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Surface effects in ferromagnetic superconductors

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The behavior of the magnetization near the surface of a semi-infinite ferromagnetic superconductor is investigated theoretically. In contrast to a previous treatment by Kotani *et al.* [Phys. Rev. B 23, 5960 (1981)] the surface is not described by means of a topological singularity of the phase of the superconducting order parameter. The physical meaning of such a surface singularity is clarified. A mean-field boundary condition $\partial m(0)/\partial x = \mu m(0)$ for the magnetization $m(x)$ is used which allows for a possible strengthening ($\mu < 0$) or weakening ($\mu > 0$) of the exchange coupling constant at the surface. The consequences of a nonzero μ for the field penetration and the magnetic surface ordering are pointed out.

I. INTRODUCTION

The interaction between the competing orderings of superconductivity and ferromagnetism has been the subject of a large number of experimental[1] and theoretical[2] investigations in recent years. In many theoretical treatments an electromagnetic coupling only is considered and the exchange interaction between rare-earth spins and conduction electrons is neglected. By adopting this point of view several interesting phenomena have been predicted and partly verified. These include formation of a spin-spiral state[3,4] or a self-induced vortex state[5] and the occurence of a relatively strong first-order transition at H_{c_1} even for high-κ superconductors.[6,7]

In a recent paper by Tachiki, Umezawa, and coworkers[8] the influence of the surface on the Meissner state of ferromagnetic superconductors has been investigated. It was found that field reversal and spontaneous magnetic surface ordering may occur at temperatures higher than the critical temperature for the bulk spin-spiral or ferromagnetic state. In this work[8] the boson method was employed to study the effect of a finite Ginzburg-Landau parameter κ. The boson theory[9] describes the electrodynamics of the superconducting state by means of a linear nonlocal relation

$$\vec{j}_s(\vec{x}) = -\frac{c}{4\pi}\int d^3y\, K^{\text{bos}}(\vec{x}-\vec{y}) \times \left[\vec{a}(\vec{y}) - \frac{\hbar c}{e}\vec{\nabla} f(\vec{y})\right] , \quad (1)$$

between supercurrent $\vec{j}_s(\vec{x})$ and vector potential $\vec{a}(\vec{x})$. Here, $K^{\text{bos}}(\vec{x})$ is the integral kernel of the boson theory usually denoted by $c(\vec{x})/\lambda_L^2$ and $f(\vec{x})$ is half the phase of the superconducting order parameter. Normal regions inside the superconductor are described by means of a non-single-valued phase function. In Ref. 8 a phase was chosen which describes the surface as a linear superposition of flux lines. The paramagnetic response of the medium is taken into account by adding to Eq. (1) a magnetization current

$$\vec{j}_m(\vec{x}) = c\,\vec{\nabla}_x \times \int d^3y\, \chi(\vec{x}-\vec{y})\,\vec{\nabla}_y \times \vec{a}(\vec{y}) , \quad (2)$$

where $\chi(\vec{x})$ is a mean-field susceptibility defined below. The boundary condition for the magnetization $\vec{m}(\vec{x})$ [defined by $\vec{j}_m(\vec{x}) = c\,\vec{\nabla} \times \vec{m}(\vec{x})$] used in Ref. 8 is $\partial m/\partial x = 0$, at the surface $x = 0$ separating the superconductor ($x > 0$) from the vacuum.

In the present work a modification and extension of the results of Ref. 8 is presented. First, the circumstances under which the surface may be treated as a topological singularity are discussed and it is shown that, at low fields, this description is inappropriate. These considerations also refer to a previous work[10] on surface effects in nonmagnetic superconductors. Secondly, the treatment of Ref. 8 is extended to include a variety of possible surface states of the magnetization. The general boundary condition[11] $\partial m(0)/\partial x = \mu m(0)$ is used, where μ—in the absence of a microscopic derivation—is treated as a phenomenological parameter. In Sec. II the boson treatment of the surface is discussed. In Sec. III various solutions of the integrodifferential equation of ferromagnetic superconductors are obtained and the consequences of a nonzero surface parameter μ are pointed out in Sec. IV. In Sec. V a short summary is given and some general remarks are made on the significance of nonlocality with regard to the elimination of unphysical features of the London model.

