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$1/f^{\gamma}$ noise in thick-film resistors as an effect of tunnel and thermally activated emissions, from measures versus frequency and temperature

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Measurements of the frequency and temperature dependence of the noise spectrum and of its frequency exponent have been performed on thick-film resistors and, together with the direct plots versus the frequency logarithm of the spectrum-frequency product, they are used to check the island model of the flicker noise. It is shown that the wide dispersion of the island relaxation times, necessary to originate the flicker noise, is due to tunnel emission and/or thermal activation processes of electrons from localized states, and to the exponential dependence of their emission probability on random variable distances and activation energies, whose distribution functions, means, and variances are determined both theoretically and experimentally.

I. PREMISE

The island model of the low-frequency excess noises[1-4] shows that these phenomena, including the flicker noise, are generated by localized states, called islands, that exchange charge carriers with the surrounding conductive medium through tunnel emission and/or thermal activation processes. This latter type of process leads to a strong temperature dependence of the conductance G of the islands and of their relaxation times τ. Furthermore, the model predicts that the product $fS_V(f,T)$ between the frequency f and the power spectral density S_V of the voltage fluctuations is related to the distribution function D of the logarithm of G, calculated at $-\ln(f/f_r)$, by the relationship

$$fS_V(f,T)=\langle(\Delta V)^2\rangle_t D(-\ln(f/f_r)) , \tag{1.1}$$

where $\langle(\Delta V)^2\rangle_t$ is the variance of the voltage fluctuations, f_r is a reference frequency, and T is the temperature. Consequently, a careful analysis of measured data of fS_V vs $\ln(f/f_r)$ over wide frequency and temperature ranges should be able to give interesting physical information of the investigated system and, together with a detailed model of the distribution function of the island conductance logarithm, should allow us to check quite easily the validity of the island theory of the flicker noise.

In this paper we will show that both these goals have been attained by means of the study of the excess noise in thick-film resistors and of a measuring system that includes a digital signal analyzer that can compute and directly display versus $\ln(f/f_r)$ not only $S_V(f)$, but also $fS_V(f)$. The investigation was performed on thick-film resistors because they have, owing to their intrinsic noncrystalline structure,[5,6] a high density of localized states[7] able to generate much $1/f^{\gamma}$ noise[8,9] which should be due to both tunnel and thermally activated emissions. Since these processes are characterized by different behaviors with respect to temperature, they can be discriminated on the basis of the island model, as will be shown in the following section.

II. DISTRIBUTION FUNCTION

The aim of this section is to specify the distribution function D that appears in Eq. (1.1). For this purpose it is adequate to remind ourselves of some results of the island theory,[1-4] according to which the conductance G and, for island size smaller than the Debye length, the capacitance C of the localized states generating the excess noises are given by

$$G=\frac{q^2}{kT}\sum_j e_jN_jf_j , \tag{2.1}$$

$$C=\frac{q^2}{kT}\sum_j N_jf_j(1-f_j) , \tag{2.2}$$

where N_j, f_j, and e_j are the state number, occupation factor, and emission coefficient, respectively, of the jth energy level E_j of the defect, q is the electron charge, and k is the Boltzmann constant.

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The probability e_j, for unit time, that an electron hops from the energy level E_j of a localized state to a given different state, localized or not, with higher energy E_e, is given by the general relationship[5]

$$e_j=\frac{1}{\tau_j}\exp\left[-2\alpha_jR_j-\frac{W_j}{kT}\right], \tag{2.3}$$

where the factor $1/\tau_j \simeq 10^{12}-10^{14}$ Hz is frequency dependent on the defect[3] and on the phonon spectrum.[5] W_j is the activation energy,[7] which in the simplest cases reduces itself to E_e-E_j, whereas according to the tunnel effect, it is[3]

$$2\alpha_jR_j=\frac{4\pi}{h}\int[2m(V-E_e)]^{1/2}d\xi\ , \tag{2.4}$$

where h is the Planck constant, m is the electron mass, and V is the potential energy.

Then, by setting[3]

$$\Theta_C=\ln(C/C_0)\ , \tag{2.5}$$

$$\Theta_G=\ln(G_0/G)\ , \tag{2.6}$$

where G_0 and $C_0=G_0/2\pi f_r$ are reference values of G and C, respectively, the distribution D to be used in (1.1) is the $D_G(\Theta_G)$ of Θ_G to be computed for $\Theta_G=-\langle\Theta_C\rangle-\ln(f/f_r)$, that is,

$$D(-\ln(f/f_r))=D_G(-\langle\Theta_C\rangle-\ln(f/f_r))\ . \tag{2.7}$$

Now let E_1 be the level of the island energy spectrum nearest to the Fermi level E_F and E_2 its highest level. If E_1 and E_2 are a few kT apart from the respective neighboring levels, or if they are the only levels of the spectrum, and $E_2>E_F+3kT$, E_1+3kT, from (2.1)–(2.6) we have

$$\Theta_C=-\,|E_1-E_F|\,/kT\ , \tag{2.8}$$

$$\Theta_G=\Theta_{\tau_2}+\Theta_{L_2}+\Theta_{W_2}+\Theta_{E_2}\ , \tag{2.9}$$

where

$$\begin{gathered}\Theta_{\tau_2}=\ln(2\pi f_r\tau_2N_2/N_1)\ ,\ \ \Theta_{L_2}=2\alpha_2R_2\ ,\\ \Theta_{W_2}=W_2/kT\ ,\ \ \Theta_{E_2}=(E_2-E_F)/kT\ .\end{gathered} \tag{2.10}$$

