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Magnetic excitations in layered media: Spin waves and the light-scattering spectrum

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We present an analysis of the spin-wave spectrum of a semi-infinite stack of ferromagnetic films, each of which is separated by a gap filled by a nonmagnetic medium. This is done within a formalism which includes the Zeeman and dipolar contributions to the spin-wave energy, with exchange omitted. We then calculate the spin-wave contribution to the Brillouin spectrum of such a system, in the backscattering geometry. The aim is to compare the spectrum for scattering from a sample with this geometry, with that from an isolated film. Two features unique to the stack appear in the spectrum. Each film, in isolation, possesses surface spin waves on its boundaries (Damon-Eshbach waves). In the layered geometry these interact to form a band of excitations of the array, which has nonvanishing component of wave vector normal to the stack. We find a feature in the spectrum associated with scattering from this band of modes; the position of the peak is controlled by dispersion introduced by interfilm interactions. Under certain conditions, the semi-infinite stack possesses a surface spin wave, whose eigenfunction is a linear superposition of individual film states, with amplitude that decays to zero as one moves down into the stack interior. This mode also produces a distinct feature in the light-scattering spectrum. These points are illustrated with a series of calculations of the spectrum, for parameters characteristic of layered ultrathin coherent structures.

I. INTRODUCTION

An intriguing development in materials science is the appearance of methods which allow one to synthesize samples in the form of a sequence of very thin layers of different material, with the thickness and composition of each element subject to precise control. When the layer thicknesses are very small, for example, on the order of a few angstroms, the resulting entity may possess unique physical properties distinct from those of the constituents of the individual layers. Most particularly, by means of a sputtering technique, one may prepare specimens from two metals, each of which is present as a layer with thickness from ten to perhaps a few hundred angstroms.[1] Samples prepared in this manner have been referred to as layered ultrathin coherent structures (LUCS).

The spectrum of elementary excitations in layered structures is of interest, most particularly if there are features unique to the sample that are also accessible to experimental study. Comparison between data, and the theory appropriate to a model structure may then allow one to assess how close the sample comes to the idealized forms one wishes to realize in practice. In LUCS samples, which are not prepared in a low-temperature environment, diffusion of atoms across the interface between two successive layers may lead to a composition profile softened substantially, from an ideal "square-wave" spatial modulation.

In the recent literature, LUCS samples have been described in which one of the two materials is nickel metal which forms a ferromagnetic film of thickness d_1, and the second is a nonmagnetic metal of thickness d_2. The idealized version of the structure is thus a finite stack of ferromagnetic films of thickness d_1, separated by magnetically "dead" layers of thickness d_2. The purpose of this paper is to explore the nature of the spin-wave spectrum of such a system, for the case where we have a semi-infinite stack of ferromagnetic films. We do this within the framework of a description valid for modes whose eigenfunctions vary slowly on the scale of the lattice constant. Then the dominant contribution to the spin-wave energy comes from dipolar and Zeeman energy, and exchange effects may be ignored. In this limit, a number of key features of the excitation spectrum may be extracted by analytic methods, and we shall see aspects unique to the LUCS structures. Since the Brillouin scattering of light in a backscattering geometry has proved a powerful probe of spin waves on metal surfaces and in films,[2–4] and the data have agreed with the theory,[5–10] we also calculate the light-scattering spectrum expected for LUCS structures.

In Sec. II we discuss the spin-wave excitations of an infinitely extended stack of films, and also for a semi-infinite one. Then we turn to the light-scattering spectrum in Sec. III. Here we require certain Green's functions which may also prove useful in other contexts. We remark that while we ignore exchange effects in the present discussion, they may be included straightforwardly with a considerable increase in algebraic complexity. From our earlier work,[5–7] we see that no difficulty in principle is encountered here. Also, the methods used here to examine the elementary excitation spectrum and to construct the Green's functions are readily extended to discuss the nature of other elementary excitations in these structures, such as the acoustical normal modes.

II. SPIN-WAVE EXCITATIONS IN A SEMI-INFINITE STACK OF FERROMAGNETIC FILMS: THE DIPOLAR REGIME

This section is devoted to the analysis of the spin-wave spectrum of a semi-infinite stack of ferromagnetic films such as that illustrated in Fig. 1. In each film the magnetization $\vec{M}_s$ and applied magnetic field $\vec{H}_0$ are parallel to the film surfaces, and parallel to the z direction. In what follows, the coordinate system is oriented so the y direction is normal to the interfaces in the structure. Finally, d_1 is the thickness of each magnetic film, and d_2 is the thickness of the nonmagnetic medium between each magnetic layer.

As remarked in Sec. I, we shall ignore exchange in the analysis presented here, and we begin by commenting on the regime of validity of this assumption. First, consider a spin wave of wave vector $\vec{Q}$ which propagates in the plane perpendicular to the magnetization, in a ferromagnet of infinite spatial extent. With frequency measured in units of magnetic field, and with D, the exchange stiffness constant, and $\vec{H}_0$, the applied magnetic field, the spin-

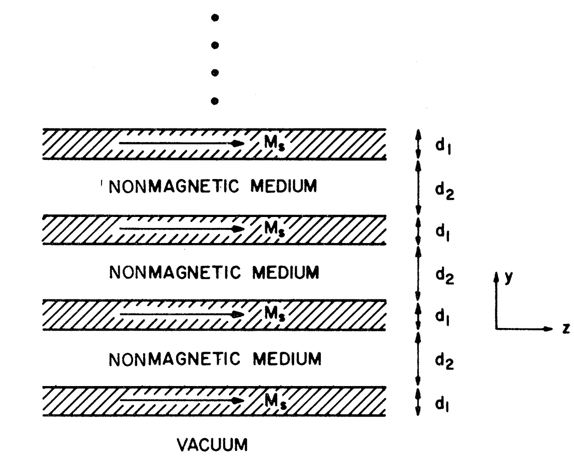

FIG. 1. Sample geometry considered in the present paper. One has a semi-infinite stack of ferromagnetic films each of thickness d_1, and they are separated by a nonmagnetic film of thickness d_2.

wave dispersion relation is given by the well-known expression

$$\Omega(\vec{Q})=[(H_0+DQ^2)(H_0+4\pi M_s+DQ^2)]^{1/2} . \tag{2.1}$$

In what follows, we assume H_0 and $4\pi M_s$ are comparable in magnitude.

In an isolated ferromagnetic film of thickness d_1, one encounters standing-spin-wave resonances with wave-vector component $Q_\perp$ normal to the surface given by $Q_\perp^{(n)} \cong n\pi/d_1$. In the light-scattering experiments of interest in Sec. III, one detects these standing mode resonances,[3,4,8] and the spin waves probed in the experiment have wave vector $\vec{Q}_{\parallel}$ parallel to the surface equal to $\vec{k}_{\parallel}^{(I)}-\vec{k}_{\parallel}^{(S)}$, where $\vec{k}_{\parallel}^{(I)}$ and $\vec{k}_{\parallel}^{(S)}$ are the projections of the wave vector of the incident and scattered photon on the plane parallel to the film surface. For all materials of interest to us, one has $DQ_{\parallel}^2$ very small compared to either H_0 or $4\pi M_s$, and if the film is so thick that $\pi^2 D/d_1^2$ is also very much smaller than these two quantities, then the influence of exchange can be safely ignored.

An isolated ferromagnetic film also has surface spin waves which propagate along the film. These modes, referred to frequently as the Damon-Eshbach modes,[11] have the dispersion relation (in the absence of exchange, and with propagation perpendicular to the field)

$$\Omega_s(Q_{\parallel})=[(H_0+2\pi M_s)^2 -4\pi^2 M_s^2 \exp(-2Q_{\parallel}d_1)]^{1/2} . \tag{2.2}$$

In film so thin that $\pi^2 D/d_1^2$ is *large* compared to either H_0 or $4\pi M_s$, the spin-wave spectrum consists of a surface mode close in frequency to Eq. (2.2), and high-frequency, exchange-dominated, standing-spin-wave modes with frequency $\Omega(\vec{Q}) \cong D(Q_\perp^{(n)})^2$. Theoretical studies show that even in very thin films of Fe or Ni exchange shifts the frequency of the surface spin wave only very slightly from the form in Eq. (2.2), for values of $Q_{||}$ of interest in light-scattering studies. Furthermore, in this very-thin-film limit, the surface-spin-wave features are by far the most intense features in the spectrum. This is clearly illustrated in the paper by Camley and Grimsditch,[8] who compare theoretical and experimental light-scattering spectra for very thin films. Note that when $Q_{||}d_1 \ll 1$, $\Omega_s(\vec{Q}_{||})$ lies very close to $[H_0(H_0+4\pi M_s)]^{1/2}$, which is the frequency of the uniform mode discussed by Kittel.[12] The surface-spin-wave eigenfunction is thus uniform across the film, so its excitation energy is influenced little by exchange.

So when $\pi^2 D/d_1^2$ is very large compared to H_0 and $4\pi M_s$, the low-lying surface spin waves are well described by a theory which ignores exchange, and these dominate the light-scattering spectrum. If a number of such films are brought together to form a stack as in Fig. 1, the surface modes on the various films interact to form a band of excitations capable of propagating normal to the interface. The dipolar theory provides a fully adequate description of this band, which we shall see contributes strongly to the light-scattering spectrum.

Thus we conclude that the theory which ignores exchange is useful both in the thin-film limit (of primary interest here), as well as in the more obvious thick-film limit. For films where $\pi^2 D/d_1^2$ is comparable in magnitude to both H_0 or $4\pi M_s$, quite clearly a full theory is required.