II. TWO POSSIBLE SURFACE DESCRIPTIONS

Let us consider the London approximation of the Ginzburg-Landau equations where a local relation between $\vec{j}_s(\vec{x})$ and $\vec{a}(\vec{x})$ is used. The presence of a flux line is accounted for by introducing a non-single-valued phase ϕ of the order parameter[12] obeying

$$\vec{\nabla} \times \vec{\nabla}\phi = 2\pi\delta^{(2)}(\vec{x})\vec{e} \ ,$$

where $\delta^{(2)}(\vec{x})$ is the two-dimensional delta function and $\vec{e}$ is the unit vector parallel to the applied field. In what follows such a ϕ—or the corresponding f of the boson theory—is referred to as a topological singularity. As is well known, the London model breaks down near the core where current and field diverge. Proper use of a nonlocal $\vec{j}_s$-$\vec{a}$ relation (see the discussion in Sec. V) eliminates these singularities. At the same time a vanishing of $\vec{j}_s(\vec{x})$ at the flux line center is found, this being a typical consequence of a vanishing order parameter. The boson method takes advantage of these facts in a most concise way. The results obtained agree reasonably well with Ginzburg-Landau calculations, where the flux line center is defined by a zero of the order parameter.

The topological singularity characterizing the surface in previous boson treatments[8,10,13] is a linear superposition of vortex singularities. The surface is actually identified with a dense array of vortices embedded in an infinite superconducting space. Consequently, it is a place where order parameter and supercurrent vanish. The latter feature is explicitly shown by the results of Refs. 8 and 10. Therefore, the topological singularity will serve as a good description provided the order parameter at the surface is sufficiently small. This is achieved either by a proper surface treatment or by application of a sufficiently high (superheating) field. In all other situations, in particular in the presence of applied fields lower than the first critical field, it will be a better approximation to assume a constant order parameter, corresponding to a regular phase which can be equated to zero. In this case, the magnetic field and the penetration depth are immediately obtained by insertion of the boson integral kernel into well-known results (see, e.g., p. 76 of Ref. 12). It seems that the representation of the surface by means of a topological singularity can, after appropriate modifications, be used to describe a normal-superconducting boundary.

The distinction between the two mentioned surface descriptions is, of course, important from a conceptual point of view. It should be added, however, that both give similar results in the case of both current reversal near a surface[10,14] and attractive interaction between flux line and surface.[13] The same holds true for the present situation. The numerical changes implied by a nonzero μ are much larger than those resulting from a different choice of the phase.

III. BASIC EQUATIONS AND ANALYTICAL RESULTS

The temperatures, measured in units of the Curie temperature T_m, characterizing the surface are t_0, where the magnetization gets an oscillatory part, and t_s, where a spontaneous surface magnetization appears. These are to be compared with t_p, the critical temperature for spin-spiral ordering.

Since a situation with a constant modulus of the superconducting order parameter is considered, one has to solve Maxwell's equation

$$\vec{\nabla} \times \vec{b}(\vec{x}) = \frac{4\pi}{c}[\vec{j}_s(\vec{x}) + \vec{j}_m(\vec{x})] \ , \tag{3}$$

subject to boundary conditions given below. The mean-field susceptibility[8,15] in the expression for $\vec{j}_m(\vec{x})$, Eq. (2), is given by its Fourier transform

$$\chi(k) = C/(T - T_m + 4\pi C + Dk^2) \ ,$$

using the notation of Ref. 8. The supercurrent $\vec{j}_s(\vec{x})$ is defined by Eq. (1) with $\vec{\nabla} f(\vec{x}) \equiv 0$. If $\vec{h}$, $\vec{b}$, $\vec{m}$ are directed along the y axis with components $h(x)$, $b(x)$, $m(x)$ and the superconductor occupies the half-space $x > 0$, Eq. (3) takes the form

$$\frac{\partial^2 a}{\partial x^2} = \int dx' K(x-x')a(x') - 4\pi \times \int dx' \frac{\partial^2 \chi(x-x')}{\partial x' \partial x} a(x') \ . \tag{4}$$

