephson junction to obtain the measured penetration depth

$$\lambda(T)=[mc^2/4\pi\rho_S(T)e^2]^{1/2}. \qquad (4.1)$$

Using experiment to calculate the superfluid density $\rho_S(T)$, we derive $\Delta(T)$ from

$$\frac{\Delta}{\Delta_0}=\frac{T}{T_0}\exp\left[-\int_{T_0}^{T}\frac{dT}{T}\frac{\rho_N}{\rho_S(T)}\right], \qquad (4.2)$$

where $\Delta_0\equiv\Delta(T_0)$ at the arbitrary reference temperature T_0 and $\rho_N\approx 0.3\times 10^{23}$ cm^{-3}. This yields a curve for $\Delta(T)$ which looks similar to that obtained theoretically (see the inset of Fig. 3). Unfortunately our results are sensitive to the normal state density, so that agreement between theory and experiment cannot be rigorously established. Despite this caveat, there does not seem to be any evidence for a sharp drop in $\Delta(T)$ near $\bar{T}_{c2}$ so that the notion that there is a first-order transition at this temperature seems well established.

We have computed dI/dV by "thermally smear-

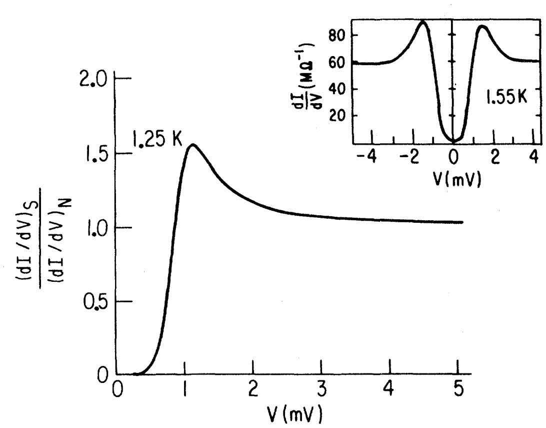

FIG. 4. Normalized conductance of $T=1.25$ K. Experimental tunneling characteristic is shown in inset (after Poppe, Ref. 6).

ing" the calculated $N_s(\omega)$ (at $T=1.25$ K) which is plotted in Fig. 3. The resulting curve is shown in Fig. 4. In the inset is plotted Poppe's[6] data at $T=1.55$ K. While the temperatures of the theoretical and experimental curves for $N_S(\omega)$ differ, it should be noted that both are found to be relatively insensitive to T at rather low temperatures. It may be seen that theory and experiment are in reasonably good agreement. dI/dV is relatively independent of V for a wide range of low voltages after which it rapidly increases to a well-defined maximum at about 1.5 mV. Even the peak values for the ratio $(dI/dV)_S/(dI/dV)_N$ correspond reasonably well. The rounded shape of dI/dV in the low-voltage region is a reflection of the small but finite density of states which we find at low ω. That this effect is seen experimentally is satisfying. However, it cannot be taken as confirmation of our theoretical results, since such an effect could derive from leakage current.

Our results have to be viewed as a first step in microscopically understanding superconductivity in the reentrant superconductors. There are many aspects of the experimental situation[6,19] which have yet to be clarified, even in the paramagnetic phase. Presumably once this is accomplished it will then be possible to undertake more complete theoretical studies of the tunneling characteristics in the "coexistent" superconducting-ferromagnetic phase.

ACKNOWLEDGMENTS

We thank A. Goldman for many useful conversations and for sending us unpublished data. We acknowledge helpful conversations with I. Schuller. L.C. is a recipient of an EXXON fellowship. This work was supported by National Research Foundation Grant No. DMR-81-15618 and National Science Foundation—Material Research Laboratories Grant No. NSF-MRL-79-24007.

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[14]To expedite convergence, Eqs. (2.6) and (2.8) were solved by guessing a value of Δ and using Eq. (2.8) to determine $N(0)V$. This was repeated until $N(0)V$ was found to equal the predetermined value of 0.342.

[15]At $T \gtrsim 3.0$ K, the sum rule (2.13) was only *approximately* satisfied. This leads to an overestimate of peak height at $T=3.0$ K (Fig. 2). The lower T results shown in the figure are more accurate.

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