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Supercurrent dependence of the microwave enhancement of the order parameter in superconducting aluminum

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We studied the dependence on supercurrent density of the enhancement by microwaves of the order parameter in superconducting aluminum films. Experiments were made on a superconducting circuit that includes an aluminum strip and the input coil of a superconducting quantum-interference device system. The experimental results are in quantitative agreement with theory with the assumption of a spread in the critical current due to inhomogeneities in the strip.

INTRODUCTION

The fact that microwave irradiation of a superconducting strip may result in enhancement of the critical current, the critical temperature, the energy gap, and the order parameter is well established.[1] Recently, we demonstrated that the enhancement by microwaves of the order parameter increases with the supercurrent density in the strip.[2] The experimental results were in qualitative agreement with theoretical calculations made by Weiss.[3] In this paper we describe our work on the enhancement of the order parameter in more detail. We show that a quantitative agreement with the theoretical calculations can be obtained if it is assumed that the critical current of the strips, which we used, has a spread over the length of the strip. The influence of this spread in the critical current on the order parameter and on its enhancement by microwave radiation as a function of supercurrent density will be indicated by separating the expression for the energy gap in a part that only depends on temperature and a part that depends on the supercurrent density. Furthermore, we give a detailed justification of the normalization of the order-parameter enhancement.[2]

EXPERIMENTAL

The experimental setup is drawn schematically in Fig. 1. The superconducting loop consists of the 2 μH input coil of an SHE SQUID system and an aluminum strip. Both are connected by a superconducting pressure contact using indium at the contact pads at the end of the strip. The strips (length: 1500 μm; width: 2–5 μm; thickness: 110 nm) were made by standard photolithographic techniques.

Both microwave radiation and an external direct current (dc) can be supplied to the strip through a 50-Ω coaxial system, which is also connected to the contact pads of the strip by indium contacts.

THEORY

The quantization condition for the superconducting loop, formed by the coil of the SQUID and the strip (see Fig. 1), can be written as[2]

$$\Phi+\frac{m^*}{(2e)^2}\oint_C \frac{\vec{j}\cdot d\vec{s}}{|\Psi^2|}=n\Phi_0 \ , \tag{1a}$$

where Φ is the total magnetic flux in the closed loop C, $\vec{j}$ is the supercurrent density, Ψ is the order parameter of the strip, and Φ_0 is the elementary flux quantum.

If the closed loop C is taken to lie deep in the leads of the superconducting niobium input coil, where j equals zero, the only contribution to the integral comes from the aluminum sample whose cross-sectional area S is small compared with the square of the penetration depth. Defining the length of the strip as L and its superconducting current as I, yields

$$\Phi+\frac{m^*}{(2e)^2}\frac{IL}{S}\frac{1}{|\Psi^2|}=n\Phi_0 \ . \tag{1b}$$

For a dirty superconductor, Ψ is directly proportional to the energy gap,[4,5]

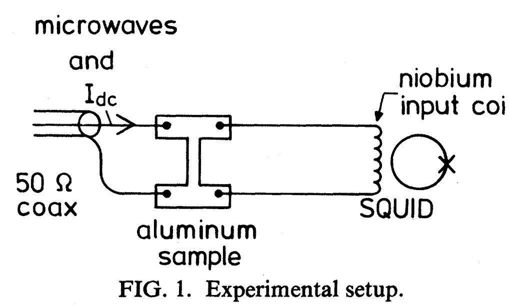

FIG. 1. Experimental setup.

$$\Psi(T,j_s/j_c(T,0),0)=A_1\Delta(T,j_s/j_c(T,0),0)\ , \tag{2a}$$

where $\Delta(T,j_s/j_c(T,0),0)$ is the gap at temperature T, normalized supercurrent density j_s, and no microwaves, $j_c(T,0)$ is the critical current density at temperature T without microwaves, and

$$A_1=[\pi m v_F l N(0)/(12\hbar k_B T_c)]^{1/2}\ . \tag{2b}$$

In this equation, v_F is the average Fermi velocity, l the mean free path, and $N(0)$ the density of states for one-spin direction at the Fermi level. The energy gap is a function of temperature and of current density and near T_c it follows from the Ginzburg-Landau equations that