Here the vector potential $\vec{a}(\vec{x}) = a(x)\vec{e}_z$ has been introduced as a new variable, replacing $b(x) = -\partial a(x)/\partial x$. The boundary conditions are

$$b(0) - 4\pi m(0) = H \ , \tag{5a}$$

$$b(\infty) = 0 \ , \tag{5b}$$

$$\frac{1}{m(0)} \left(\frac{\partial m(x)}{\partial x} \right)_{x=0} = \mu \ . \tag{5c}$$

According to the Ginzburg-Landau boundary condition[11,16] (5c), the surface state is characterized by a temperature-independent parameter μ, which allows for a possible weakening ($\mu > 0$) or strengthening

($\mu < 0$) of the magnetization at the surface, compared to that in the bulk.[17] Since, to the present author's knowledge, little is known on the actual value of μ in the rare-earth compounds, a solution for arbitrary μ is required. It can be obtained by a generalization of the London-limit calculations of Ref. 8. Before doing this, however, it is illustrative to consider two simple analytical solutions.

Equation (4), with integrations performed from $-\infty$ to $+\infty$, is only valid in an infinite medium but can be adapted to a half-space by requiring $a(x)$ to be either an even or an odd function of x. Let us first take $a(x)=a(-x)$, an assumption commonly used to describe specular reflection[12] of the current carriers at the surface. Fourier transformation of Eq. (4) yields

$$-k^2a(k)-2\left(\frac{\partial a}{\partial x}\right)_{x=0}=[K(k)-4\pi k^2\chi(k)]a(k) \ ,$$

from which the following fields are obtained:

$$b(x)=\frac{2H}{\pi}\int_0^\infty dk\frac{k\sin kx}{k^2[1-4\pi\chi(k)]+K(k)} \ , \qquad (6)$$

$$m(x)=\frac{2H}{\pi}\int_0^\infty dk\frac{k\chi(k)\sin kx}{k^2[1-4\pi\chi(k)]+K(k)} \ , \qquad (7)$$

$$j_s(x)=-\frac{cH}{2\pi^2}\int_0^\infty dk\frac{K(k)\cos kx}{k^2[1-4\pi\chi(k)]+K(k)} \ . \qquad (8)$$

Because $m(0)=0$, Eqs. (6)–(8) constitute a solution for $\mu=\infty$. The temperatures t_0 and t_p, being determined by the roots of the equation

$$k^2[1-4\pi\chi(k)]+K(k)=0 \ , \qquad (9)$$

agree with the results of Ref. 8 [provided the boson kernel $K(k)=c(k)/\lambda_L^2$ is used]. The critical temperature t_s differs from Ref. 8 and is equal to t_p for all values of κ.

Let now $a(x)$ be an odd function of x with a discontinuity at $x=0$. The Fourier transform of Eq. (4) now reads

$$-k^2a(k)-2ik\lim_{x\to 0+}a(x)=[K(k)-4\pi k^2\chi(k)]a(k) \ . \qquad (10)$$

The fields $b(x)$, $m(x)$, and $j_s(x)$ obtained from Eq. (10) agree exactly with those of the boson method [Eqs. (4.8), (4.11), and (4.12) of Ref. 8]. We shall come back to this coincidence in the last section. The boson result corresponds to $\partial m(0)/\partial x=0$, i.e., $\mu=0$, but in addition contains, as shown in the last section, the implicit assumption of a vanishing order parameter at the surface. As a consequence, $j_s(0)=0$.