Since the variables Θ_{τ_2}, Θ_{L_2}, Θ_{W_2}, and Θ_{E_2} are affected by many random independent parameters, it appears to be reasonable to consider them as uncorrelated and, according to the central-limit theorem, as normally distributed.[10] Therefore, according to (2.9), Θ_G also is a normal random variable with mean and variance obtained by adding the means and variances, respectively, of Θ_{τ_2}, Θ_{L_2}, Θ_{W_2}, and Θ_{E_2}.[10] Consequently, the distribution D becomes

$$D(-\ln(f/f_r))=\sum_i D_i(-\ln(f/f_r))\ , \tag{2.11}$$

where

$$D_i=\frac{\alpha_i}{(2\pi)^{1/2}\psi_i}\exp\left[-\frac{1}{2\psi_i^2}\left[\ln\frac{f}{f_r}+\Theta_i\right]^2\right], \tag{2.12}$$

$$\begin{aligned}\Theta_i={}&\langle\Theta_{\tau_{2i}}\rangle+\langle\Theta_{L_{2i}}\rangle+\langle\Theta_{W_{2i}}\rangle\\ &+\langle\Theta_{E_{2i}}\rangle+\langle\Theta_{C_i}\rangle\ ,\end{aligned} \tag{2.13}$$

$$\begin{aligned}\psi_i^2={}&\langle(\Delta\Theta_{\tau_{2i}})^2\rangle+\langle(\Delta\Theta_{L_{2i}})^2\rangle+\langle(\Delta\Theta_{W_{2i}})^2\rangle\\ &+\langle(\Delta\Theta_{E_{2i}})^2\rangle(1-\delta_{E_1E_2})\ .\end{aligned} \tag{2.14}$$

The index i designates the ith set of islands, whose relative fraction is α_i, which are characterized by the same type of emission process. $\delta_{E_1E_2}=0$ for $E_1\neq E_2$ and $\delta_{E_1E_2}=1$ for $E_1=E_2$ also allows us to take into account, by means of (2.11)–(2.14), the case of islands having a single energy level[3]; in this case the relaxation time $\tau=C/G$, from (2.1) and (2.2), becomes $\tau=1/e_1$ and thus

$$D(-\ln(f/f_r))=D_\tau(-\ln(f/f_r))\ ,$$

$D_\tau(\Theta_\tau)$ being the distribution function of $\Theta_\tau=\ln(2\pi f_r\tau)$.[3]

Only in this case of single-level islands and of simple tunnel emission (tun), i.e., for $W_2=0$, from (2.3)–(2.6), (2.8)–(2.10), (2.13), and (2.14) we find that the quantities

$$\Theta_i=\langle\Theta_{\tau_{2i}}\rangle+\langle\Theta_{L_{2i}}\rangle=\ln(2\pi f_r\tau_{0i})=\Theta_{\text{tun}}$$

and

$$\psi_i^2=\langle(\Delta\Theta_{\tau_{2i}})^2\rangle+\langle(\Delta\Theta_{L_{2i}})^2\rangle=\psi_{0i}^2=\psi_{0\text{tun}}^2$$

become independent of the temperature. In all the other cases, which we will refer to here as thermally activated emission, Θ_i and ψ_i^2, from the same relationships, are temperature dependent according to the equations

$$\Theta_i=\ln(2\pi f_r\tau_{0i})+\phi_i/kT \tag{2.15}$$

$$\psi_i^2=\psi_{0i}^2+\sigma_i^2/(kT)^2\ , \tag{2.16}$$

where the quantities

$$\phi_i=\langle W_{2i}\rangle+\langle E_{2i}-E_F\rangle-\langle\,|E_{1i}-E_F|\,\rangle\ , \tag{2.17}$$

$$\sigma_i^2 = \langle (\Delta W_{2i})^2 \rangle + \langle (\Delta E_{2i})^2 \rangle (1-\delta_{E_1 E_2}) \ . \qquad (2.18)$$

Likewise, the parameters τ_{0i} and ψ_{0i}^2 defined above are independent of T if $E_{1i} > E_F$ or if E_F is independent of T itself. Otherwise, for $E_F \simeq E_{F0} - \gamma_T kT$,

$$\ln(2\pi f_r \tau_{0i}) = \langle \Theta_{\tau_{2i}} \rangle + \langle \Theta_{L_{2i}} \rangle - 2\gamma_T$$

and

$$\phi_i = \langle W_{2i} \rangle + \langle E_{2i} - E_{F0} = - \langle E_{F0} - E_{1i} \rangle \ .$$

Moreover, by indicating S_{Vi} as the spectrum of noise generated by the ith process, so that

$$S_{Vi} = \langle (\Delta V)^2 \rangle_t D_i / f \ , \qquad (2.19)$$

from (1.1) and (2.11) the spectrum of the total noise becomes

$$S_V(f,T) = \sum_i S_{Vi} \ . \qquad (2.20)$$

Its local frequency exponent, defined as

$$\gamma(f,T) \equiv -\partial \ln[S_V f_r / \langle (\Delta V^2) \rangle_t] / \partial \ln(f/f_r)$$

from (1.1), becomes

$$\gamma(f,T) = 1 - \frac{d \ln D}{d \ln(f/f_r)} \ , \qquad (2.21)$$

which, according to (2.11) and (2.12), reduces itself to the form[3]

$$\gamma \simeq \gamma_i = 1 + \frac{1}{\psi_i^2} \left[\ln \frac{f}{f_r} + \Theta_i \right] \qquad (2.22)$$

when, in particular frequency and temperature ranges, the ith process causes a D_i to prevail on the other distributions.