We consider first the description of spin-wave excitations in an infinitely extended stack, then we turn to the semi-infinite array illustrated in Fig. 1. In the magnetostatic limit, we consider the demagnetizing field $\vec{h}_d(\vec{x},t)$ generated by the spin motion, which has vanishing curl,

$$\vec{\nabla}\times\vec{h}_d(\vec{x},t)=0\ , \tag{2.3}$$

and so one writes

$$\vec{h}_d(\vec{x},t)=-\vec{\nabla}\varphi_M(\vec{x},t)\ , \tag{2.4}$$

where $\varphi_M(\vec{x},t)$ is the magnetic potential. If $\vec{M}(\vec{x},t)$ is the time and spatially varying magnetization associated with the spins, we require the field $\vec{b}=\vec{h}_d+4\pi\vec{M}$ which has vanishing divergence:

$$\vec{\nabla}\cdot\vec{b}=0\ . \tag{2.5}$$

In the magnetic medium $\vec{b}$ and $\vec{h}_d$ are related by the magnetic susceptibility tensor which, in the long-wavelength limit, depends only on the frequency Ω of the spin motion. Thus

$$\vec{b}=\vec{h}_d+4\pi\overleftrightarrow{\chi}(\Omega)\vec{h}_d\ , \tag{2.6}$$

where the nonvanishing elements of the tensor $\overleftrightarrow{\chi}(\Omega)$ in the ferromagnet are

$$\chi_{xx}(\Omega)=\chi_{yy}(\Omega)\equiv\chi_1(\Omega)=\frac{M_sH_0}{H_0^2-\Omega^2-i2\Omega\Gamma} \tag{2.7a}$$

and

$$\chi_{xy}(\Omega)=-\chi_{yx}(\Omega)=i\chi_2(\Omega)\ , \tag{2.7b}$$

where

$$\chi_2(\Omega)=\frac{M_s\Omega}{H_0^2-\Omega^2-i2\Omega\Gamma}\ . \tag{2.7c}$$

Throughout the paper we measure frequency in units of magnetic field, and Γ is a phenomenological spin-damping time. It is easy to show that in the magnetic films, in the coordinate system of Fig. 1, φ_M satisfies

$$\nabla^2\varphi_M+4\pi\chi_1(\Omega)\left[\frac{\partial^2\varphi_M}{\partial x^2}+\frac{\partial^2\varphi_M}{\partial y^2}\right]=0\ , \tag{2.8}$$

an anisotropic form of Laplace's equations sometimes referred to as the Walker equation. In the nonmagnetic film and in the vacuum, in essence $\chi_1\equiv0$, so we have

$$\nabla^2\varphi_M=0\ . \tag{2.9}$$

One basic task in this section is to solve this set of equations subject to the constraint that tangential components of $\vec{h}_d$ are conserved across each interface, along with normal $\vec{b}$. Translational invariance in the two directions parallel to the interface (within the xz plane) allows us to seek solutions of the form

$$\begin{aligned}\varphi_M(x,y,z)&=e^{iQ_xx}e^{iQ_zz}\Phi_M(y)\\&=e^{i\vec{Q}_{||}\cdot\vec{x}_{||}}\Phi_M(y)\ ,\end{aligned} \tag{2.10}$$

where the subscript $||$ is appended to vectors which lie in the xz plane.

A. The infinite stack of ferromagnetic films

We begin by considering an infinitely extended stack of ferromagnetic films of thickness d_1 with nonmagnetic spacer material of thickness d_2 interspersed, as in Fig. 1. Now we have a Bloch theorem which applies to the variation of the magnetic potential in the direction normal to the interfaces. We thus seek solutions of the equations above for which $\Phi_M(y)$ satisfies

$$\Phi_M(y+L)=e^{iQ_\perp L}\Phi_M(y) , \tag{2.11}$$

where $L=d_1+d_2$ is the length of a "unit cell" in the direction normal to the interfaces. We arrange the geometry as follows. The nth unit cell consists of a ferromagnet film which lies between $y=nL$ and $y=nL+d_1$, followed by a nonmagnetic film between $y=nL+d_1$ and $y=(n+1)L$. We then write

$$\Phi_M(y)=e^{iQ_\perp y}u(y) , \tag{2.12}$$

where $u(y)$, in analogy with the form used in electron energy-band theory, is necessarily periodic:

$$u(y+nL)\equiv u(y) . \tag{2.13}$$

Now inside each nonmagnetic film, we have

$$\left[\frac{\partial^2}{\partial y^2}-Q_{||}^2\right]\Phi_M(y)=0 ,$$

so $\Phi_M(y)$ is a linear combination of $\exp(+Q_{||}y)$ and $\exp(-Q_{||}y)$. This allows us to write $u(y)$ in the form

$$u(y)=e^{-iQ_\perp(y-nL)}(u^{(+)}e^{Q_{||}[y-(n+1)L]}+u^{(-)}e^{-Q_{||}(y-nL-d_1)}) , \quad nL+d_1\le y\le(n+1)L . \tag{2.14}$$

The variation of $\Phi_M(y)$ within each ferromagnetic film may be discussed in a similar manner. We shall write

$$Q_x=Q_{||}\cos(\psi) , \tag{2.15a}$$

$$Q_z=Q_{||}\sin(\psi) , \tag{2.15b}$$

where ψ is the angle between $\vec{Q}_{||}$ and the x axis. Then within each ferromagnetic film, we may write

$$u(y)=e^{-iQ_\perp(y-nL)}(v^{(+)}e^{+\tilde{Q}(y-nL-d_1)}+v^{(-)}e^{-\tilde{Q}(y-nL)}) , \tag{2.16}$$

where

$$\tilde{Q}=Q_{||}\left[\frac{\Omega^2-\Omega_m^2}{\Omega^2-\Omega_M^2}\right]^{1/2} , \quad \mathrm{Im}\tilde{Q}>0 \tag{2.17}$$

and with the damping constant set equal to zero,

$$\Omega_M^2=H_0(H_0+4\pi M_s)=H_0B , \tag{2.18a}$$

$$\Omega_m^2=H_0(H_0+4\pi M_s\cos^2\psi) . \tag{2.18b}$$

If we have $\Omega>\Omega_M$ or $\Omega<\Omega_m$, $\tilde{Q}_{||}$ is real while if $\Omega_m<\Omega<\Omega_M$, $\tilde{Q}$ is pure imaginary. As indicated in Eq. (2.17), in the latter case, we choose $\mathrm{Im}\tilde{Q}$ positive. If we consider an infinitely extended ferromagnet, and examine spin waves with wave vector

$$\vec{Q}=Q_{||}(\hat{x}\cos\psi+\hat{z}\sin\psi)+\hat{y}Q_\perp ,$$

where $Q_{||}$ and ψ are fixed and $Q_\perp$ is allowed to vary, then Ω_m is the smallest spin-wave frequency ($Q_\perp=0$) and Ω_M is the maximum ($Q_\perp=\infty$), so Ω_m and Ω_M are the lower and upper bounds of the bulk spin-wave manifold.

We now must turn to the boundary conditions at the various interfaces. Continuity of $\Phi_M(y)$ ensures continuity of h_d, and application of this condition at $y=nL$ leads to

$$v^{(+)}e^{-\tilde{Q}d_1}+v^{(-)}=e^{-iQ_\perp L}(u^{(+)}+u^{(-)}e^{-Q_{||}d_2}) , \tag{2.19}$$

while the same condition at $y=nL+d_1$ gives us

$$v^{(+)}+v^{(-)}e^{-\tilde{Q}d_1}=u^{(+)}e^{-Q_{||}d_2}+u^{(-)} . \tag{2.20}$$

Then continuity of normal $\vec{b}(b_y)$ at $y=nL$ provides a third relation:

$$[\tilde{Q}(1+4\pi\chi_1)+4\pi Q_{||}\cos\psi\chi_2]v^{(+)}e^{-\tilde{Q}d_1}-[\tilde{Q}(1+4\pi\chi_1)-4\pi Q_{||}\cos\psi\chi_2]v^{(-)}=Q_{||}e^{-iQ_\perp L}[u^{(+)}-u^{(-)}e^{-Q_{||}d_2}] , \tag{2.21}$$

and the final condition is conservation of normal $\vec{b}$ at $y=nL+d_1$:

$$[\tilde{Q}(1+4\pi\chi_1)+4\pi Q_{||}\cos\psi\chi_2]v^{(+)}-[\tilde{Q}(1+4\pi\chi_1)-4\pi Q_{||}\cos\psi\chi_2]v^{(-)}e^{-\tilde{Q}d_1}=Q_{||}[u^{(+)}e^{-Q_{||}d_2}-u^{(-)}] . \tag{2.22}$$

We thus have four homogeneous equations in the four amplitudes $v^{(+)}$, $v^{(-)}$, $u^{(+)}$, and $u^{(-)}$. The equations admit a nontrivial solution only if the determinant formed from the coefficients vanishes, and this requirement provides us with a dispersion relation for waves which propagate normal to the plane interfaces; i.e., for a particular choice of $\vec{Q}_{||}$ and $Q_\perp$, the determinant vanishes for one particular frequency $\Omega=\Omega(\vec{Q}_{||},Q_\perp)$. In general, this dispersion relation must be studied by numerical methods, but there are special limits where the problem admits an analytic solution. We focus our attention on these in the present paper. We continue the general discussion, and then turn to the special limits shortly.

It is convenient to introduce $r=\tilde{Q}/Q_{||}$, and two quantities $\Lambda^{(\pm)}$ defined by

$$\Lambda^{(\pm)}=r(1+4\pi\chi_1)\mp 4\pi\chi_2\cos\psi \ . \tag{2.23}$$

Then the condition that Eqs. (2.19)—(2.22) admits a nontrivial solution may be written

$$\sinh(\tilde{Q}d_1)\sinh(Q_{||}d_2)(1+\Lambda^{(+)}\Lambda^{(-)})+[\cosh(\tilde{Q}d_1)\cosh(Q_{||}d_2)-\cos(Q_\perp L)][\Lambda^{(+)}+\Lambda^{(-)}]=0 \ . \tag{2.24}$$

An important special case is $\psi=0$, so that $\vec{Q}_{||}$ is directed normal to $\vec{M}_s$. Such orientations of $\vec{Q}_{||}$ are commonly probed in light-scattering experiments. For $\psi=0$, $r=1$, and we have

$$\Omega^2(\vec{Q}_{||},Q_\perp)=\frac{H_0B}{1+\Delta(Q_{||},Q_\perp)}+\frac{1}{2}\left[H_0^2+B^2\right]\frac{\Delta(Q_{||},Q_\perp)}{1+\Delta(Q_{||},Q_\perp)} \ , \tag{2.25}$$

where, as above, $B=H_0+4\pi M_s$, and

$$\Delta(Q_{||},Q_\perp)=\frac{\sinh(Q_{||}d_1)\sinh(Q_{||}d_2)}{\cosh(Q_{||}d_1)\cosh(Q_{||}d_2)-\cos(Q_\perp L)} \ . \tag{2.26}$$

Note that $Q_\perp$ is to range from 0 to π/L, which is the Brillouin-zone boundary for the present problem.