A solution is needed which avoids this unphysical feature and at the same time takes into consideration a variety of possible surface states μ. It is obtained by writing $b(x)$ in the form

$$b(x)=\sum_l b_l e^{ik_lx} \ ,$$

where k_l are the complex roots of Eq. (9). This method of adaption of Eq. (4) to the half-space is different from the two preceding ones. Each component of $b(x)$ is a solution of Eq. (4) in an infinite medium. The constants b_l are then to be determined from the boundary conditions (5a)–(5c). In the absence of superconductivity this method of solution gives the mean-field result[16]: $t_s=1+d\mu^2$ for $\mu<0$, and $t_s=1$ for $\mu>0$ ($d=D\lambda_L^2/T_m$). For the integral kernel we choose for simplicity[18] the anaytical approximation[8] of the boson theory:

$$K(k)=\lambda_L^{-2}e^{-0.447(k\xi)^2} \ .$$

For the temperature range considered here the coherence length ξ, the London penetration depth λ_L, and the Ginzburg-Landau parameter $\kappa=\lambda_L/\zeta$ may be treated as constants. The parameters used are $\bar{c}=4\pi C/T_m=2$ and $d=D/\lambda_L^2T_m=0.01$. For $t>t_0$, Eq. (9) has purely imaginary roots $k_{1,2}=\pm i\gamma_1$, $k_{3,4}=\pm i\gamma_2$. In the range $t_0>t>t_p$, complex roots appear which are denoted by $k_{1,2}=\pm p_1\pm ip_2$, $k_{3,4}=\pm p_1\mp ip_2$. The values of γ_1, γ_2 and p_1, p_2 enter the analytical expressions for $b(x)$, $m(x)$, $j_s(x)$, and the effective penetration depth

$$\lambda_{\rm eff}=\frac{1}{b(0)}\int_0^\infty dx\, b(x) \ , \qquad (11)$$

calculated by means of the boundary conditions.

For $t>t_0$, one obtains

$$b(x)=H\eta[\gamma_1^2(\gamma_2+\mu)(\gamma_2^2-e^{R_3\gamma_2^2})e^{-\gamma_1\bar{x}}-\gamma_2^2(\gamma_1+\mu)(\gamma_1^2-e^{R_3\gamma_1^2})e^{-\gamma_2\bar{x}}] \ , \qquad (12)$$

$$m(x)=(H/4\pi)\eta(\gamma_1^2-e^{R_3\gamma_1^2})(\gamma_2^2-e^{R_3\gamma_2^2})[(\gamma_2+\mu)e^{-\gamma_1\bar{x}}-(\gamma_1+\mu)e^{-\gamma_2\bar{x}}] \ , \qquad (13)$$

$$j_s(x)=-(cH/4\pi\lambda_L)\eta[\gamma_1(\gamma_2+\mu)(\gamma_2^2-e^{R_3\gamma_2^2})e^{R_3\gamma_1^2}e^{-\gamma_1\bar{x}}-\gamma_2(\gamma_1+\mu)(\gamma_1^2-e^{R_3\gamma_1^2})e^{R_3\gamma_2^2}e^{-\gamma_2\bar{x}}] \ , \qquad (14)$$

$$\lambda_{\rm eff}=\lambda_L\frac{\gamma_1(\gamma_2+\mu)(\gamma_2^2-e^{R_3\gamma_2^2})-\gamma_2(\gamma_1+\mu)(\gamma_1^2-e^{R_3\gamma_1^2})}{\gamma_1^2(\gamma_2+\mu)(\gamma_2^2-e^{R_3\gamma_2^2})-\gamma_2^2(\gamma_1+\mu)(\gamma_1^2-e^{R_3\gamma_1^2})} \ , \qquad (15)$$

where $\bar{x}=x/\lambda_L$, $R_3=0.447/\kappa^2$, and

$$\eta=[\gamma_2 e^{R_3\gamma_1^2}(\gamma_2^2-e^{R_3\gamma_2^2}+\mu\gamma_2)-\gamma_1 e^{R_3\gamma_2^2}(\gamma_1^2-e^{R_3\gamma_1^2}+\mu\gamma_1)]^{-1} .$$