III. EXPERIMENTS

A. Methods and samples

The objective of our experiments was to check, by means of detailed measurements of fS_V, S_V, and γ vs f and T, the previous analysis and, hence, to support the island model of the flicker noise and, implicitly, the distribution of relaxation times as its source.[11]

For this purpose we have taken advantage of the capabilities of the currently available digital signal analyzers, including the one used in our investigation; particularly relevant is the possibility of obtaining quickly and directly, over many frequency decades, quasicontinuous and accurate plots not only of $S_V(f)$ but of $fS_V(f)$ too. In particular, reliable values of the frequency exponent γ can be obtained; this result is remarkable since a detailed study of γ, and of its dependence on temperature and frequency, is of paramount relevance for a correct interpretation of flicker-noise phenomena.

Such accurate and extended measures of γ, now possible and easy, should prove experimentally, in agreement with the island model,[3] that an exponent γ, which is independent of f and T on their wide intervals and which is rigorously equal to 1, is a mere mathematical abstraction. That is, although the tunnel emission might yield a γ very near to 1 on many frequency decades and on extended temperature intervals, in our opinion a "pure" $1/f$ noise should not exist physically.

Our measurements were performed in the frequency band from 10^{-2} to 2.5×10^4 Hz and over the temperature range from 80 to 750 K. The sample temperature was preset in temperature steps of 25 K by means of a liquid-nitrogen cryogenic system with a stability of ± 0.5 K in the range from 80 to 300 K, and by means of a microfurnace characterized by a stability[12] of ± 0.1 K in the range from 300 to 800 K.

Magnetic and electric shields, nickel-cadmium batteries, metal-film bias resistors, and low-noise preamplifiers[9] were used to have a low background noise S_B: $S_B = 2 \times 10^{-16}$ V^2/Hz between 10 Hz and 25 kHz and $S_B = 10^{-15}$ V^2/Hz at 1 Hz; its value is much smaller than the measured flicker noise of our samples on the whole frequency band we have explored. Thus in the present investigation we did not resort to another facility of the dual-channel input analyzer sometimes used in order to obtain a further reduction of S_B (of 15 dB), namely, the computation of the cross spectrum of the resistor voltage fluctuations picked up by two independent preamplifiers.

The measurements were performed on several thick-film resistors obtained from commercially available inks, screened and fired on 96% alumina substrates, and processed according to their respective manufacturer instructions. In an explorative investigation we included resistors based on ruthenium or iridium dioxides or ruthenates. Samples with resistance R_0 in the range from a few kΩ to 15 kΩ at $T = 300$ K were prepared to fit the conditions for more easy and precise measurements of noise with the instrumentation used. The samples were provided with prefired Pd-Au thick-film-conductor terminations. Preliminary checks were made to be sure that the contact noise of the samples was negligible. Moreover, four Ohmic contacts on the resistor metal terminations were obtained with a thermocompression microwelding system: two external contacts for driving the bias current and two internal contacts in order to pick up the voltage fluctuations. It should be noted that the measurements were performed at unusually high temperatures for thick-film resistors.[13,14] After the noise measurements taken at the highest temperatures ($T > 650$ K), which last a few hours, in some cases we observed a change in R_0 of some parts per thousand, in agreement with extrapolated data on the stability of these types of resistors reported in the literature.[13,14]

B. Results

The results reported in this paper concern mainly IrO_2-based resistors; in fact, after an explorative investigation on several types of resistors we concentrated our attention on the samples that provide more complete evidence of the effects of tunnel and thermally activated emissions considered in the previous analysis of the distribution function D. However, we found that the general features of the reported results [e.g., structures in the fS_V vs $\ln(f/f_r)$ plots and in those of γ vs T] are common to about all the thick-film resistors that we have examined, even if only in some cases the numerical values of parameters such as the activation energy for thermally activated processes, its variance, etc., can be obtained. In fact, the possibility of assigning numerical values to such parameters is offered only in the case where maxima of the fS_V vs $\ln(f/f_r)$ plots are found, for a wide temperature range, in the explored frequency band, as will be pointed out in the case illustrated.

Figure 1 shows the graphs of the noise-power spectral density S_V vs f directly given by the measuring apparatus after 1200 averages for two selected cases. It should be remarked that the ac-coupled preamplifier roll-off has been corrected by computing and plotting, by means of the digital signal analyzer, the ratio between the noise spectrum and the squared frequency response of the preamplifier previously stored in the memory of the instrument.

In Fig. 2 a series of measured spectra on the IrO_2-based resistors over a wide temperature range are reported. For clarity the statistical fluctuations of S_V, visible in Fig. 1, have been omitted in Fig. 2. Here an "average" frequency exponent γ which is temperature dependent is clearly evident, as already reported,[8,9] but data in the form presented in Fig. 3 are extremely useful for the analysis of the physical information contained in the spectra.