$$\Delta^2(T,j_s/j_c(T,0),0)=\Delta^2(T,0,0)F(j_s/j_c(T,0))\ , \tag{3a}$$

$$\Delta(T,0,0)=\beta_0^{-1/2}k_B T_c(1-T/T_c)^{1/2}\ , \tag{3b}$$

$$F(j_s/j_c(T,0))=\frac{2}{3}\left[\frac{1}{2}+\cos\left[\frac{1}{3}\tan^{-1}\frac{2[j_s/j_c(T,0)]\{1-[j_s/j_c(T,0)]^2\}^{1/2}}{1-2[j_s/j_c(T,0)]^2}\right]\right]\ , \tag{3c}$$

with

$$0\le\tan^{-1}x\le\pi$$

and

$$\beta_0=7\zeta(3)/8\pi^2\ .$$

Figure 2(a) gives a plot of $F(j_s/j_c(T,0))$ vs $j_s/j_c(T,0)$. The critical-current density at temperature T and in the absence of microwaves is

$$j_c(T,0)=A_2(1-T/T_c)^{3/2} \tag{4a}$$

with

$$A_2=\beta_0^{-1}(2\sqrt{2}/9)eN(0)k_B T_c(k_B T_c l v_F \pi/\hbar)^{1/2}\ . \tag{4b}$$

With the use of Eqs. (2)–(4), Eq. (1b) can be rewritten as

$$\Phi+\frac{Cj_s}{(1-T/T_c)F(j_s/j_c(T,0))}=n\Phi_0\ , \tag{5a}$$

where

$$C=\frac{m^*}{(2e)^2}L\beta_0\frac{1}{(A_1 k_B T_c)^2}\ . \tag{5b}$$

The properties of a spatially-homogeneous superconductor under the influence of a weak microwave field have been studied by Eliashberg.[6,7] The gap equation can be written as[3]

$$\frac{\Delta^2(T,j_s,\alpha)}{\Delta^2(T,0,0)}=1-\frac{4}{27}\left[\frac{j_s}{j_c(T,0)}\right]^2\frac{\Delta^4(T,0,0)}{\Delta^4(T,j_s,\alpha)}+a_1\alpha\tau_E\ . \tag{6a}$$

In this equation τ_E is the inelastic-scattering time and

$$\alpha=(v_F l/3)(e/\hbar)^2 A_\omega^2 \tag{6b}$$

characterizes the microwave irradiation, with frequency ω and real amplitude A_ω of the vector potential of this radiation in the London gauge. The parameter a_1 is given by

$$a_1=\frac{\pi}{2}\left[\frac{\hbar\omega}{\Delta(T,0,0)}\right]^2\times\left[-1+\frac{1}{2\pi\beta_0}\frac{k_B T_c}{\hbar\omega}g\left[\frac{\Delta(T,j_s,\alpha)}{\hbar\omega}\right]\right]\ , \tag{6c}$$

where g is a function involving elliptic integrals of the first and third kind.[3] We use the quantity δ, introduced by Weiss,[3] which describes the enhancement of the gap

$$\frac{\Delta^2(T,j_s,\alpha)}{\Delta^2(T,j_s,0)}=1+\delta(j_s,\alpha)\ . \tag{7}$$

The enhancement depends on temperature, microwave power, and supercurrent density and can be calculated from Eq. (6a) as a function of $a_1\alpha\tau_E$. Two examples are shown in Fig. 2(b) (solid lines).

In the above discussion we have assumed that the strip is spatially homogeneous. However, in practice the properties of a strip always vary more or less along the strip. We therefore investigated the case in which the local critical-current value has a distribution along the strip. This may be caused by variations in the cross-sectional dimensions along the strip. The measured critical current of the strip as a whole, of course, is determined by the "weakest spot" where the local critical current is minimum. The order-parameter measurement relies on the integral used in Eq. (1) and is therefore an average over the whole strip. Thus, variations in the local critical current along the strip have a different influence on the experimental critical current of the strip