For $t<t_0$, one obtains

$$b(x)=-H\chi e^{-p_2\bar{x}}\{e^{\alpha}(p_1^2+p_2^2)^2[p_1\cos p_1\bar{x}+(p_2+\mu)\sin p_1\bar{x}]+p_1(p_1^2+p_2^2+2\mu p_2)\cos(p_1\bar{x}+\beta)$$
$$-[p_2(p_1^2+p_2^2)-\mu(p_1^2-p_2^2)]\sin(p_1\bar{x}+\beta)\} , \tag{16}$$

$$m(x)=-(H/4\pi)\chi e^{-p_2\bar{x}}\{[p_1\cos p_1\bar{x}+(p_2+\mu)\sin p_1\bar{x}]$$
$$\times[e^{\alpha}(p_1^2+p_2^2)^2+e^{-\alpha}+2(p_1^2-p_2^2)\cos\beta-4p_1p_2\sin\beta]\} , \tag{17}$$

$$j_s(x)=(cH/4\pi\lambda_L)\chi e^{-p_2\bar{x}}\{(p_1^2+p_2^2)[p_1(2p_2+\mu)\cos(p_1\bar{x}-\beta)-(p_1^2-p_2^2-\mu p_2)\sin(p_1\bar{x}-\beta)]$$
$$+e^{-\alpha}[\mu p_1\cos p_1\bar{x}-(p_1^2+p_2^2+\mu p_2)\sin p_1\bar{x}]\} , \tag{18}$$

$$\lambda_{\rm eff}=\lambda_L\frac{p_1(2p_2+\mu)(p_1^2+p_2^2)+e^{-\alpha}[\mu p_1\cos\beta-(p_1^2+p_2^2+\mu p_2)\sin\beta]}{p_1(p_1^2+p_2^2)^2+e^{-\alpha}\{p_1(p_1^2+p_2^2+2\mu p_2)\cos\beta+[\lambda(p_1^2-p_2^2)-p_2(p_1^2+p_2^2)]\sin\beta\}} , \tag{19}$$

where $\alpha=R_3(p_1^2-p_2^2)$, $\beta=2R_3p_1p_2$, and

$$\chi=[p_1(p_1^2-2\mu p_2-3p_2^2)\cos\beta+(p_2^3-3p_1^2p_2+\mu p_2^2-\mu p_1^2)\sin\beta+p_1e^{-\alpha}]^{-1} .$$

IV. NUMERICAL RESULTS

Some results of the numerical solution of Eq. (9) for various κ are shown in Fig. 1. The temperatures t_0 and t_p are defined by $p_1=0$ and $p_2=0$, respectively. Both of them are related to bulk properties, since at t_0 an attractive part in the interaction between flux lines appears.

For $\mu=0$ the results (12)–(19) agree with those of the boson theory only in the London limit. Due to the smallness of ξ the suppression of the order parameter at the surface does not play any role in this limit. In contrast to Ref. 8, $j_s(x)$ as given by Eqs. (14) and (18) does not vanish at $x=0$ but adjusts itself according to the boundary conditions as shown in Fig. 2. As a further illustration of the differences between Ref. 8 and the present treatment, $b(x)$ for $\mu=0$ and two values of κ is reported in Fig. 3. There is a common tendency for $b(x)$ to increase with decreasing κ which is however much stronger for the boson result. This may be understood by considering that for κ fixed, the suppression of the order parameter will cause the magnetic field to penetrate deeper into the interior (see Fig. 4). This makes the surface more "normal" and reduces the difference between t_s and the transition temperature $t=1$. For $\mu\to\infty$ Eqs. (12)–(19) and Eqs. (6)–(8) differ in detail but yield identical characteristic temperatures.

The value $\mu=0$ will be appropriate if the exchange constant at the surface is equal to its bulk value. Then the magnetic surface ordering is solely due to the imperfect diamagnetic response of the superconductor. If $\mu\neq0$ the second mechanism, mentioned above, of purely magnetic origin is superimposed. A positive or negative μ weakens or strengthens the fields $m(x)$, $b(x)$ within a magnetic correlation dis-