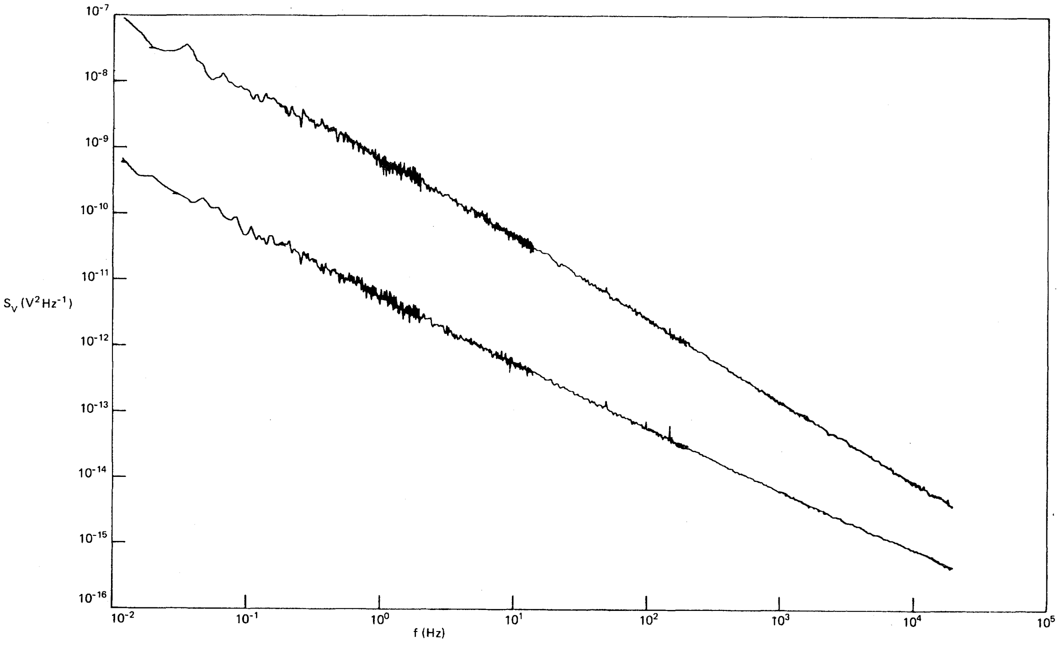

FIG. 1. Power spectra density of flicker noise given by the digital signal analyzer: The upper plot concerns a IrO_2-based thick-film resistor, extensively analyzed in the present investigation, measured at $T = 556$ K; the lower plot concerns a ruthenate-based resistor, measured at $T = 300$ K. In both cases the spectra were recorded after 1200 averages.

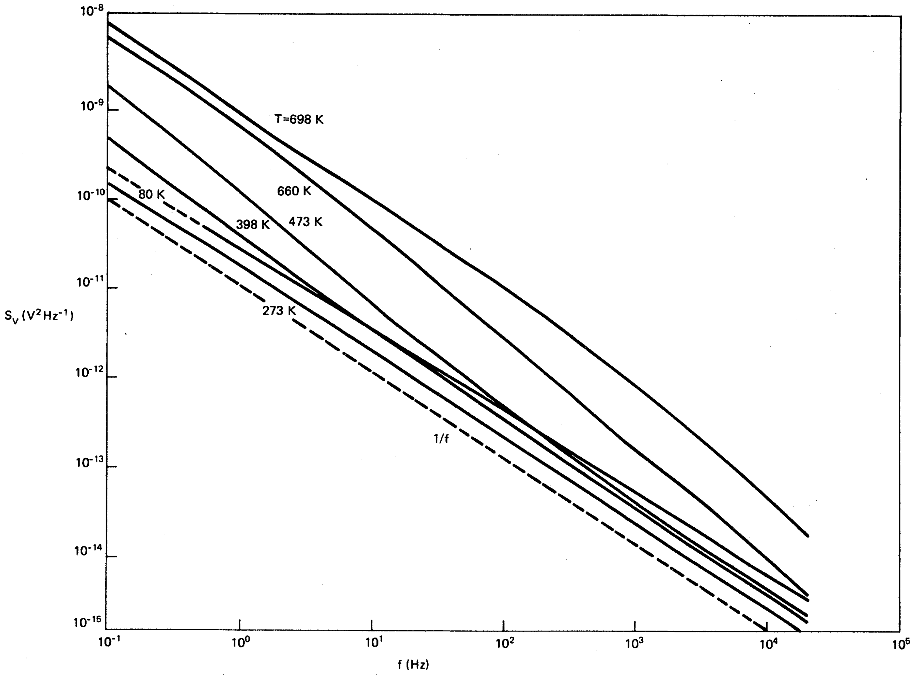

FIG. 2. Power spectral density of flicker noise as a function of frequency for different temperatures. The sample is a IrO_2-based resistor with a sheet resistivity of 10 kΩ/□ and a resistance $R_0 = 11.55$ kΩ at 300 K. For clarity only a few spectra have been reported.

As already mentioned the plots of fS_V vs $\ln(f/f_r)$ of Fig. 3 were directly computed and displayed by the digital signal analyzer. For the specific case of Fig. 3, a reference frequency $f_r = 1$ Hz was chosen.

If we compare the experimental findings with the expectation theory we can envisage a picture of the emission processes generating the flicker noise in the sample considered. Such a picture presents a tunnel emission process with parameters

$$\Theta_{\text{tun}} = \ln(2\pi f_r \tau_{0\text{tun}})$$

and $\psi_{\text{tun}} = \psi_{0\text{tun}}$ independent of T, and two thermally activated emission processes $A1$ and $A2$ characterized by the parameters $\Theta_i = \Theta_{a1}$, $\psi_i = \psi_{a1}$ and $\Theta_i = \Theta_{a2}$, $\psi_i = \psi_{a2}$, respectively, of the type (2.15) and (2.16), with $\phi_{a1} < \phi_{a2}$.

This picture, in fact, is suggested by the slight dependence of fS_V, and then of γ [see Fig. 6(a)], on temperature for $T < 400$ K, which can be ascribed to the prevalence of the tunnel emission process at lower temperatures. The lack of a maximum of fS_V in the measured frequency band hampered the computation, by means of (2.11) and (2.12), of Θ_{tun} and ψ_{tun}. In contrast for $T > 550$ K a maximum of fS_V at a frequency $f_M(T)$ emerges in the measured frequency band, which can be associated to the $A1$ process.