It is striking that the dispersion relation is symmetric with respect to interchange of d_1 and d_2. If either d_1 or d_2 become small, in the sense that $Q_{||}d_{1,2}$ is very small compared to unity, then the manifold of waves collapses around H_0B, i.e., $\Delta(Q_{||},Q_\perp)$ vanishes. Later we quote a more general form of the dispersion relation valid when both $Q_{||}d_1$ and $Q_{||}d_2$ are small.

The remark in the previous paragraph shows that when either $Q_{||}d_1$ or $Q_{||}d_2$ is small, there is very little dispersion. Maximum dispersion occurs when $d_1=d_2$. Then when $Q_\perp=0$, the frequency $\Omega(Q_{||},0)$ equals $\frac{1}{2}(H_0+B)$, a value identical to the frequency of the Damon-Eshbach surface spin wave on a semi-infinite ferromagnetic material. [See Eq. (2.2) with $d_1\rightarrow\infty$.] Note that with $d_1=d_2$, this value of $\Omega(Q_{||},0)$ is obtained for any choice of $Q_{||}d_1$.

Another interesting limit of Eq. (2.26) is $Q_{||}d_2\gg 1$, with $Q_{||}d_1$ arbitrary; i.e., we have a stack of ferromagnetic films separated by thick layers of a nonmagnetic medium. Then Eq. (2.26) becomes

$$\begin{aligned}\Omega^2(Q_{||},Q_\perp)=&(H_0+2\pi M_s)^2-4\pi^2M_s^2e^{-2Q_{||}d_1}\\&+e^{-Q_{||}d_2}\cos(Q_\perp L)\{e^{-Q_{||}d_1}\tanh(Q_{||}d_1)[H_0^2+B^2-2H_0Be^{-Q_{||}d_1}\cosh(Q_{||}d_1)]\} \ .\end{aligned} \tag{2.27}$$

By comparing Eq. (2.27) with Eq. (2.2), it is evident that we have a band of excitations which propagate normal to the stack, and the band has its physical origin in the coupling of surface spin waves localized on the various interfaces. This limit is analogous to the tight-binding limit of the electron energy-band problem, where the surface spin wave associated with an isolated film is the analog of the Wannier function which becomes highly localized around a given lattice site.

The case where $\psi\neq 0$ generally requires numerical analysis of Eq. (2.24). However, when $Q_{||}d_1$ and $Q_{||}d_2$ are both small, once again an analytic solution of the problem may be obtained. Here we find

$$\Omega^2(\vec{Q}_{||},Q_\perp)=H_0^2+\frac{4\pi H_0M_sd_2}{d_1+d_2}+\frac{4\pi H_0M_sd_1}{d_1+d_2}\frac{(Q_\perp^2+Q_{||}^2\cos^2\psi)}{Q_\perp^2+Q_{||}^2}+16\pi^2M_s^2\cos^2(\psi)\frac{d_1d_2Q_{||}^2}{(Q_{||}^2+Q_\perp^2)(d_1+d_2)^2} \ , \tag{2.28}$$

which for $\psi=0$ reduces to the simpler form

$$\Omega^2(\vec{Q}_{||},Q_\perp)=H_0^2+\frac{4\pi H_0M_sd_2}{d_1+d_2}+\frac{4\pi H_0M_sd_1}{d_1+d_2}\frac{Q_\perp^2}{Q_\perp^2+Q_{||}^2} \ . \tag{2.29}$$

B. The semi-infinite stack of ferromagnetic films

We next turn to the semi-infinite stack of ferromagnetic films. In this case we no longer expect the Bloch theorem to hold since we no longer have perfect periodicity in the direction normal to the stack. As a result, Eqs. (2.14) and (2.16) are not appropriate.

We now look for surface-wave solutions, i.e., solutions which are localized near the surface of the stack and which decay exponentially as one travels through the layers away from the surface. Thus for $\varphi_M(y)$ in the nonmagnetic film we take

$$\varphi_M(y)=e^{-\alpha nL}(u^{(+)}e^{Q_{\parallel}[y-(n+1)L]}+u^{(-)}e^{-Q_{\parallel}(y-nL-d_1)}),\quad nL+d_1\leq y\leq (n+1)L \tag{2.30}$$

and in the ferromagnetic film we have

$$\varphi_M(y)=e^{-\alpha nL}(v^{(+)}e^{\tilde{Q}(y-nL-d_1)}+v^{(-)}e^{-\tilde{Q}(y-nL)}),\quad nL\leq y\leq nL+d_1 \ . \tag{2.31}$$

The constant α governs the exponential decay as one penetrates into the stack. It is readily seen that the forms for $\varphi_M(y)$ given in Eqs. (2.30) and (2.31) satisfy the differential equations for $\varphi_M(y)$ in the nonmagnetic and ferromagnetic films, respectively. There is also a magnetic potential outside the stack which is set up by the motion of the spins in the ferromagnetic layers. Thus for $\varphi_M(y)$ in this region we assume a solution of the form

$$\varphi_M(y)=\varphi_0 e^{+Q_{\parallel}y},\quad y<0 \ . \tag{2.32}$$

This potential decays to zero as $y\to-\infty$ and satisfies the differential equation for $\varphi_M(y)$ outside the stack.

We proceed by matching the expressions for the magnetic potential in the various regions through the use of the boundary conditions. To obtain the surface-wave dispersion relation, it is sufficient to consider the boundary conditions along three interfaces: (1) $y=nL$, (2) $y=nL+d_1$, and (3) $y=0$, the end of the semi-infinite stack.

The application of the boundary conditions of $y=nL$ and $y=nL+d_1$ gives a set of four equations for $u^{(+)}$, $u^{(-)}$, $v^{(+)}$, and $v^{(-)}$ which are identical to Eqs. (2.19)–(2.22) except that $iQ_\perp$ is replaced by $-\alpha$ everywhere. The variables $u^{(+)}$ and $u^{(-)}$, the amplitudes in the nonmagnetic film, may be eliminated from this set of four equations. One obtains two equations in the two unknowns, $v^{(+)}$ and $v^{(-)}$:

$$(1+\Lambda^{(-)})(1-e^{-\tilde{Q}d_1}e^{-Q_{\parallel}d_2}e^{-\alpha L})v^{(+)}+(1-\Lambda^{(+)})(e^{-\tilde{Q}d_1}-e^{-Q_{\parallel}d_2}e^{-\alpha L})v^{(-)}=0 \tag{2.33}$$

and

$$(1-\Lambda^{(-)})(1-e^{\tilde{Q}d_1}e^{-Q_{\parallel}d_2}e^{\alpha L})v^{(+)}+(1+\Lambda^{(+)})(e^{\tilde{Q}d_1}-e^{-Q_{\parallel}d_2}e^{\alpha L})v^{(-)}=0 \ . \tag{2.34}$$

The solvability condition for these two equations gives an expression for α in terms of $Q_{\parallel}$ and Ω.

$$\cosh(\alpha L)=\frac{1+\Lambda^{(+)}\Lambda^{(-)}}{\Lambda^{(+)}+\Lambda^{(-)}}\sinh(\tilde{Q}d_1)\sinh(Q_{\parallel}d_2)+\cosh(\tilde{Q}d_1)\cosh(Q_{\parallel}d_2) \ . \tag{2.35}$$

The remaining boundary conditions connect the amplitude of the potential outside the stack to the amplitudes of the potential in the outermost ferromagnetic film. The continuity of φ_M at $y=0$ gives

$$\varphi_0=v^{(+)}e^{-\tilde{Q}d_1}+v^{(-)} \ . \tag{2.36}$$

Continuity of b_y at $y=0$ gives

$$\varphi_0=\Lambda^{(-)}e^{-\tilde{Q}d_1}v^{(+)}-\Lambda^{(+)}v^{(-)} \ . \tag{2.37}$$

By subtracting (2.37) from (2.36) we obtain an equation involving only $v^{(+)}$ and $v^{(-)}$:

$$(1-\Lambda^{(-)})e^{-\tilde{Q}d_1}v^{(+)}+(1+\Lambda^{(+)})v^{(-)}=0 \ . \tag{2.38}$$

Upon combining (2.34) and (2.38) we obtain the dispersion relation for the surface mode:

$$(1-\Lambda^{(-)})(1+\Lambda^{(+)})\sinh(\tilde{Q}d_1)=0 \ . \tag{2.39}$$

There are several different ways to satisfy this equation but not all of them will correspond to true surface modes of the semi-infinite stack.

We examine the following three cases:

(1) $1-\Lambda^{(-)}=0$. One can show from Eq. (2.34) that $v^{(-)}=0$. Then from Eq. (2.33) one finds

$$\alpha L=-(\tilde{Q}d_1+Q_{\parallel}d_2) \ . \tag{2.40}$$

Since α must be positive in order to have exponential decay, and since $\tilde{Q}$ and $Q_{\parallel}$ are positive, we conclude that $1-\Lambda^{(-)}=0$ is not a good surface-wave solution.

(2) $1+\Lambda^{(+)}=0$. From Eq. (2.34) one can show that in this case $v^{(+)}=0$. Then from Eq. (2.33) one

finds

$$\alpha L = \tilde{Q} d_1 - Q_{\|} d_2 \ . \tag{2.41}$$

This clearly can be positive if $\tilde{Q}d_1$ is larger than $Q_{\|}d_2$. Thus this is a true surface-wave solution. The condition $1+\Lambda^{(+)}=0$ may be written

$$\Omega = \tfrac{1}{2}(H_0/\cos\theta + B\cos\theta) \ , \tag{2.42}$$

which is just the dispersion relation for a surface spin wave on a semi-infinite ferromagnet. Physically, this solution corresponds to a surface wave of the semi-infinite stack composed of surface waves in each ferromagnetic film. It is remarkable that its frequency coincides precisely with that of the semi-infinite simple ferromagnet.