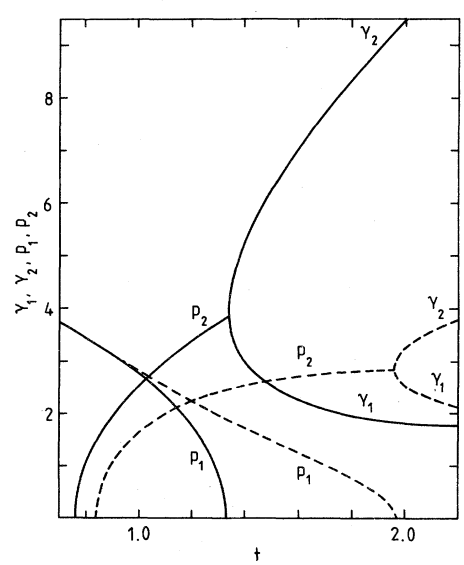

FIG. 1. Temperature dependence of γ_1, γ_2, p_1, p_2 for two different values of the Ginzburg-Landau parameter, $\kappa=5$ (solid line) and $\kappa=2$ (dashed line).

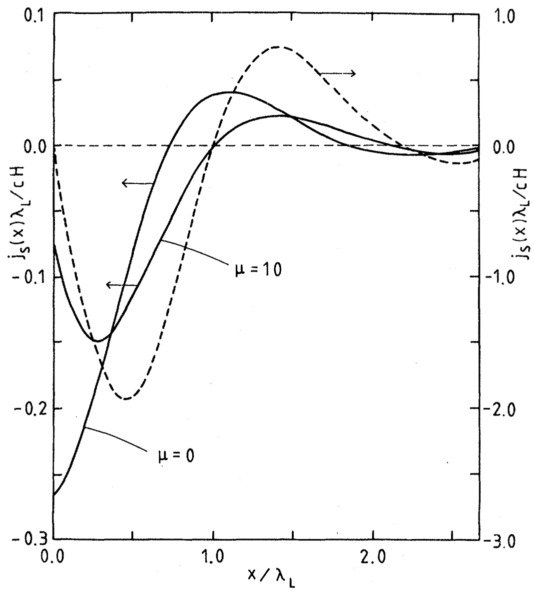

FIG. 2. Supercurrent $j_s(x)$ for reduced temperature $t=1$, Ginzburg-Landau parameter $\kappa=2$, and surface parameter $\mu=0$ and $\mu=10$. For comparison with the $\mu=0$ curve, $j_s(x)$ according to Ref. 8 for $t=1$, $\kappa=2$ is plotted as a dashed line.

tance $\xi_m=[d/(t-1+\bar{c})]^{-1/2}$ from the surface. This behavior is illustrated in Fig. 3, curve C and Fig. 5, curve C. The κ dependence of t_0, t_p and the κ dependence of t_s for different μ are plotted in Fig. 6. The critical temperature t_s approaches t_p for high values of μ and increases with decreasing μ. The

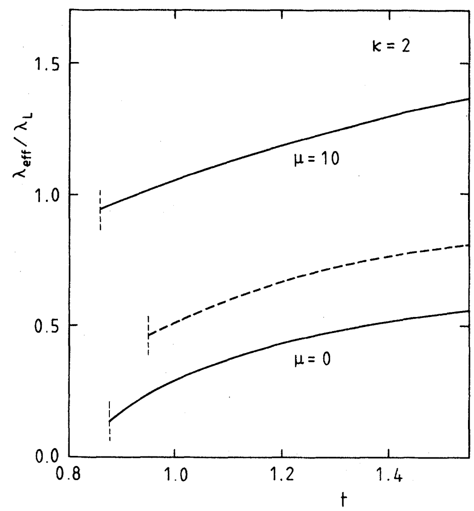

FIG. 4. Temperature dependence of the effective penetration depth λ_{eff} for $\kappa=2$, $\mu=0$, and $\mu=10$. The dashed line shows the result of Ref. 8 for $\kappa=2$, $\mu=0$. At the left cutoff the transition to the ferromagnetic surface state occurs.

result of Ref. 8 for t_s (not shown in Fig. 6) lies somewhat above the corresponding curve $t_s(0)$; this accounts for the large difference in field amplitudes shown in Fig. 3. If the exchange constant in the surface layer becomes much larger than in the bulk, then t_s may even exceed t_0. The μ dependence of t_s has already been reported in Ref. 8 for the London limit. For μ appreciably smaller than zero, t_s is