From (1.1), (2.11), and (2.12) we have the relationship

$$\Theta_{a1}(T) \simeq -\ln[f_M(T)/f_r] \,, \qquad (3.1)$$

which allows us to calculate Θ_{a1}; the plot of Θ_{a1} vs $1/T$, reported in Fig. 4(a), gives, according to Eq. (2.15), $\tau_{0a1} = 2.28 \times 10^{-14}$ sec and $\phi_{a1} = 1.52$ eV. Moreover, by making f^* the frequency for which $f^* S_V(f^*) = f_M S_V(f_M) e^{-1/2}$, from Eqs. (1.1), (2.11), (2.12), and (3.1), we obtain the relationship

$$[\psi_{a1}(T)]^2 \simeq \{\ln[f^*(T)/f_M(T)]\}^2 \,, \qquad (3.2)$$

whose plot versus $1/T^2$, reported in Fig. 4(b), yields, according to (2.16), $\psi_{0a1} = 2.48$ and $\sigma_{a1} = 0.14$ eV.

The self-consistency of the values of the parameter τ_{0a1}, ϕ_{a1}, ψ_{0a1}, and σ_{a1} is remarkable. Moreover, the value found for τ_{0a1} is in close agreement with theoretical previsions.[7,11]

It should be noted that ϕ_{a1} can be interpreted as a minimum value of the mean barrier height for electron tunneling between metal-oxide particles in the film. In fact, according to (2.17), in the ex-

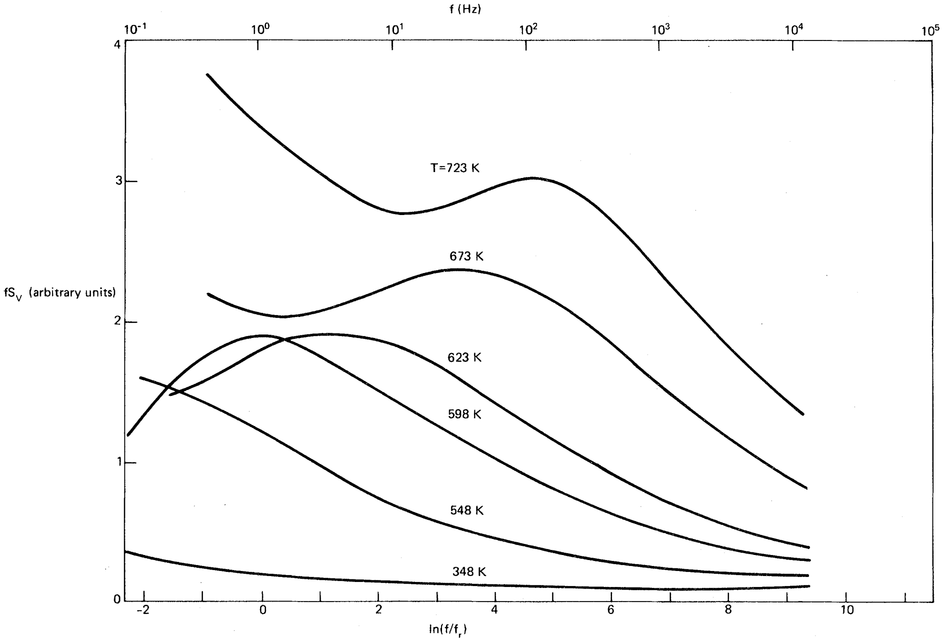

FIG. 3. Spectra of flicker noise of the sample of Fig. 2, presented in the form fS_V vs $\ln(f/f_r)$; the digital signal analyzer has computed the data in order to present the spectra in this form. The chosen reference frequency f_r is 1 Hz. Only a few spectra, recorded at many temperatures with increments of 25 K, are reported for clarity.

treme case where $\langle |E_{1a1}-E_F|\rangle \simeq 0$ it is $\phi_{a1} \simeq \langle E_{ea1}-E_F\rangle$, i.e., ϕ_{a1} represents in this case the mean distance from the Fermi level of the external energy level E_{ea1} in which the electron hops. In the better case E_{ea1} should be in the "conduction band" of the glassy interparticle material of the resistor. In this framework a high value of $\phi_{a1}=1.52$ eV is in very good agreement with previous considerations of the electrical transport properties of thick-film resistors.[6,15,16]

Even if it seems premature to associate the localized states so evidenced with specific impurities or defects in the resistor film, we note that the presence of deep energy levels in the glassy material is required in any self-consistent model of electrical transport in thick-film resistors[6,15,16]; moreover, the presence of such localized states emerged also from measurements of high-frequency response in thick-film resistors.[17] The presence of a second thermally activated emission process, with $\phi_{a2} > \phi_{a1}$, is implied by the appearance of a minimum in the plot of fS_V vs f (Fig. 3) at a frequency lower than f_M (at any given value of T) when the temperature goes over $T=650$ K.

A detailed cross comparison of theoretical and experimental data is now possible. First of all we observe that the theory predicts a maximum of the plot $S_{Vi}(f)$ vs T at a temperature

$$T_{Mi}(f) \simeq -\phi_i / k \ln(2\pi\tau_{0i} f) \ . \tag{3.3}$$

This relationship is a first-order approximation which follows from Eqs. (2.12), (2.15), (2.16), and (2.19), and from the remark that the variance $\langle(\Delta V)^2\rangle_t$ is so slightly dependent on the temperature in comparison with $D(T)$ (which is an exponential function of T) that $\langle(\Delta V)^2\rangle_t$ can be assumed constant in the product $\langle(\Delta V)^2\rangle_t D$.