(3) $\sinh(\tilde{Q}d_1)=0$. This equation may be satisfied if

$$\tilde{Q} = \frac{in\pi}{d_1}, \quad n=0,1,2,3,\ldots, \ . \tag{2.43}$$

Such a solution corresponds to a bulk standing wave in each ferromagnetic film. The dispersion relation corresponding to Eq. (2.43) is

$$\Omega^2 = H_0(H_0+4\pi M_s) - \frac{4\pi M_s H_0 \sin^2\psi}{1+\left[\dfrac{n\pi}{Q_{\|}d_1}\right]^2} \ . \tag{2.44}$$

The decay parameter α may be calculated from Eq. (2.35):

$$\cosh(\alpha L) = (-1)^n \cosh(Q_{\|}d_2) \ . \tag{2.45}$$

In order to satisfy this equation for real, positive α, one is restricted to even values of n. We then see $\alpha L = Q_{\|}d_2/L$. If n is odd, then one finds

$$\alpha L = Q_{\|}(d_2/L) + i\pi(2n+1)/L \ ,$$

and the imaginary part produces a 180° phase shift as we move from film to film. This solution thus corresponds to bulk standing waves in each ferromagnetic film, where the amplitude of the standing waves is decreased exponentially as one moves away from the surface of the stack of films. For n odd, we also have solutions for which $\alpha L = Q_{\|}d_z + i\pi$. The two types of allowed surface modes for the semi-infinite stack are illustrated in Fig. 2.

We now turn to some numerical examples to illustrate the dispersion relations for the bulk and surface spin waves. As a model system we consider alternating layers of ferromagnetic Ni ($M_s = 480$ G) on nonmagnetic Mo. The applied field H_0 is 1000 G. This system has been studied experimentally recently through Brillouin scattering.[13]

First consider propagation perpendicular to the

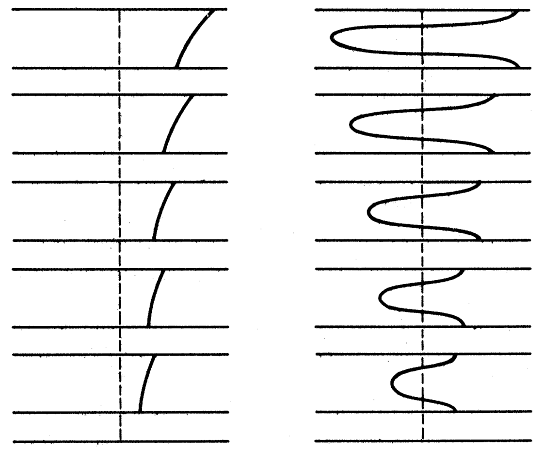

FIG. 2. Illustration of the two types of surface modes. In (a) we have a surface mode of the stack composed of *surface* modes in each ferromagnetic film. In (b) we have a surface mode of the stack composed of bulk modes in each ferromagnetic film.

applied field. In Fig. 3 we present results for the frequency of the various modes versus the ratio d_1/d_2. Results are plotted for three different values of $Q_{\|}d_1$. The general features common to all three sets of curves are as follows:

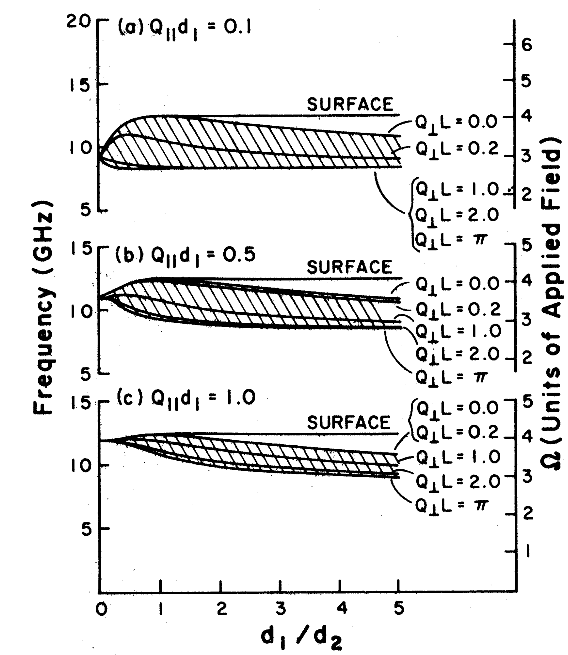

FIG. 3. Frequency of various modes vs the ratio d_1/d_2. Bulk modes of the stack are shown with a shaded region the surface modes are shown by a solid line.

(1) There is a band of "bulk states" (bulk states for the stack—in each ferromagnetic film there is a surface-wave-like mode) with a maximum range in frequency from H_0B to $\frac{1}{2}(H_0+B)$.

(2) In general, as $Q_\perp L$ increases, the frequency of the mode decreases. For the values of $Q_{\parallel}d_1$ used here, the density of states is largest near $Q_\perp L=\pi$. As $Q_\perp d_1$ is increased, the density of states becomes more uniform over the allowed frequency range.

(3) There is a surface mode for which the *frequency* is independent of the ratio of d_1/d_2, and equal to that of the Damon-Eshbach frequency of the semi-infinite magnet. This mode exists, however, only if $d_1>d_2$.

We also note that for $d_1=d_2$, as $Q_{\parallel}d_1$ increases, the modes near $Q_\perp L=\pi$ are shifted up in frequency. Since the density of states is large near $Q_\perp L=\pi$ one would imagine that in a light-scattering experiment the scattering from these states should dominate. In this case, a shift in frequency should be observed as $Q_{\parallel}d_1$ increases, and we will see later that this is so, when we present our detailed calculations of the light-scattering spectrum.

In Fig. 4 we present results for frequency versus the ratio d_1/d_2 for different modes. Propagation is now at a 30° angle with respect to the x axis, and we have taken $Q_{\parallel}d_1=1.0$. In addition to the bulk modes there are several surface modes. One at high frequency is at the Damon-Eshbach frequency given by Eq. (2.42). In addition we see other surface modes *below* the bulk band. As mentioned earlier, the low-frequency modes are surface modes of the stack, but composed of standing bulk waves in each ferromagnetic film, as illustrated in Fig. 2(b).

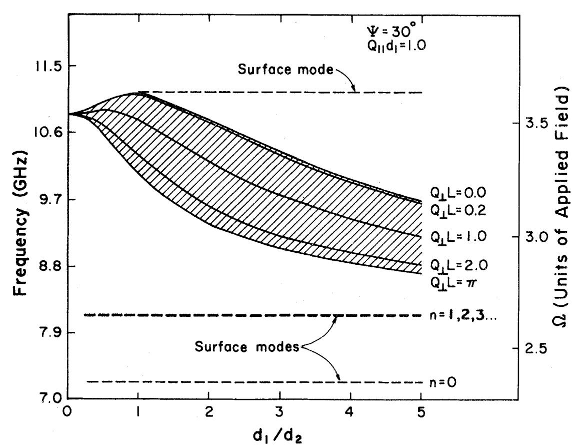

FIG. 4. Frequency of various modes vs the ratio d_1/d_2. Bulk modes of the stack are shown with a solid line, surface modes of the stack have a dashed line. Modes with $n\geq 1$ lie very close in frequency, and cannot be resolved on the scale of the graph.

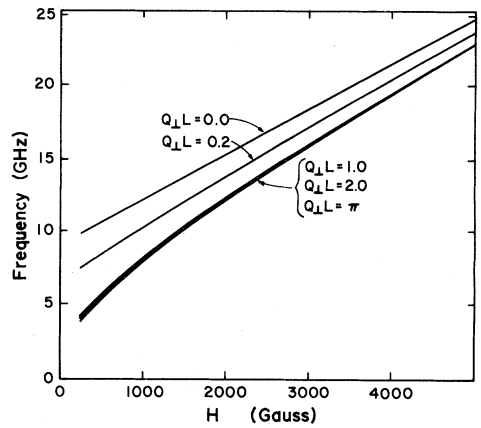

FIG. 5. Frequency of different bulk modes vs applied field.

In Fig. 5 we present a plot of frequency versus the applied field for different values of $Q_\perp$. For this example we have set $Q_\perp d_1=Q_{\parallel}d_2=0.1$, and propagation is perpendicular to the fields. We see here that as $Q_\perp L$ is increased the frequency decreases. Over most of the range of field values here, the frequency varies nearly linearly with applied field. A similar plot to Fig. 5, but with $Q_{\parallel}d_1=Q_{\parallel}d_2=1.0$ would show the $Q_\perp L=0$ line of the same position as in Fig. 5, but the $Q_\perp L=\pi$ line would be upshifted by about 2 GHz from its position in Fig. 5.

III. CALCULATION OF RESPONSE FUNCTIONS AND LIGHT-SCATTERING SPECTRUM

In order to calculate the light-scattering spectrum we need the appropriate set of response functions or Green's functions for the layered structure. These functions give the response of the system to an external perturbation.

In Sec. II we wrote the equations for the magnetic scalar potential in terms of a frequency-dependent susceptibility. Here it is more convenient to begin with the equations of motion for the transverse components of magnetization and the scalar potential. The equation of motion for $M_x(\vec{x},t)$ and $M_y(\vec{x},t)$ are given by the Bloch equations,

$$\frac{dM_i}{dt}(\vec{x},t)=[\vec{M}(\vec{x},t)\times\vec{h}_T(\vec{x},t)]_i\ , \tag{3.1}$$

where $\vec{h}_T(\vec{x},t)$ is the total effective field acting on a spin at $\vec{x}$ at time t. Now $\vec{h}_T(\vec{x},t)$ has three contributions: (1) the applied field H_0 in the z direction, (2) the demagnetizing field $\vec{h}_d(\vec{x},t)$ set up by the spin motion, and (3) an externally applied field $\vec{h}_e(\vec{x},t)$ which varies in space and time. The demagnetizing field may be calculated from Maxwell's equations, which in the absence of retardation has the form

$$\vec{\nabla}\times\vec{h}_d(\vec{x},t)=\vec{\nabla}\cdot[\vec{h}_d(\vec{x},t)+4\pi\vec{M}(\vec{x},t)]=0 . \quad (3.2)$$