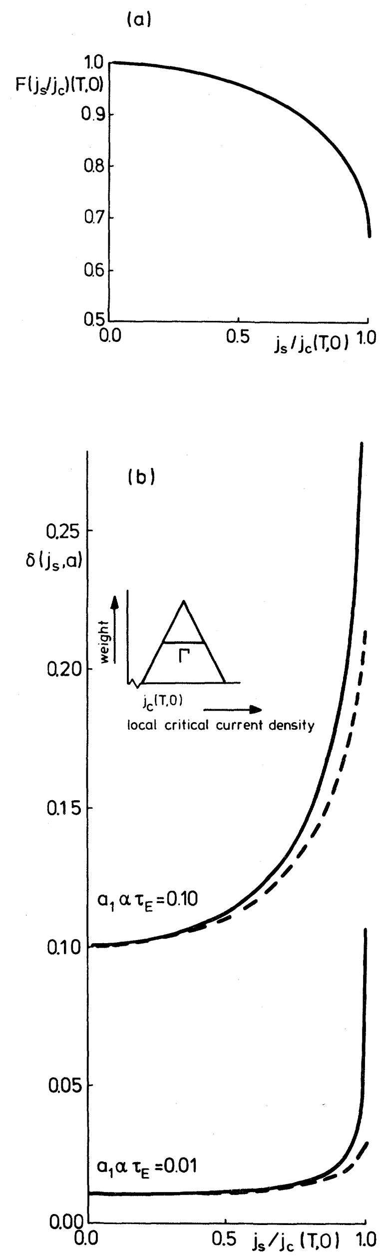

FIG. 2. (a) Dependence of $F(j_s/j_c(T,0))$ on $j_s/j_c(T,0)$ [Eq. (3c)]. (b) Influence of the distribution of critical-current values on the relative gap enhancement as a function of critical-current density. The inset shows the distribution function used. Solid line: without spread in critical current; dashed line: spread included ($\Gamma=7.5\%$).

and the phase difference over the strip caused by the supercurrent. To describe the variations in the critical current we used a normalized triangular distribution function with full width at half maximum Γ. [See inset of Fig. 2(b).] It should be noted that the distribution of the local critical currents only affects the supercurrent-dependent part, $F(j_s/j_c(T,0))$, of the expression for the energy gap [Eq. (3)]. For a given value j_s of the supercurrent density, the size of the effect will be most pronounced if $j_s \simeq j_c(T,0)$, since then F varies most steeply with $j_s/j_c(T,0)$ [see Fig. 2(a)]. In Fig. 2(b) the resulting modification of the enhancement of the gap $\delta(j_s,\alpha)$ is shown. It is clear that the effect is only important for relatively large values of $j_s/j_c(T,0)$.

MEASUREMENTS AND DISCUSSION

Three different types of experiments will be discussed here:

(i) First we study the temperature dependence of the total magnetic flux without microwaves at a constant current density. This experiment yields the temperature dependence of the order parameter and moreover values for the temperature-dependent parameter

$$[m^*/(2e)^2](L/S)(1/|\Psi^2|)\ ,$$

which are needed to determine the relative increase of the order parameter at given temperature [see (iii)], can be obtained. The temperature dependence of the order parameter, as follows from Eq. (1a), was already studied in Ref. 8 for cylinders of tin. In our experiment we start by supplying a constant current to the input wires to the sample and the SQUID coil at a temperature between the critical temperatures of the sample and the SQUID coil. The sample is normal, the coil superconducting, and the applied current will therefore flow through the coil. Now the sample is cooled from above its critical temperature to far below it and a superconducting loop is formed by the coil and the sample. The external current is then switched off and consequently a persistent supercurrent flows in the loop. If the temperature is now increased, the total magnetic flux Φ will change due to the temperature dependence of the order parameter Ψ, as described in Eq. (1b). This change of the flux is measured directly by the output of the SQUID. Figure 3 shows an example for a strip of width 2 μm at a constant current of 0.25 mA. The moment the temperature reaches the value where the constant current equals the temperature-dependent critical current of the sample, the strip becomes normal and the measurement is stopped.

The experimental results can be explained by the temperature and critical-current dependence of the order parameter, Eqs. (2) and (3). The dashed lines in Fig. 3 are the theoretical calculations (the curves were normalized at $T=1.180$ K), in which the distribution of local critical-current values is also taken

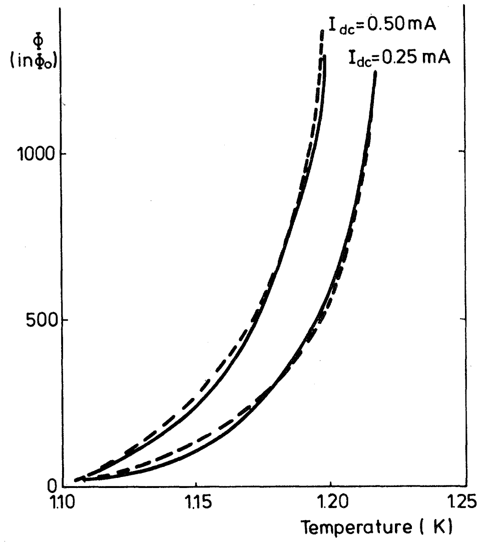

FIG. 3. Dependence of the flux on temperature at a constant value for the dc current. The solid lines are the experimental results and the dashed lines are the theoretical calculations.