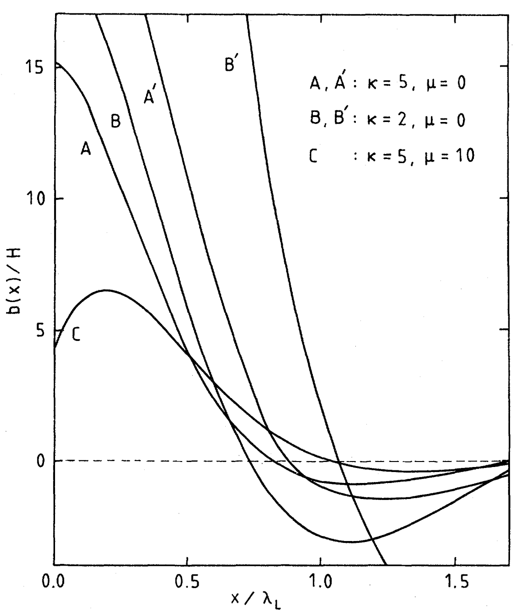

FIG. 3. Magnetic induction $b(x)$ at reduced temperature $t=1$ according to the present treatment (curves A, B, C) and according to Ref. 8 (curves A′, B′).

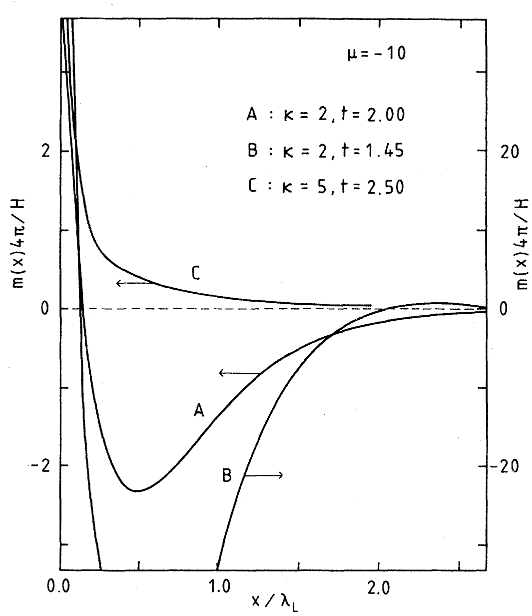

FIG. 5. Behavior of magnetization $m(x)$ for different κ and t in the case of strong enhancement of magnetic order at the surface.

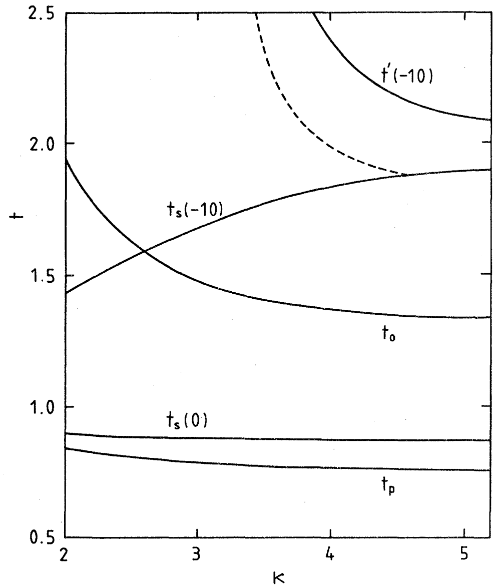

FIG. 6. Characteristic temperatures t_0, t_p, $t_s(\mu)$ (for $\mu=0$ and $\mu=-10$), and $t'(\mu)$ (for $\mu=-10$) as a function of the Ginzburg-Landau parameter κ. The critical temperature $t_s(10)$ lies very close to t_p and is not shown in the graph. At the dashed line the effective penetration depth vanishes for $\mu=-10$.

found to decrease with decreasing κ in contrast to its behavior for $\mu>0$.