Equation (3.3) gives, for $f=1$ Hz and the values of $\phi_{a1}=1.52$ eV and $\tau_{0a1}=2.28\times10^{-14}$ sec previously evaluated, $T_{Ma1}(1 \text{ Hz})=595$ K, in very good agreement with the experimental value $T_{Ma1}=600$ K [see Fig. 5(a)]. Furthermore, we can compare experimental and theoretical plots of S_V/V^2 vs T ($V=18.5$, V being the bias voltage of the sample) for some specific values of f (e.g., $f=1$ Hz and $f=1$ kHz) as shown in Fig. 5.

The theoretical plots of S_{Va1} are calculated with Eqs. (2.12), (2.15), (2.16), and (2.19); the experimental plot of S_{Va1} is obtained from the rough experimental data after subtraction of $S_{V\text{tun}}(f)$, due to the tunnel emission process, which appears at $T<400$ K and is basically temperature independent. The theoretical and experimental data of $S_{Va1}(1 \text{ Hz})$ are made coincident at $T=T_{Ma1}(1 \text{ Hz})$ and this superposition gives

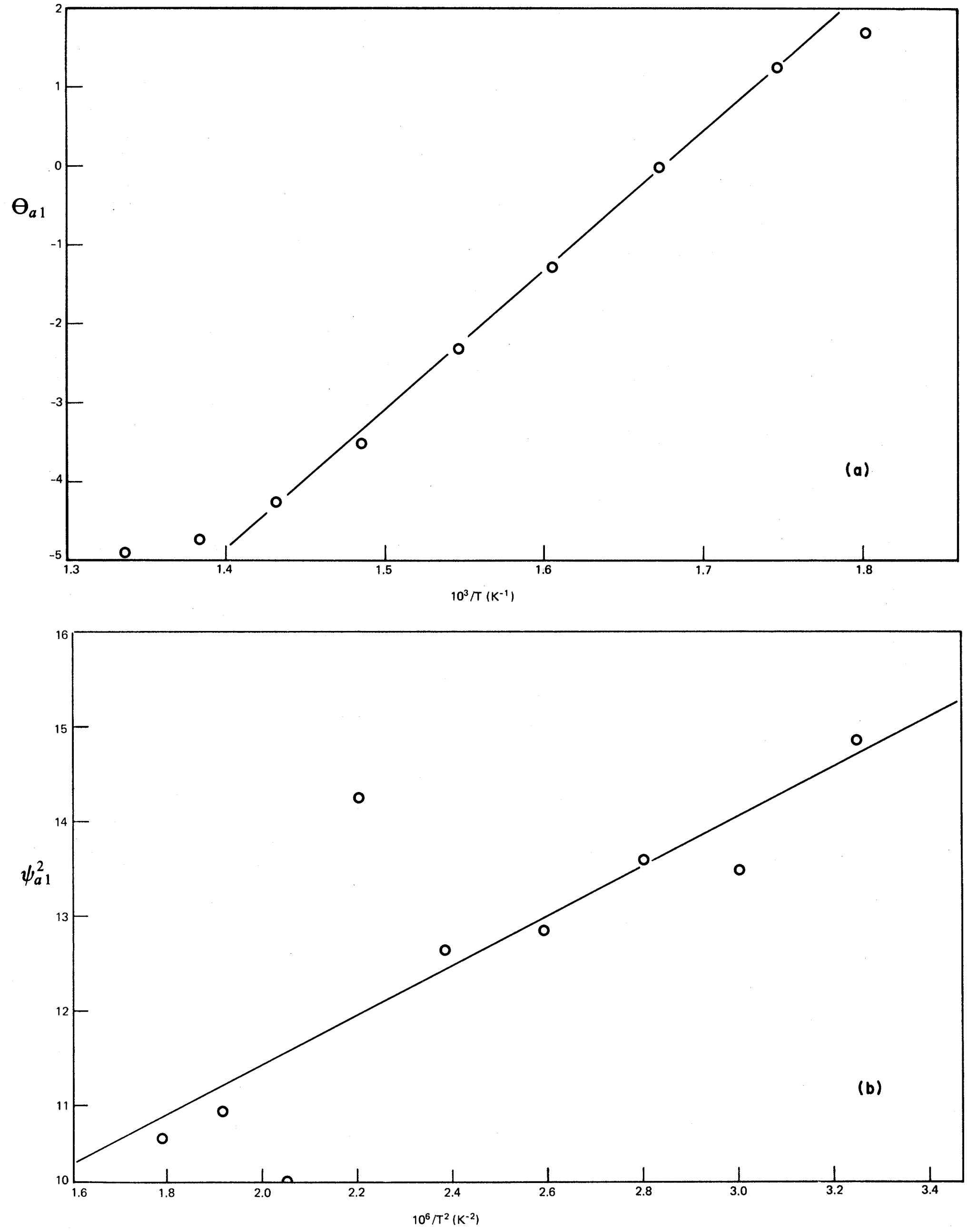

FIG. 4. (a) Θ_{a1} vs $1/T$; (b) ψ^2_{a1} vs $1/T^2$.

$$\alpha_{a1}\langle(\Delta V)^2\rangle_t/V^2=1.29\times10^{-13} ,$$

which is used in the equations listed above in order to calculate the whole dashed curve shown in Fig. 5(a). The agreement of the shape of this curve with the experimental one is very satisfactory up to the temperature $T=630$ K, over which the second thermally activated emission $A\,2$ is operating.

