PHYSICAL REVIEW B VOLUME 27, NUMBER 3 1 FEBRUARY 1983

Relation between the condensate fraction and the surface tension of superfluid ^{4}He

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(Received 16 July 1982)

The variation of the superfluid order parameter at the liquid surface contributes to the surface tension in proportion to the condensate density. This yields a prediction of the condensate fraction as a function of temperature when proper account is taken of the temperature dependence of the normal components of the surface tension, the largest of which is due to surface modes of vibration. Good agreement is obtained with recent values of the condensate fraction from neutron and x-ray scattering measurements.

By now it is established that a fundamental characterization of superfluidity is the breaking of a gauge symmetry for temperatures $T < T_\lambda$.[1] In the case of superfluid ^{4}He (and superconductors) this occurs through the appearance of a complex scalar order parameter ψ which, for ^{4}He, is identified with the macroscopic occupation of a single-particle state, so that $|\psi|^2$ is the density of particles ρ_0 in the so-called condensate state.[2] A very general description of such systems is provided by gauge theory plus the Higgs mechanism for broken symmetry.[3–6] The simplest Higgs potential gives rise to the well-known[7,8] (and formally equivalent) Ginzburg-Landau and Gross-Pitaevskii (GP)[9,10] equations, whose predictions, for instance, of quantized vorticity[8] are completely verified in the superconducting and superfluid states. Because a first-principles derivation of the GP equation does not exist for ^{4}He, we justify its use here on the basis of its general origin from gauge symmetry as outlined above. The associated Higgs energy then contributes an additional term at the boundary of superfluid regions that is proportional to the condensate fraction $n_0 = \rho_0/\rho$, where ρ is the total density. As shown below, this permits n_0 to be evaluated in terms of the experimental surface tension σ_e, the superfluid coherence length ξ, and theoretical estimates of the nonsuperfluid contribution to the temperature dependence of $\sigma_e(T)$ between $T=0$ and $T=T_\lambda$.

The physical state of interest here is static and has a fixed gauge so it is sufficient to employ the Higgs energy with only the scalar field:

$$E = \frac{\hbar^2 \rho_0 \xi}{2m^2} \int d^3x [|\vec{\nabla}\phi|^2 + \tfrac{1}{2}(|\phi|^2 - 1)^2] , \quad (1)$$

where m is the mass of a ^{4}He atom, distances are in units of ξ, and $\phi = \psi/\sqrt{\rho_0}$. The surface energy σ_s associated with Eq. (1) for a half space, where $|\phi| \to 1$ in the interior and $\phi = 0$ on the planar boundary, is known to be[8]

$$\sigma_s = \frac{\sqrt{2}}{3}\left(\frac{\hbar}{m}\right)^2 \frac{\rho_0}{\xi} . \quad (2)$$

This is the superfluid component of the surface energy, i.e., the component due to variation of the order parameter at the boundary. In addition, there remains the nonsuperfluid component σ_n, so

$$\sigma_e = \sigma_s + \sigma_n . \quad (3)$$

Using $\sigma_s(T_\lambda) = 0$, Eqs. (2) and (3) imply

$$n_0(T) = \frac{3}{\sqrt{2}}\left(\frac{m}{\hbar}\right)^2 \frac{\xi(T)}{\rho}[\sigma_e(T) - \sigma_e(T_\lambda) + \sigma_n(T_\lambda) - \sigma_n(T)] . \quad (4)$$

Therefore, if $\xi(T)$ and $\sigma_n(T_\lambda) - \sigma_n(T)$ are known, the condensate fraction can be found from measurements of the surface energy.

The coherence length ξ, as inferred from the measured dynamics of vortex rings,[11–13] is taken to be constant and equal to 1×10^{-8} cm (except near T_λ, as discussed below).

The major temperature dependence to σ_n is expected to come from surface modes of vibration inasmuch as ρ is nearly constant below T_λ and phonon-roton effects on the surface energy are small.[14,15] A theory by Atkins[16] of $\sigma_n(T)$ due to surface modes gives

$$\sigma_n(T_\lambda) - \sigma_n(T) = (-39.7 + 6.50\,T^{7/3}) \times 10^{-3}\ \mathrm{erg\,cm^{-2}} . \quad (5)$$

This theory treats helium as an incompressible liquid whose surface modes have a classical dispersion $\omega^2 \propto k^3$. Using Eq. (5) and the values of $\sigma_e(T)$ given by Eckardt *et al.*,[17] the condensate fraction takes the form given by the dashed curve in Fig. 1, which also shows the recent experimental n_0 values given by Sears, Svensson, Martel, and Woods.[18]

A more elaborate theory of the surface modes by Edwards *et al.*[19] contains parameters that relate to the dependence of the surface energy on surface curvature, which affects the surface vibrational frequencies. Using Ebner and Saam's values[20] for these