The first part of Eq. (3.2) means we may write $\vec{h}_d$ in terms of a scalar potential φ

$$\vec{h}_d(\vec{x},t)=-\vec{\nabla}\varphi_M(\vec{x},t) .$$

In Eqs. (3.1) and (3.2) we use the approximation of spin-wave theory and replace M_z by the saturation magnetization M_s. Also we linearize the equations by dropping terms which are nonlinear in M_x, M_y, or φ_M. We introduce a compact notation where $u_1=M_x$, $u_2=M_y$, and $u_3=\varphi_M$. Also we let $f_1=+M_s h_{e_y}$, $f_2=-M_s h_{e_x}$, and $f_3=0$. We also assume a time variation of $e^{-i\Omega t}$ for M_x, M_y, φ_M, and $\vec{h}_e$. In this case the equations of motion may be written in the compact form

$$\sum_{j=1}^{3} L_{ij}(\vec{x})u_j(\vec{x})=f_i(\vec{x}) . \quad (3.3)$$

The matrix L is given by

$$L=\begin{bmatrix} i\Omega & H_0 & M_s\partial/\partial y \\ -H_0 & i\Omega & -M_x\partial/\partial x \\ -4\pi\partial/\partial x & -4\pi\partial/\partial y & \nabla^2 \end{bmatrix} . \quad (3.4)$$

The formal solution of Eq. (3.3) is found by introducing an array of Green's functions $g_{ij}(\vec{x},\vec{x}',\Omega)$ which satisfy

$$\sum_{k=1}^{3} L_{ik}(\vec{x})g_{kj}(\vec{x},\vec{x}';\Omega)=\delta_{ij}\delta(\vec{x}-\vec{x}') . \quad (3.5)$$

The solution to Eq. (3.3) may then be written in terms of the Green's tensor

$$u_i(\vec{x})=\sum_{j=1}^{3}\int d^3x' g_{ij}(\vec{x},\vec{x}';\Omega)f_j(\vec{x}') . \quad (3.6)$$

We may exploit the translational invariance parallel to the surface by Fourier transforming out the dependence on $\vec{x}_{||}$ and $\vec{x}'_{||}$. We thus expand the Green's tensor and the δ function as follows:

$$g_{ij}(x,x';\Omega)=\int \frac{d^2Q_{||}}{(2\pi)^2}g_{ij}(y,y';Q_{||}\Omega) \times e^{i\vec{Q}_{||}\cdot(\vec{x}_{||}-\vec{x}'_{||})} , \quad (3.7)$$

$$\delta(\vec{x}-\vec{x}')=\int \frac{d^2Q_{||}}{(2\pi)^2}\delta(y-y')e^{i\vec{Q}_{||}\cdot(\vec{x}_{||}-\vec{x}'_{||})} . \quad (3.8)$$

In the interest of a compact notation, we will write $g_{ij}(y,y';Q_{||},\Omega)$ simply as $g_{ij}(y,y')$ in what follows. The equations for $g_{ij}(y,y')$ may then be written

$$\begin{bmatrix} i\Omega & H_0 & M_s\partial/\partial y \\ -H_0 & i\Omega & -iM_sQ_x \\ -4\pi i Q_x & -4\pi\partial/\partial y & \partial^2/\partial y^2-Q_{||}^2 \end{bmatrix}\begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}=\begin{bmatrix} \delta(y-y') & 0 & 0 \\ 0 & \delta(y-y') & 0 \\ 0 & 0 & 0 \end{bmatrix} . \quad (3.9)$$

In earlier papers we have solved Eq. (3.9) for the semi-infinite geometry and thin-film geometry using a direct method. There we first solved the inhomogenous problem for a ferromagnet which was infinitely extended in all three spatial directions. In that case, $g_{ij}(y,y')$ is a function of $y-y'$ only and one may do a Fourier transform of Eq. (3.9) to reduce the set of differential equations to an algebraic set of equations. To obtain solutions which satisfied the boundary conditions we added, to the solution of the inhomogeneous problem in an infinite ferromagnet, solutions of the homogenous one in the confined geometry under consideration.

The method described in the preceding paragraph does not work in the case of the layered system. The reason for this is that, even for the infinitely extended layered system, $g_{ij}(y,y')$ is not a function of $y-y'$ since there is no infinitesimal translational invariance in the direction perpendicular to the layers.

We thus employ a different method in this paper. For all values of $y\neq y'$, the δ function vanishes identically, and elements of the Green's tensor obey homogeneous differential equations. We will find solutions to these homogeneous equations (we have found these solutions already in Sec. II) in the regions above ($y>y'$) and below ($y<y'$) point y' where the argument of the δ function vanishes. These solutions will already satisfy the boundary conditions at each interface and at the surface of the semi-infinite stack. To fix the coefficients in front of the solutions for the two regions $y>y'$ and $y<y'$ we will match the solutions across the singularity provided by the δ function at $y=y'$. For the problem of interest here it will be sufficient to consider y' fixed in one particular ferromagnetic film.

Consider now the set of equations involving g_{11}, g_{21}, and g_{31}. It is easy to show that g_{31} obeys the following differential equation:

$$[\partial^2/\partial y^2-Q_x^2-Q_z^2/(1+4\pi\chi_1)]g_{31}(y,y') = \frac{4\pi\left[H_0\frac{\partial}{\partial y}-Q_x\Omega\right]}{(4\pi M_sH_0+H_0^2-\Omega^2)}\delta(y-y') . \quad (3.10)$$

The homogeneous version of this equation is just the Eq. (2.8) satisfied by φ_M in each ferromagnetic film in the layered stack. We thus introduce two functions $\varphi^>(y)$ and $\varphi^<(y)$ which satisfy the equations of motion and boundary conditions for φ_M in the regions of $y>y'$ and $y<y'$, respectively. Explicit forms for $\varphi^>(y)$ and $\varphi^<(y)$ in both the ferromagnetic and nonmagnetic films will be found later through the use of Eqs. (2.14) and (2.16). Thus, we expect the solution of Eq. (3.10) to be of the form

$$g_{31}(y,y')=\varphi^>(y)A(y')\Theta(y-y')+\varphi^<(y)B(y')\Theta(y'-y)\ . \qquad (3.11)$$

While the prefactors A and B are clearly independent of y, we shall find they are functions of y', the location of the singularity in the δ function. The particular values of A and B must be chosen so that $g_{31}(y,y')$ will satisfy Eq. (3.10). We note that Eq. (3.10) is somewhat unusual in that the right-hand side contains not only a δ function, but the derivative of a δ function as well. It is well known that the equation

$$\left[k^2+\frac{\partial^2}{\partial y^2}\right]\Phi(y,y')=\delta(y-y') \qquad (3.12)$$

has a solution

$$\Phi(y,y')=\frac{1}{W}[\varphi^>(y)\varphi^<(y')\Theta(y-y')+\varphi^<(y)\varphi^>(y')\Theta(y'-y)]\ , \qquad (3.13)$$

where the Wronskian is

$$W=\left.\frac{\partial}{\partial y}\varphi^>(y)\varphi^<(y')-\frac{\partial}{\partial y}\varphi^<(y)\varphi^>(y')\right|_{y=y'}\ . \qquad (3.14)$$

and $\varphi^>(y)$ and $\varphi^<(y)$ are the solutions of

$$\left[k^2+\frac{\partial^2}{\partial y^2}\right]\varphi(y)=0$$

in the region $y>y'$ and $y<y'$, respectively.

It is also easy to show that if we consider

$$\Phi(y,y')=-\frac{1}{W}\left[\varphi^>(y)\left[\frac{\partial}{\partial y'}\varphi^<(y')\right]\Theta(y-y')+\varphi^<(y)\left[\frac{\partial}{\partial y'}\varphi^>(y')\right]\Theta(y'-y)\right]\ . \qquad (3.15)$$

Then

$$\left[k^2+\frac{\partial^2}{\partial y^2}\right]\Phi=\frac{\partial}{\partial y}\delta(y-y')\ . \qquad (3.16)$$

Thus in order to solve Eq. (3.10) we need only to add the solutions of Eqs. (3.13) and (3.15) with appropriate constants to obtain the prefactor of the δ function in Eq. (3.10). We thus find the following solution for $g_{31}(y,y')$:

$$g_{31}(y-y')=\frac{4\pi}{W\omega_1^2}\left[\varphi^>(y)\left[-H_0\frac{\partial}{\partial y'}\varphi^<(y')-Q_x\Omega\varphi^<(y')\right]\Theta(y-y')+\varphi^<(y)\left[-H_0\frac{\partial}{\partial y'}\varphi^>(y')-Q_x\Omega\varphi^>(y')\right]\Theta(y'-y)\right]\ , \qquad (3.17)$$

where $\omega_1^2\equiv H_0(H_0+4\pi M_s)-\Omega^2$. We note from the homogeneous equations of motion that

$$m_y(y)=\frac{M_s}{\Omega^2-H_0^2}\left[H_0\frac{\partial}{\partial y}+\Omega Q_x\right]\varphi(y)\ . \qquad (3.18)$$

The function $g_{31}(y,y')$ may thus be compactly written

$$g_{31}(y,y')=-\frac{4\pi(\Omega^2-H_0)}{WM_s\omega_1^2}[\varphi^>(y)m_y^<(y')\Theta(y-y')+\varphi^<(y)m_y^>(y')\Theta(y'-y)]\ . \qquad (3.19)$$

The remaining two functions g_{21} and g_{11} may be found in terms of g_{31} through Eq. (3.9).