The same procedure for $f=1$ kHz gives $T_{Ma1}=780$ K, i.e., a maximum at a temperature higher than the upper limit of the experimental temperature range, in agreement with experimental data [Fig. 5(b)]; moreover, the previously described procedure gives a very good agreement between the theoretical and experimental curves of $S_{Va1}(1\text{ kHz})$ with a value of

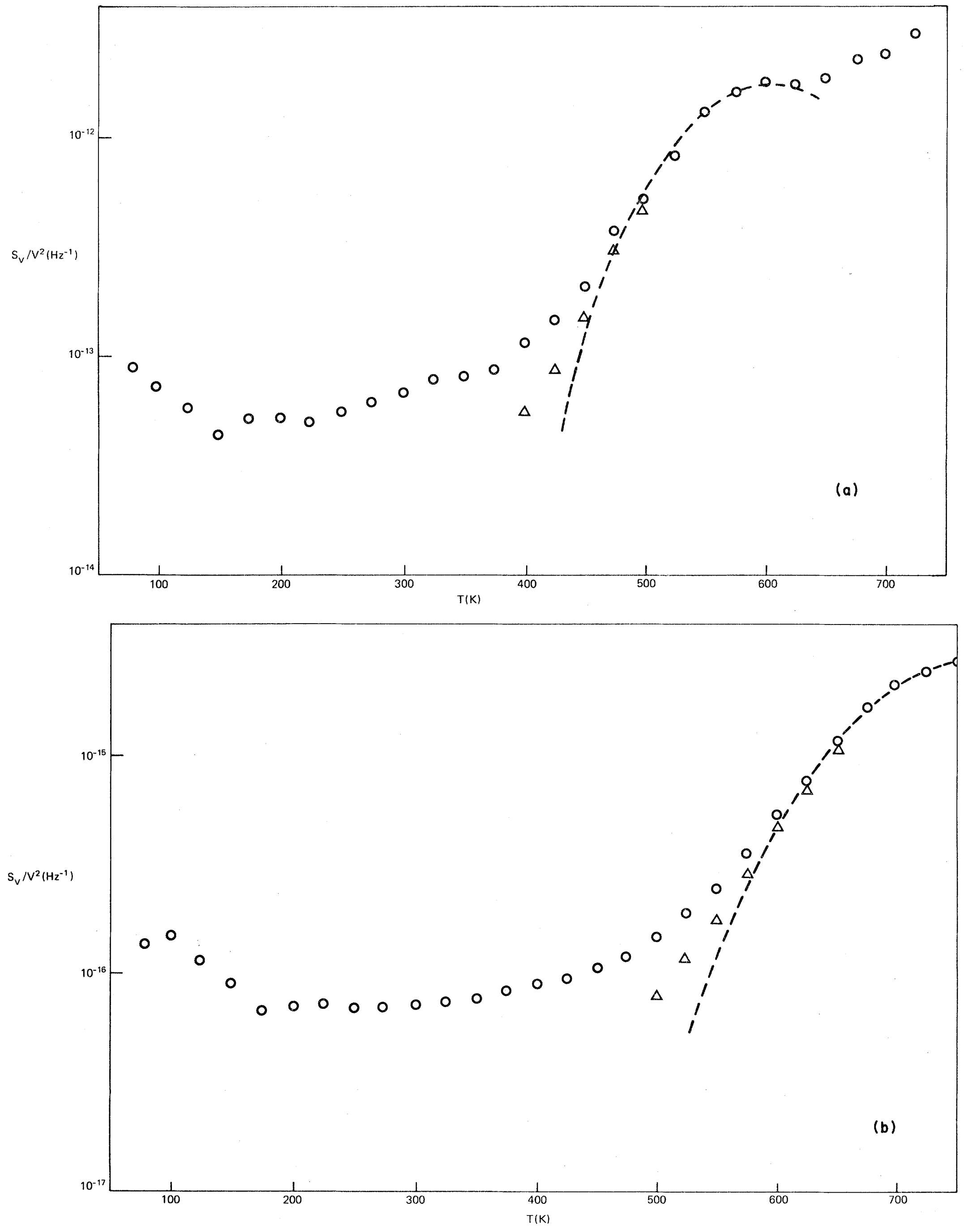

FIG. 5. (a) Normalized noise spectrum S_V/V^2 vs temperature at $f=1$ Hz; open dots are experimental data; triangles represent $S_{Va1}/V^2=(S_V-S_{V\text{tun}})/V^2$ and are obtained from experimental data after subtraction of the contribution $S_{V\text{tun}}$, nearly constant with the temperature; the dashed lines represent theoretical values of $(S_V-S_{V\text{tun}})/V^2$ (see text). (b) the same as in (a) for $f=1$ kHz.

$$\alpha_{a1}\langle(\Delta V)^2\rangle_t/V^2=1.76\times 10^{-13}\ ,$$

which compares well with the value 1.29×10^{-13} used in the case of $f=1$ Hz.

We note that plots of S_V vs T similar to those shown in Fig. 5 have been obtained for thin-film resistors,[11] and we suggest that also for these samples the data can be easily explained with the island