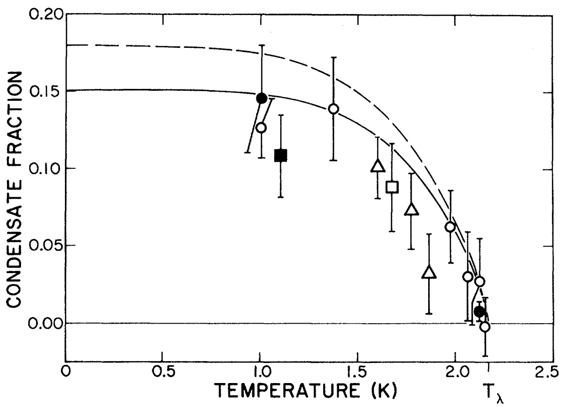

FIG. 1. Predictions of the superfluid condensate fraction based on Eq. (4) and the surface tension measurements of Ref. 17 for two versions of the surface mode energies: dashed line, Atkins (Ref. 16); solid line, Edwards *et al.* (Ref. 19), with parameters from Ref. 20. The experimental points, taken from Ref. 18, are the results of x-ray (open square) and neutron scattering (other symbols) measurements of various workers.

parameters, Eckardt *et al.*[17] calculated the surface entropy, which can be extrapolated and numerically integrated to find $\sigma_n(T_\lambda)-\sigma_n(T)$ and, consequently, n_0, as shown by the solid curve in Fig. 1.

This completes the argument in its simplest form. However, a number of points bearing on the accuracy and implications of this method should be mentioned.

(1) The coherence length ξ is not currently known to better than about $\pm 15\%$, both because of differences in experimental results[11–13] on vortex rings and because of the use, in the experimental analysis, of various classical formulas which differ somewhat from the formulas for GP vortex rings.[21,22] Variational calculations[23] of vortex rings using a model many-body wave function also predict a core size near 1×10^{-8} cm and agree with the dynamical experiments.[24] Any error in our estimate of ξ would merely scale the curves in Fig. 1.

(2) Very near T_λ, $\xi(T)=\xi_0|\epsilon|^{-2/3}$, where $\epsilon=(T/T_\lambda-1)$.[25] When this is used in Eq. (4), along with the σ_e data of Magerlein and Sanders,[26] one can infer a critical exponent for n_0 that is roughly consistent with the expected value of $\frac{2}{3}$. The inference is hampered by unexpected features of shifting and rounding in the data, which require separate explanation.[27] Also, there is an alternative, but not necessarily exclusive, interpretation of the critical behavior of σ_e.[28] Sobyanin's theory[29] for the surface energy near T_λ is formally similar to that proposed here, the chief differences being his use of ρ_s/ρ as the order parameter and some uncertainty in the coefficients of the free energy when expanded in powers of ρ_s/ρ.

(3) A correct theory of $\sigma_n(T)$ would obey $d\sigma_n/d\epsilon=d\sigma_e/d\epsilon$ for $\epsilon=0^+$ (eliminating critical effects). However, $d\sigma_e/d\epsilon=-0.19$ erg cm^{-2},[26] while $d\sigma_n/d\epsilon\approx-0.11$ erg cm^{-2}, which shows there is additional temperature variation that my extrapolation has missed. Correcting this would lower the $n_0(T)$ prediction, by increasing $|\sigma_n(T_\lambda)-\sigma_n(T)|$, but by much less than the error in the derivative.

(4) The variation of the order parameter should contribute to the surface energy of other quantum fluids and, indeed, it accurately accounts for *all* the surface energy between the A and B phases of superfluid ^{3}He.[30] It is probably the dominant component of the interfacial energy of separated ^{3}He-^{4}He mixtures and should be detectable at the liquid-vacuum interface in superfluid ^{3}He at T_c if the effect of the density anomaly[31] on the surface energy can be subtracted. Systems involving ^{3}He presumably have the simplicity of highly damped, and hence negligible, surface modes.

(5) The previous calculations[8,21] of the superfluid helium surface energy at $T=0$ that used Eq. (2) substituted the total density for ρ_0. Although this is justified for a weakly interacting Bose gas, the system for which the GP equation is typically derived, and although the result is close to the experimental value $\sigma_e(0)=0.378$ erg cm^{-2}, the view of this paper is that such a substitution is unjustified and the agreement is fortuitous and meaningless for liquid helium. The only order parameter or symmetry field with a microscopic basis for ^{4}He is the expectation value of a single-particle state, proportional to $(n_0)^{1/2}$.[2] This in no way precludes the participation of the total superfluid density in current flow caused by spatial variation of the gauge; then the total kinetic energy is indeed proportional to ρ_s.[32]

ACKNOWLEDGMENTS

It is a pleasure to thank Michael Cohen, David Pines, and John Wheatley for discussions and Eric Svensson for a copy of the data in Fig. 1 prior to publication. This work was supported by the U. S. Department of Energy.

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