into account [see also point (iii)]. This distribution of critical-current values is only important if the constant current is larger than about 0.7 times the temperature-dependent critical current (i.e., for $T \gtrsim 1.205$ K for $I=0.25$ mA and $T>1.180$ K for $I \simeq 0.50$ mA). It can be seen that the correspondence between the theoretical calculations and the experimental values is good. From the temperature derivative of Φ the value of

$$[m^*/(2e)^2](L/S)(1/|\Psi^2|)$$

as a function of temperature can be determined. This value is used for the determination of the absolute value of the order-parameter enhancement.

(ii) A second type of experiment concerns the critical current of the sample both with and without microwaves. In this case both a dc current and microwaves can be supplied to the sample via the coax system. The measurement is started with no persistent current in the superconducting loop below T_c. As long as the externally applied dc current is lower than the critical current of the sample, it will flow almost completely through the strip as its inductance is much smaller than the inductance of the input coil. However, the small current through the input coil ($\sim 1/1000 I_{dc}$) can be detected with the SQUID. Then the externally supplied current is slowly increased and the moment it equals the critical current the strip becomes normal and the externally supplied current switches from flowing through the strip to flowing through the input coil. This switching occurs very rapidly and the SQUID system cannot follow it. In practice, the SQUID only shows a small, readily observable discontinuity in its output each time this process occurs. Consequently, the strip becomes superconducting again and the whole process repeats itself. So, the flux measured by the SQUID as a function of the slowly increasing dc current will show a discontinuity each time the dc current reaches the critical current. Measurements of the critical current both with and without microwaves using this method gave the same results as experiments, in which I—V characteristics of strips were used to determine the critical current. The important point here is that the enhancement by microwaves of both the critical current and the order parameter [see (iii)] can be measured on the same sample and in the same experimental setup. Our measurements have already been discussed in connection with a comparison between the enhancement by microwaves of the order parameter and of the critical current.[9]

(iii) The third and main type of experiment concerns the enhancement by microwaves of the order parameter. As was shown in Ref. 2, it follows from Eq. (1) that a change in the order parameter due to microwave radiation or temperature variation (with $\delta\Psi \ll \Psi$) is related to a change in the total flux given by

$$\delta\Phi = \frac{m^*}{(2e)^2}\frac{IL}{S}\frac{2}{|\Psi^2|}\frac{\delta\Psi}{\Psi} . \tag{8}$$

Experimentally the setup was as follows: Both an external dc current and microwaves are supplied to the sample by the coax system. The microwaves are chopped with a chopper frequency ~ 200 Hz. The difference in flux with and without microwaves as measured by the SQUID is detected with a synchronous detector. Its output as a function of the slowly increasing dc current I_{dc} is then determined at various temperatures and power levels of the microwaves. As the microwaves are chopped, only changes in Φ related to the microwaves are detected and not the changes due to the slowly varying dc current. The temperature-dependent proportionality constant between $\delta\Phi$ and $\delta\Psi/\Psi$ was obtained from the temperature dependence of the flux at constant dc current [see (i)].

In Fig. 4 we give experimental values of the relative value of the order-parameter enhancement $\delta\Psi/\Psi$ as a function of the current ratio I/I_c, where I_c is the critical current without microwaves. Both the influence of temperature changes and that of increasing microwave power are shown. A marked increase of $\delta\Psi/\Psi$ is observed with increasing supercurrent density. This observation is in agreement with theory.[3] However, the absolute value of $\delta\Psi/\Psi$ at the critical-current density is much lower than ex-