The full set of Green's functions necessary for the light-scattering calculation may be found by similar methods. The results are the following:

$$G_{11}(y,y')=\frac{i\Omega\delta(y-y')}{\omega_1^2}-\frac{4\pi(\Omega^2-H_0^2)}{WM_s\omega_1^2}[m_x^>(y)m_y^<(y')\Theta(y-y')+m_x^<(y)m_y^>(y')\Theta(y'-y)]\ , \qquad (3.20)$$

$$G_{21}(y,y')=\frac{H_0\delta(y-y')}{\omega_1^2}-\frac{4\pi(\Omega^2-H_0^2)}{WM_s\omega_1}[m_y^>(y)m_y^<(y')\Theta(y-y')+m_y^<(y)m_y^>(y')\Theta(y'-y)]\ , \tag{3.21}$$

$$G_{31}(y,y')=-\frac{4\pi(\Omega^2-H_0^2)}{WM_s\omega_1^2}[\varphi^>(y)m_y^<(y')\Theta(y-y')+\varphi^<(y)m_y^>(y')\Theta(y'-y)]\ , \tag{3.22}$$

$$G_{12}(y,y')=-\frac{B\delta(y-y')}{\omega_1^2}-\frac{4\pi(\Omega^2-H_0^2)}{WM_s\omega_1^2}[m_x^>(y)m_x^<(y')\Theta(y-y')+m_x^<(y)m_x^>(y')\Theta(y'-y)]\ , \tag{3.23}$$

$$G_{22}(y,y')=\frac{i\Omega\delta(y-y')}{\omega_1^2}-\frac{4\pi(\Omega^2-H_0^2)}{WM_s\omega_1^2}[m_y^>(y)m_x^<(y')\Theta(y-y')+m_y^<(y)m_x^>(y')\Theta(y'-y)]\ , \tag{3.24}$$

$$G_{32}(y,y')=-\frac{4\pi(\Omega^2-H_0^2)}{WM_s\omega_1^2}[\varphi^>(y)m_x^<(y')\Theta(y-y')+\varphi^<(y)m_x^>(y')\Theta(y'-y)]\ . \tag{3.25}$$

We note that within the ferromagnetic films, the Wronskian W is independent of y. Also, m_x may be found in terms of φ through the homogeneous equations of motion:

$$m_x(y)=\frac{iM_s}{\Omega^2-H_0^2}\left[H_0Q_x+\Omega\frac{\partial}{\partial y}\right]\varphi(y)\ . \tag{3.26}$$

The formal solutions for the Green's functions given above are essentially independent of geometry. If we wish to consider a semi-infinite ferromagnet, then we must only substitute in Eqs. (3.20)—(3.25) the forms for $m_x^>(y),m_x^<(y),m_y^>(y),m_y^<(y),\varphi^<(y)$, which are solutions for waves in the semi-infinite ferromagnet. One may show that Eqs. (3.20)—(3.25) do in fact reproduce the correct Green's functions for the infinite ferromagnet, the semi-infinite ferromagnet, and for a single isolated film.

For the layered structure considered here $\varphi^>(y)$ will be a wave in the region $y>y'$ propagating toward $+\infty$. Similarly $\varphi^<(y)$ is composed of two waves: One propagates away from the singularity in the δ function at y' toward the surface, and one is a reflected wave which propagates away from the surface. Thus in the ferromagnet we set

$$\varphi^>(y)=e^{i\vec{Q}_{\parallel}\cdot\vec{x}_{\parallel}}e^{+iQ_{\perp}nL}(v_+^>e^{\tilde{Q}(y-nL-d_1)}+v_-^>e^{-\tilde{Q}(y-nL)}) \tag{3.27}$$

and

$$\varphi^<(y)=e^{i\vec{Q}_{\parallel}\cdot\vec{x}_{\parallel}}e^{-iQ_{\perp}nL}(v_+^Ie^{\tilde{Q}(y-nL-d_1)}+v_-^Ie^{-\tilde{Q}(y-nL)})+e^{i\vec{Q}_{\parallel}\cdot\vec{x}_{\parallel}}e^{+iQ_{\perp}nL}(v_+^Re^{\tilde{Q}(y-nL-d_1)}+v_-^Re^{-\tilde{Q}(y-nL)})\ . \tag{3.28}$$

The form of these equations is chosen so that $\varphi^>(y)$ and $\varphi^<(y)$ satisfy the differential equation for $\varphi(y)$ in the ferromagnet, as we saw in Sec. II. As we have seen then $Q_{\perp}L$ may be found for a given $Q_{\parallel}$ and Ω through Eq. (2.24). Also v_+ and v_- are related to each other through the boundary conditions at $y=nL$ and $y=nL+d_1$. From Eqs. (2.15)—(2.22) it can be shown that

$$v_+^{R,>}=A^Rv_-^{R,>} \tag{3.29}$$

and

$$v_+^I=A^Iv_-^I\ , \tag{3.30}$$

where

$$A^R=\left[\frac{e^{\tilde{Q}d_1}-e^{-iQ_{\perp}L}e^{-Q_{\parallel}d_2}}{1-e^{-iQ_{\perp}L}e^{\tilde{Q}d_1}e^{-Q_{\parallel}d_2}}\right]\frac{(1+\Lambda^{(+)})}{(\Lambda^{(-)}-1)}\ , \tag{3.31}$$

$$A^I=\left[\frac{e^{\tilde{Q}d_1}-e^{+iQ_{\perp}L}e^{-Q_{\parallel}d_2}}{1-e^{+iQ_{\perp}L}e^{\tilde{Q}d_1}e^{-Q_{\parallel}d_2}}\right]\frac{(1+\Lambda^{(+)})}{(\Lambda^{(-)}-1)}\ . \tag{3.32}$$

The equations for $\varphi^<(y)$ and $\varphi^>(y)$ in the nonmagnetic material may also be obtained through Eqs. (2.19)—(2.22). However, we will not need these results in the present paper.

Thus in the expressions for $\varphi^{>}(y)$ and $\varphi^{<}(y)$ we have only three arbitrary constants, $v_{-}^{(R)}$, $v_{-}^{(I)}$, and $v_{-}^{\geq}$. To completely determine the Green's function, we need only to find $v_{-}^{(R)}$ in terms of $v_{-}^{(I)}$. This is done through the boundary conditions at the surface of the semi-infinite stack. In the region outside the stack ($y<0$) there is a magnetic potential set up by the motion of the spins inside the ferromagnet. This potential has the form

$$\varphi_0(y)=v_0 e^{i\vec{Q}_{\parallel}\cdot\vec{x}_{\parallel}} e^{+Q_{\parallel}y} .$$

The boundary conditions are the continuity of

$$[\varphi(y)]_{y=0+}=[\varphi_0(y)]_{y=0-} \tag{3.33}$$

and the continuity of b_y at the surface

$$\left[-\frac{\partial}{\partial y}\varphi^{<}(y)+4\pi M_y(y)\right]_{y=0+} = \left[-\frac{\partial}{\partial y}\varphi_0(y)\right]_{y=0-} . \tag{3.34}$$

From the preceding two equations, we may obtain the following expression for v_{-}^{R} in terms of v_{-}^{I}. Let

$$A=\frac{(\Lambda^{(-)}-1)A^{I}e^{-\tilde{Q}d_1}-(1+\Lambda^{(+)})}{(1-\Lambda^{(-)})A^{R}e^{-\tilde{Q}d_1}+(1+\Lambda^{(+)})} ; \tag{3.35}$$

then

$$v_{-}^{R}=Av_{-}^{I} . \tag{3.36}$$

With these results, explicit expressions for the Green's functions may be obtained for the semi-infinite layered material. Since these equations are derivable from the preceding work, and since the equations are lengthy, we do not give the final form for the Green's functions here.

Having found the Green's functions, we may directly calculate the scattering of light by spin-wave fluctuations in the ferromagnets. The detailed theory for such a calculation has been given by the current authors in Refs. 5 and 7.

The geometry of the light-scattering experiment is illustrated in Fig. 6. The applied field and saturation magnetization are parallel to the z axis. The incident laser light, with wave vector $\vec{k}^{0}$ and frequency ω_0, makes an angle θ_0 with respect to the surface normal. The scattered light, with wave vector $\vec{k}^{s}$ and frequency ω_s, makes an angle θ_s with respect to the surface normal.

In Ref. 5 it was shown that the intensity of the scattered light was proportional to

$$\begin{aligned}S(\vec{Q}_{\parallel},\Omega)=\mathrm{Im}\{ \int_0^{\infty} dy' \int_0^{\infty} dy'' \exp[i(\Delta k_{\perp}y'-\Delta k_{\perp}^{*}y'')][&r_{\perp\perp}g_{21}(\vec{Q}_{\parallel},\Omega;y'',y')+r_{\parallel\perp}g_{11}(\vec{Q}_{\parallel},\Omega;y'',y')\\ &-r_{\parallel\perp}^{*}g_{22}(\vec{Q}_{\parallel},\Omega;y'',y)-r_{\parallel\parallel}g_{12}(\vec{Q}_{\parallel},\Omega;y'',y')]\} .\end{aligned} \tag{3.37}$$

In the above expression $\vec{Q}_{\parallel}$ is the wave vector of the spin wave created or destroyed in the light-scattering experiment. Thus

$$\vec{Q}_{\parallel}=\vec{k}_{\parallel}^{(0)}-\vec{k}_{\parallel}^{(s)} , \tag{3.38}$$

where $\vec{k}_{\parallel}^{(0)}$ and $\vec{k}_{\parallel}^{(s)}$ are the components of the wave vector parallel to the surface for the incident and scattered radiation, respectively. Similarly,

$$\Omega=\omega_0-\omega_s . \tag{3.39}$$

Thus for a Stokes process (creation of a spin wave) Ω is positive. For an anti-Stokes process (destruction of a spin wave), Ω is negative. In Eq. (3.37), $r_{\perp\perp}, r_{\parallel\perp}, r_{\parallel\parallel}$ are factors that depend on the light-scattering geometry and dielectric properties of the ferromagnet. The definitions are given in Refs. 5 and 6. In essence, these are the Fresnel coefficients which describe transmission of the incident and scattered photon through the sample surface. The quantity $\Delta k_{\perp}$ is the sum of the wave-vector component perpendicular to the surface for the incident and scattered light in the medium:

$$\Delta k_{\perp}=k_{\perp>}^{(0)}+k_{\perp>}^{(s)} , \tag{3.40}$$

where

$$k_{\perp>}^{(0),(s)}=\left[\frac{\omega_0}{c}\right]\left[\epsilon-\sin^2\theta_{0,s}\right]^{1/2}, \mathrm{Im}k_{\perp>}^{(0),(s)}>0 \tag{3.41}$$

and ϵ is the complex dielectric constant of the ferromagnetic.