As a second consequence of large negative values of μ, a nonmonotonic behavior of $b(x)$ and $m(x)$ may already occur at temperatures $t>t_0$. It will be present for those values of μ, for which $\gamma_2+\mu$ [appearing in the first term within the brackets of Eq. (12) and (13)] changes its sign with decreasing temperature. The temperature $t'(\mu)$ where this happens is plotted in Fig. 6 for $\mu=-10$. Figure 5 shows the behavior of $m(x)$ in the temperature regions $t>t'$ (curve C, monotonic decrease), $t_0<t<t'$ (curve A, occurrence of a single minimum), and $t<t_0$ (curve B, additional occurrence of small oscillations). Since γ_2 decreases strongly with decreasing κ, in low-κ superconductors the field reversal at t' may occur for somewhat less extreme values of μ. As is to be expected from the distribution of $b(x)$, the effective penetration depth λ_{eff} is maximal for $\mu=\infty$ and decreases with decreasing μ. For $\mu=-10$ it already reaches zero at the dashed line in Fig. 6. Its enhancement for $\mu=10$ as compared to the case $\mu=0$ is shown in Fig. 4.

V. CONCLUDING REMARKS

The present treatment of surface effects in ferromagnetic superconductors is based on the linear, electrodynamical approach of Ref. 8. Its validity is of course not restricted to a particular integral kernel $K(\vec{x})$. It has been pointed out that—within this picture—two modifications have to be made in order to come somewhat closer to a realistic description. The first of these concerns the difference between a surface and a linear array of flux lines, the second the possibility of arbitrary magnetic surface states. In view of the unknown μ and of a possible influence of fluctuations and exchange coupling, no realistic fit of the parameters has been attempted. However, some general features which will be helpful in determining the actual value of μ in the rare-earth compounds have been pointed out. For an extension of the present theory to higher fields and in particular for a calculation of the superheating field, local, or for a more accurate description, nonlocal Ginzburg-Landau functionals must be used as a starting point.

Some shortcomings of the boson surface treatment have recently been pointed out[19] by the authors of Ref. 8. The physical origin of these shortcomings was, however, not analyzed. This paper[19] also contains boundary conditions for $m(x)$, which are slightly more general than those of Ref. 8. The method of solution employed did not, however, allow calculation of the temperature t_s, whose value indicates most sensitively a possible modification of the exchange coupling constant near the surface.

In Sec. III the results of the boson method were rederived by postulating a discontinuity of the vector potential at the surface. To understand this, one has to recall that as a result of gauge invariance, a singular part, say $\vec{a}_1$, of the vector potential may equivalently be introduced instead of a singular phase of the order parameter. Treating a flux line as a cylindrical hole with radius zero, $\vec{a}_1$ is uniquely determined by fluxoid quantization [$\vec{a}_1=(\phi_0/2\pi r)\vec{e}$, where ϕ_0 is the quantum of flux] and agrees exactly with the corresponding term $-(\hbar c/e)\vec{\nabla} f$ of the boson method. For a surface constructed as a linear distribution of flux lines, a step function or a discontinuous $\vec{a}$ must be introduced. This explains the above coincidence. In addition, one concludes that all the results of the boson theory regarding flux lines[20] may be derived from the Bardeen-Cooper-Schrieffer (BCS) theory as well provided $c(k)/\lambda_L^2$ is replaced by the BCS kernel.

The most satisfying feature of the boson results is the vanishing of the supercurrent at the flux line center revealing the vanishing of the order parameter there. This feature is due to the nonlocal electrodynamics and disappears in the London limit; it will be present for any $K(k)$ decreasing sufficiently strong for large k. Nonlocality explains why a theory linear in $\vec{a}$ is able to describe, at least qualitatively correctly, a situation with strongly varying order parameter,[21] a fact not accepted by all workers in this field. A further example for the disappearance of singularities due to inclusion of nonlocality may be found in the field of continuum mechanics.[22]

ACKNOWLEDGMENTS

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[17]See Ref. 16 for an interesting discussion on the difference in boundary conditions for the superconducting and magnetic order parameters.

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[20]See, for example, F. Mancini, M. Tachiki, and H. Umezawa, Physica (Utrecht) 94B, 1 (1978).

[21]Compare the slightly different explanation of the success of the boson theory given in the introduction of the second paper quoted in Ref. 6.

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