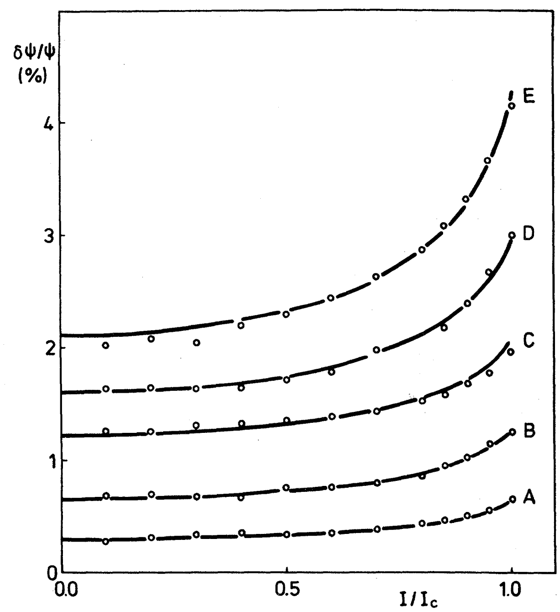

FIG. 4. Relative increase of the order parameter, $\delta\Psi/\Psi$, as a function of the normalized supercurrent. The solid lines are the fits to the data using the method discussed in the text with $\Gamma=7.5\%$. A denotes $T/T_c=0.951$, $P=3$ μW; B denotes $T/T_c=0.969$, $P=3$ μW; C denotes $T/T_c=0.978$, $P=3$ μW; D denotes $T/T_c=0.951$, $P=15$ μW; and E denotes $T/T_c=0.951$, $P=30$ μW. The power given is the microwave power delivered to the coax system.

pected from Eq. (6) on the assumption of a fixed critical-current density. It should be remarked that the microwave field strength in the sample is unknown, since the only experimentally accessible quantity is the microwave power delivered to the coax system. We suggest the following explanation for the too low values of $\delta\Psi/\Psi$ at the critical-current density: In scanning electron microscopy experiments on the samples studied here we observed that the width of the sample over its length varies by about 10–15%. From this we conclude that when the current reaches the critical value of the weakest spot (i.e., the smallest part of the strip) the value is still below the critical current of the wider parts. Since the measured value of $\delta\Psi/\Psi$ is an average over the whole strip its value at a given ratio of I/I_c contains contributions for a distribution of local I/I_c values. This is especially important at the critical-current density value, where theory[3] predicts a very steep increase in $\delta\Psi/\Psi$. As we already showed in Fig. 2(b), the inclusion of a distribution of critical-current values has a quite pronounced effect on the absolute enhancement of the order parameter. We chose a very simple shape for the distribution: A normalized triangle whose full width at half maximum Γ was the only free parameter. We fitted many curves of $\delta\Psi/\Psi$ vs I/I_c with only Γ and $a_1\alpha\tau_E$ as free parameters. As is shown in Fig. 4 good agreement between experimental values and modified theory can be obtained for a constant value of Γ. For freshly prepared samples we always observed a value of $\sim 7.5\%$ for Γ, which can already be explained with the spread observed in the width of the strip. After several months, however, the increase of $\delta\Psi/\Psi$ with current density becomes weaker, and the critical current becomes smaller. This could be reproduced by increasing the width of the distribution function. The origin of the increase of the width Γ is probably a slight inhomogeneous oxidation of the film leading to a lowering of the experimental critical current and an increase of the relative variation of the local critical-current values.

According to theory[10] the parameter $a_1\alpha\tau_E$ is proportional to the microwave power at constant temperature and to $(T_c-T)^{-n}$ with $1<n<1.5$ at constant microwave power. [See also Eq. (6).] It should be remarked that Entin-Wohlman derived $n=1.3$,[11] but inclusion of heating effects into Eq. (6) (Ref. 1) can increase the value to $n=1.5$. Experimentally these predictions are easiest to check by measuring $\delta\Psi/\Psi$ at $I\approx 0$. (Note that the influence of the spread in critical-current values is then negligible.) A noteworthy point that emerges from Fig. 4 is that temperature and microwave-power variations can indeed be used independently to change $\delta\Psi/\Psi$ values. For the temperature dependence at several power levels we observe $n=1.6\pm 0.1$, i.e., a somewhat steeper dependence than expected. The dependence on microwave power at constant temperature is linear up to values of the power above which the critical-current enhancement is also no longer linear in the power, i.e., up to power levels that result in a critical current about 1.5 times the value without microwaves. Above that power level the dependence is sublinear.

In conclusion, it can be said that the experimentally determined dependence of the enhancement by microwaves of the order parameter on supercurrent density can be quantitatively explained by including in the theory the assumption of a spread in critical-current values over the length of the superconducting strip.

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