The materials in the LUCS structures under consideration here are metals and have a very small skin depth, the order of 100—200 Å. As a result, $\Delta k_{\perp}$ will have a large positive imaginary part, and from Eq. (3.37) we see that the contribution from large y'' or y' will be small. We will then only require the

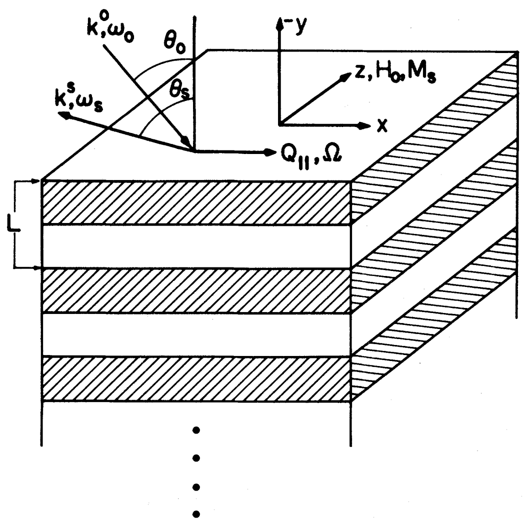

FIG. 6. Geometry of the light-scattering experiment. The incident light, with wave vector $\vec{k}^{(0)}$ and frequency ω_0, strikes the surface at an angle θ_0 with respect to the surface normal. The scattered light has wave vector $\vec{k}^{(s)}$ and frequency ω_s.

Green's functions where y'' and y' are both in the outermost layer. Physically, this corresponds to the fact that the laser light in the metallic LUCS structure is so highly attenuated that it only samples magnetic fluctuations in the outermost ferromagnetic film. Of course, despite this the light probes normal modes of the entire structure by coupling to that portion of the normal-mode eigenfunction which resides in the outer film.

We present below expressions for the elements of the Green's functions which are necessary for the light-scattering calculation in the limit that y and y' are both in the outermost ferromagnetic film:

$$F_1=\frac{iM_sQ_{\|}}{\Omega^2-H_0^2}\left[\frac{Q_x}{Q_{\|}}H_0+\Omega\right]A^R e^{-\tilde{Q}d_1}, \tag{3.42}$$

$$F_2=\frac{iM_sQ_{\|}}{\Omega^2-H_0^2}\left[\frac{Q_x}{Q_{\|}}H_0-\Omega\right], \tag{3.43}$$

$$F_3=\frac{iM_sQ_{\|}}{\Omega^2-H_0^2}\left[\frac{Q_x}{Q_{\|}}H_0+\Omega\right]\left[A^I+A^RA\right]e^{-\tilde{Q}d_1}, \tag{3.44}$$

$$F_4=\frac{iM_sQ_{\|}}{\Omega^2-H_0^2}\left[\frac{Q_x}{Q_{\|}}H_0-\Omega\right](1+A), \tag{3.45}$$

$$F_5=-i\frac{Q_x}{Q_{\|}}F_1,\quad F_7=-i\frac{Q_x}{Q_{\|}}F_3, \tag{3.46}$$

$$F_6=i\frac{Q_x}{Q_{\|}}F_2,\quad F_8=+i\frac{Q_x}{Q_{\|}}F_4. \tag{3.47}$$

Then $m_x^>(y), m_x^<(y), m_y^>(y), m_y^<(y)$ may be written

$$m_x^>(y)=F_1e^{\tilde{Q}y}+F_2e^{-\tilde{Q}y}, \tag{3.48}$$

$$m_x^<(y)=F_3e^{\tilde{Q}y}+F_4e^{-\tilde{Q}y}, \tag{3.49}$$

$$m_y^>(y)=F_5e^{\tilde{Q}y}+F_6e^{-\tilde{Q}y}, \tag{3.50}$$

$$m_y^<(y)=F_7e^{\tilde{Q}y}+F_8e^{-\tilde{Q}y}, \tag{3.51}$$

and we have

$$\begin{aligned} g_{11}(y,y')=\frac{1}{\omega_1^2}\Bigg\{ & i\Omega\delta(y-y')-\frac{4\pi(\Omega^2-H_0^2)}{WM_s}[(F_1F_7e^{\tilde{Q}(y+y')}+F_2F_8e^{-\tilde{Q}(y+y')} \\ & +F_1F_8e^{\tilde{Q}(y-y')}+F_2F_7e^{-\tilde{Q}(y-y')})\Theta(y-y') \\ & +(F_3F_5e^{\tilde{Q}(y+y')}+F_4F_6e^{-\tilde{Q}(y+y')} \\ & +F_3F_6e^{\tilde{Q}(y-y')}+F_4F_5e^{-\tilde{Q}(y-y')})\Theta(y'-y)]\Bigg\}, \end{aligned} \tag{3.52}$$

$$\begin{aligned} g_{21}(y,y')=\frac{1}{\omega_1^2}\Bigg\{ & H_0\delta(y-y')-\frac{4\pi(\Omega^2-H_0^2)}{WM_s}(F_5F_7e^{\tilde{Q}(y+y')}+F_6F_8e^{-\tilde{Q}(y+y')} \\ & +F_5F_8e^{\tilde{Q}|y-y'|}+F_6F_7e^{-\tilde{Q}|y-y'|})\Bigg\}, \end{aligned} \tag{3.53}$$

$$g_{12}(y,y')=\frac{1}{\omega_1^2}\left[-B\delta(y-y')-\frac{4\pi(\Omega^2-H_0^2)}{WM_s}(F_1F_3e^{\tilde{Q}(y+y')}+F_2F_4e^{-\tilde{Q}(y+y')}+F_1F_4e^{\tilde{Q}|y-y'|}+F_2F_3e^{-\tilde{Q}|y-y'|})\right], \tag{3.54}$$

$$g_{22}(y,y')=\frac{1}{\omega_1^2}\left\{i\Omega\delta(y-y')-\frac{4\pi(\Omega^2-H_0^2)}{WM_s}\left[(F_3F_5e^{\tilde{Q}(y+y')}+F_6F_4e^{-\tilde{Q}(y+y')}+F_5F_4e^{+\tilde{Q}(y-y')}+F_6F_3e^{-\tilde{Q}(y-y')})\Theta(y-y')+(F_1F_7e^{\tilde{Q}(y+y')}+F_2F_8e^{-\tilde{Q}(y+y')}+F_7F_2e^{\tilde{Q}(y-y')}+F_1F_8e^{-\tilde{Q}(y-y')})\Theta(y'-y)\right]\right\}, \tag{3.55}$$

and the Wronskian is given by

$$W=2\tilde{Q}(A^R-A^I)e^{-\tilde{Q}d_1}, \tag{3.56}$$

where

$$\omega_1^2=H_0(H_0+4\pi M_s)-\Omega^2 .$$

With the above results, Eq. (3.37) may be easily integrated to provide the light-scattering spectrum.

In Fig. 7 we plot the theoretical light-scattering spectrum for scattering from a semi-infinite layered structure. The incident beam is at a 45° angle with respect to the surface normal, and the scattered light which is collected is that backscattered along the direction of incidence. In this figure and the figures which follow, the plane of incidence for the laser light is perpendicular to the surface and to $\vec{H}_0$ and $\vec{M}_s$. The spin waves which are probed in this geometry are those propagating perpendicular to $\vec{H}_0$. The structure is chosen so that $d_1=d_2$. The applied field is $H_0=1$ kG. The dotted lines at $\omega_0-\omega_s=\pm 2.7$ give the frequency of the bulk spin waves in an infinite ferromagnet $\Omega_b^2=H_0(H_0+4\pi M_s)$. The dotted lines at $\omega_0-\omega_s=\pm 4.02$ give the frequency of surface spin waves on a semi-infinite ferromagnet: $\Omega_s=(H_0+2\pi M_s)$. We see in this figure that for small $Q_{\parallel}d_1$ ($d_1=100$ Å), there is a peak near Ω_b on both the Stokes and anti-Stokes sides. As d_1 is increased, a broad peak emerges from Ω_b and begins moving toward Ω_s. This broad peak represents scattering from the bulk spin waves of the *layered system*. In Sec. II we saw that the density of states for these bulk waves was greatest near the low-frequency side, where $Q_{\perp}L=\pi$. This explains the shape of the peak moving toward Ω_s. There is a sharp increase in intensity at the low-frequency edge of the bulk band, and then as the density of states is reduced there is a reduction of intensity at higher frequencies. On the Stokes side, we have drawn above each spectrum a box which indicates the width of the bulk spin-wave band for that choice of d_1. This box has been calculated using the dispersion relation given in Sec. II. As we saw in Sec. II (Fig. 3), as $Q_{\parallel}d_1$ increases the width of the bulk band decreases, and the low-frequency edge of the band moves up in frequency. All these features are easily seen in the calculated light-scattering spec-

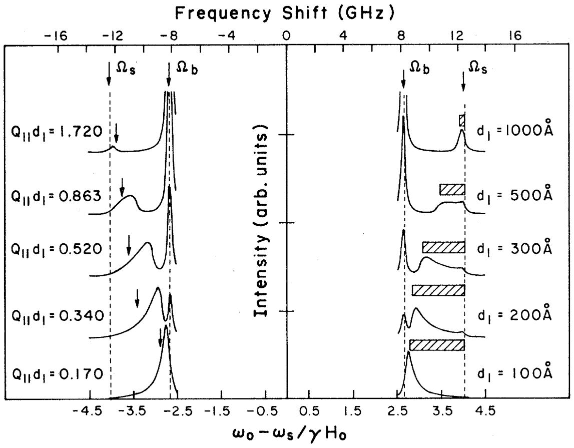

FIG. 7. Light-scattering spectrum from a LUCS structure of alternating layers of Ni and Mo for different values of d_1. Here $d_1=d_2$. The box over each spectrum on the Stokes side gives the frequency range of bulk spin waves in the layered structure. The arrow over each spectrum on the anti-Stokes side gives the frequency of a surface spin wave on an isolated film of thickness d_1.

trum. On the anti-Stokes side, we have drawn arrows representing the frequency of a surface spin wave on an isolated film of thickness d_1. It is clear that the position of the peaks is not in accordance with the results for an isolated film, i.e., the features present in the light-scattering spectrum are influenced strongly by interactions between the ferromagnetic films in the semi-infinite stack.

In Fig. 7 and the figures which follow, we always see a peak at Ω_b. We believe this is an artifact of our model, which includes only dipolar coupling and ignores exchange interactions. In this limit *all* bulk waves in a ferromagnetic film which propagate perpendicular to the magnetization lie at frequency Ω_b. When exchange is included, each bulk wave is shifted up in frequency. As remarked earlier, the bulk modes then have frequencies given roughly by

$$\Omega^2=\left\{H_0+D\left[Q_{\|}^2+\left[\frac{n\pi}{d_1}\right]^2\right]\right\}\times\left\{H_0+D\left[Q_{\|}^2+\left[\frac{n\pi}{d_1}\right]^2\right]+4\pi M_s\right\},\quad n=1,2,3,\ldots, . \tag{3.57}$$

In Ni the exchange constant D is 3.08×10^{-9} G cm^2, and it is easy to see that even for $d_1=1000$ Å only a few bulk modes lie within the range of frequency explored here. Thus if exchange were properly included, there would be no large peak at $\Omega_b=2.7$ GHz. The inclusion of exchange will not alter the nature of the peak which represents scattering from bulk modes of the layered structure. The bulk modes of the layered structure are composed of surface modes in each ferromagnetic film. The introduction of exchange coupling only modestly influences the properties of the surface wave, because of the slow spatial variation of the transverse components of magnetization for these modes.

In Fig. 8 we present several light-scattering spectra for the case $d_1\neq d_2$. We again consider a backscattering geometry for the laser light, with the incident beam at $\theta_0=45°$. In the bottom curve where $d_1=3d_2$ we see on the Stokes side three peaks. One peak at $\omega_0-\omega_s=2.7$ is the spurious peak due to the lack of exchange in the calculation. The peak of $\omega_0-\omega_s=2.9$ is again due to the large density of states of the low-frequency edge of the bulk spin-wave band for the layered structure. Finally at $\omega_0-\omega_s=4.02$ we see a peak at the surface-spin-wave frequency for a semi-infinite Ni sample. This peak is due to scattering from the surface wave of the layered structure. From Eq. (2.41) we see that this mode only exists if $\tilde{Q}d_1>Q_{\|}d_2$. For propagation perpendicular to H_0, $\tilde{Q}=Q_{\|}$, so this condition reduces to $d_1>d_2$ for the surface mode of the layered structure to exist. In the middle set of curves ($H_0=1000$ G) we have reversed the applied field, along with the magnetization, with scattering geometry fixed and we see that the feature at $\omega_0-\omega_s=4.02$ changes from the Stokes to the anti-Stokes side. This is characteristic of surface spin waves on ferromagnetic materials, and indicates the nonreciprocal nature of the mode.[5] Finally in the upper curve we consider the case $d_1=\frac{1}{3}d_2$. In this case $d_1<d_2$, and we see no peak at the surface-spin-wave frequency. We thus predict a spectrum for the case where $d_1>d_2$ that is strikingly different than when $d_1<d_2$.

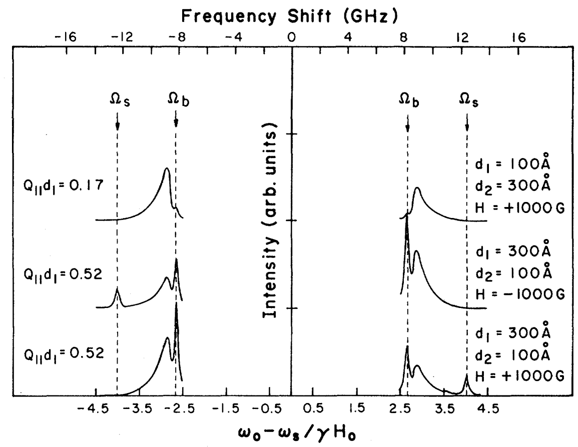

FIG. 8. Light-scattering spectrum from a LUCS structure of alternating layers of Ni and Mo. Here $d_1\neq d_2$. We see a peak at Ω_s in the case $d_1>d_2$ due to scattering from surface waves of the layered structure. For $d_1<d_2$ there is no peak at Ω_s.

Finally we wish to point out that large Stokes—anti-Stokes intensity asymmetries will occur for light scattering from the LUCS structures. This asymmetry is strongly dependent on the angles θ_0 and θ_s and the choice of polarization. In Fig. 9 we present results for the light-scattering spectrum for $\theta_0=74°$, $\theta_s=34°$ and the incident electric field is polarized parallel to the plane of incidence. We see that the features on the Stokes side are significantly larger than on the anti-Stokes side. This is in agreement with recent experiments. A complete discussion of the Stokes—anti-Stokes intensity ratio for light scattering from magnetic excitations has been given recently.[6,9]

IV. SUMMARY AND GENERAL REMARKS

We summarize the main results of the paper below.

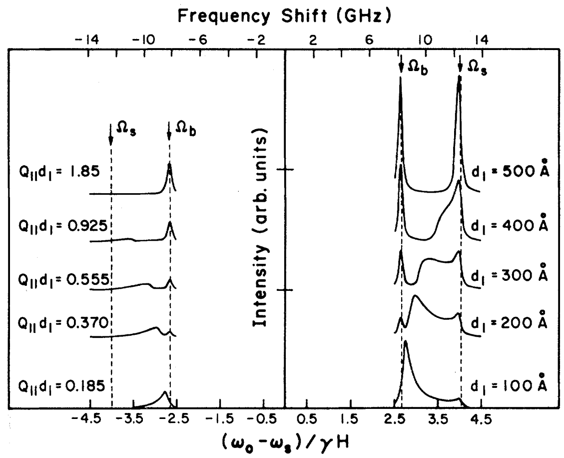

FIG. 9. Light-scattering spectrum from a LUCS structure of alternating layers of Ni and Mo. Here $d_1 = d_2$, and the applied field is $H_0 = 1$ kG. For the geometry of the light scattering ($\theta_0 = 74°$, $\theta_s = 84°$), there is a large asymmetry between the intensity of the Stokes side and on the anti-Stokes side.

(1) We have derived the dispersion relation for bulk and surface spin waves in layered ferromagnetic structures to find aspects unique to the layered structure.

(2) The bulk waves in the layered structure are composed of surface waves in each ferromagnetic film. The surface waves in the layered structure may be composed of surface waves in each ferromagnetic film, or composed of bulk waves in each ferromagnetic film, as discussed in Sec. II. In the first case the frequency of the surface modes of the LUCS lies above the frequency for the bulk band for the LUCS. In the second case the frequency of the surface modes lies below the frequency of the bulk band.

(3) The existence of the surface modes for the layered structure depends on the ratio d_1/d_2. For propagation perpendicular to the applied field we must have $d_1 > d_2$ for surface modes to exist, and the frequency of the surface mode is that of a surface spin wave on a semi-infinite ferromagnet.

(4) We have obtained the Green's functions for layered ferromagnetic structures, and used these response functions to calculate the light-scattering spectrum from ferromagnetic LUCS. We find that the light-scattering spectrum will give detailed information on those properties of the excitations unique to ferromagnetic layered structures.

At the time of this writing, no light-scattering data exists in the literature for these structures, though experiments are underway and preliminary results are in hand.[13] These structures have been studied extensively through use of electron spin resonance, however. Herring and White[14] have presented a theory of the electron-spin-resonance signal expected from a structure with spatially inhomogeneous magnetization, and find the simple and appealing result

$$\omega^2 = H_0 \left[H_0 + 4\pi \frac{\langle M^2 \rangle}{\langle M \rangle} \right] . \tag{4.1}$$

We may inquire if the results presented here are consistent with this relation.

In an electron-spin-resonance experiment, we may expect $\vec{Q}_{\|}$ and $Q_\perp$, as introduced in Sec. II, to be very small in the sense that $|\vec{Q}_{\|}| d_{1,2}$ and $Q_\perp d_{1,2}$ will be small compared to unity. If we consider a sample in the form of a slab of width W and thickness L, where both W and L are large compared to d_1 and d_2, and furthermore let W and L be small compared to a microwave wavelength, then one may expect $|\vec{Q}_{\|}|$ to be the order of π/W, and $Q_\perp$ to be the order of π/L or the inverse of the microwave skin depth, whichever is larger. The mode excited in a resonance experiment is then described by Eq. (2.25) or (2.29). For typical sample geometries, W will be large compared to either L or the microwave skin depth, so we are in the limit $Q_\perp \gg |\vec{Q}_{\|}|$. Then both Eqs. (2.28) and (2.29) reduce to

$$\Omega^2 = H_0(H_0 + 4\pi M_s) , \tag{4.2}$$

where M_s is not the average magnetization in the LUCS sample, but rather that within one ferromagnetic film; i.e., Eq. (4.2) is the ferromagnetic resonance frequency (Kittel frequency) of an *isolated* ferromagnetic film. For the particular magnetization profile we have used in our analysis, the result of Herring and White also reduces to Eq. (4.2), so the two analyses would predict the same result for the electron-spin-resonance frequency. Of course, the analysis of Herring and White has the virtue that it leads to a result applicable not only to the "square-wave"-modulated magnetization profile used throughout the present paper, but to other more complex spatial variations as well.

We have learned recently that Grunberg[15] has also completed a study of spin-wave excitations in layered media, though at the time of this writing, we have no detailed account of his results.

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[1]For example, see Z. Q. Zheng, C. M. Falco, J. B. Ketterson, and I. K. Schuller, Appl. Phys. Lett. 38, 424 (1981).
[2]J. R. Sandercock and W. Wettling, J. Appl. Phys. 50, 7784 (1979).
[3]F. Metawe and P. Grünberg, J. Magn. Magn. Mater. 9, 80 (1978).
[4]M. Grimsditch, A. P. Malozemoff, and A. Brunsch, Phys. Rev. Lett. 43, 711 (1979).
[5]R. E. Camley and D. L. Mills, Phys. Rev. B 18, 4821 (1978).
[6]R. E. Camley and D. L. Mills, Solid State Commun. 28, 321 (1979).
[7]R. E. Camley, Talat S. Rahman, and D. L. Mills, Phys. Rev. B 23, 1226 (1981).
[8]R. E. Camley and M. Grimsditch, Phys. Rev. B 22, 5420 (1980).
[9]R. E. Camley, P. Grünberg, and C. M. Mayr, Phys. Rev. B 26, 2609 (1982).
[10]P. Grünberg, M. G. Cottam, W. Vach, C. Mayr, and R. E. Camley J. Appl. Phys. 53, 2078 (1982).
[11]R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids 19, 308 (1961).
[12]See the discussion by C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1971).
[13]M. Grimsditch, private communication.
[14]C. Herring and R. M. White, Phys. Rev. B 22, 1465 (1980).
[15]P. Grünberg, private communications.