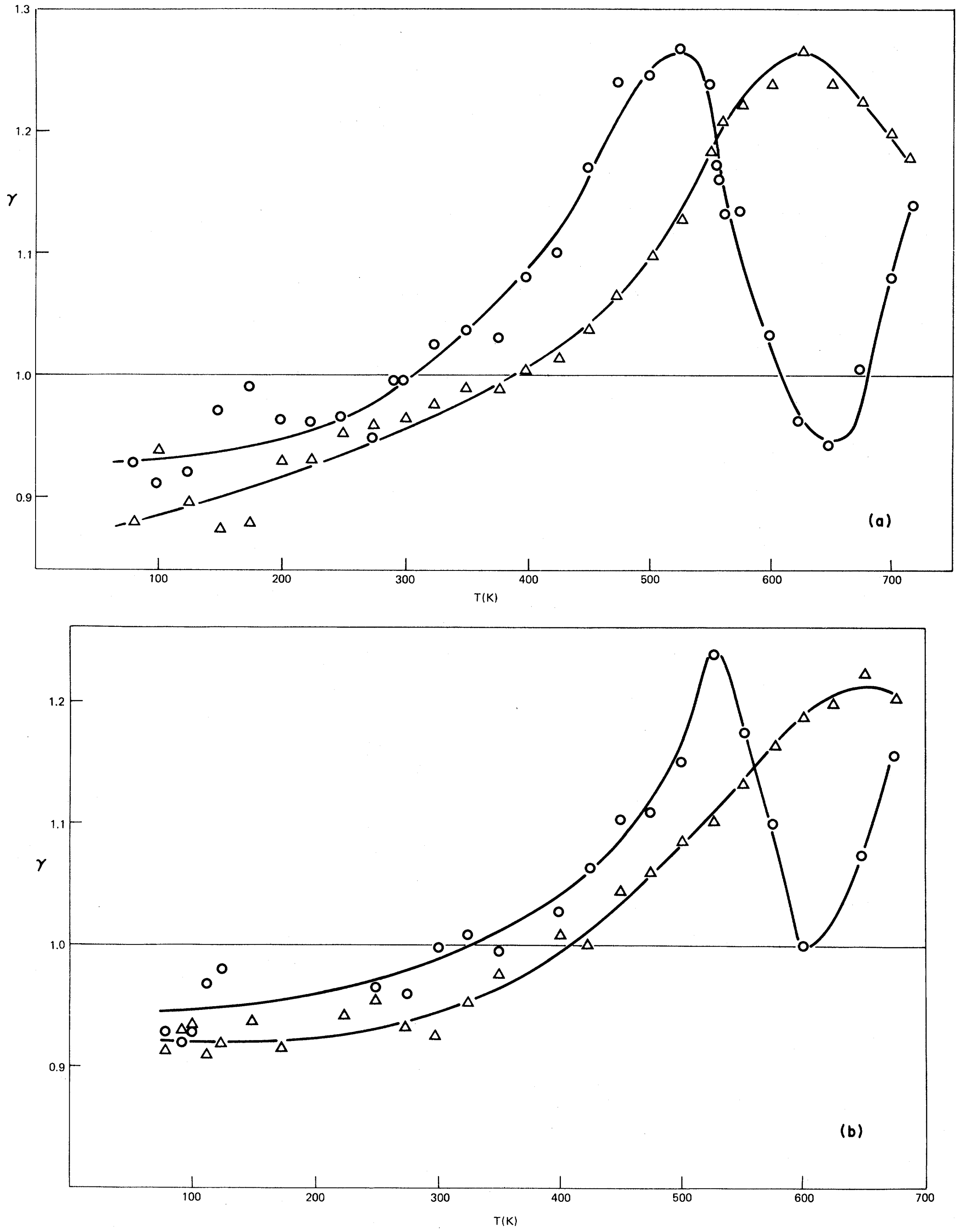

FIG. 6. Local frequency exponent = of the noise spectra vs temperature at the frequency $f = 1$ Hz (open dots) and $f = 1$ kHz (triangles); (a) for the IrO_2-based resistors and (b) for a ruthenate-based resistor.

model and two thermally activated emission processes.

A further check of the model and of the self-consistency of the data is given by the analysis of the local frequency exponent $\gamma(f,T)$. The experimental values of γ vs T for two frequencies ($f = 1$ Hz and $f = 1$ kHz) are shown in Fig. 6. According to the model [see Eqs. (2.15), (2.22), and (3.3)] we find that at $T = T_{Ma1}(1\text{ Hz}) = 595$ K, we should have $\gamma_{a1}(1\text{ Hz}) = 1$; the requirement is in excellent agreement with the experimental behavior of γ(1 Hz), which equals the unity at 600 K [see Fig. 6(a)];

this value of the temperature is really in the range where the $A1$ process prevails. The further increase of $\gamma(1\ \mathrm{Hz})$ at temperatures higher than 650 K again reflects the onset of a second thermally activated emission process. On the other hand, the nearly constant values of $\gamma(1\ \mathrm{Hz}) \simeq \gamma(1\ \mathrm{kHz}) \simeq 0.95$ for $T < 300$ K reflect the fact that tunnel emission processes are mainly effective in the lower temperature range.

The equality of the values of γ for $f = 1$ Hz and $f = 1$ kHz at $T = 556$ K means that at this temperature the flicker noise is characterized by an exponent γ effectively constant over many frequency decades; this is just what we have in Fig. 1 where the slope of the upper spectrum is constant over at least 4.5 frequency decades.

As mentioned previously, not all the resistors investigated exhibit a flicker noise with thermally activated emission processes prevailing in a net way over the tunnel emission. For this reason experimental data of noise in these resistors are less suitable for a detailed comparison with theory and for an evaluation of the parameters characterizing the localized states from which the noise itself arises. However, the presence of at least one thermally activated process could be argued for by the data of γ vs T in almost all the resistors examined.

An example of this situation is shown in Fig. 6(b) where the plots of $\gamma(1\ \mathrm{Hz})$ and $\gamma(1\ \mathrm{kHz})$ vs T of a ruthenate-based resistor are reported. Also in this sample large changes of γ as a function of frequency and temperature are detected owing to the sensitivity of the local frequency exponent to the charge carrier emission processes and to their temperature dependence.

IV. CONCLUSIONS

The island model identifies the origin of the flicker, burst, and generation-recombination noises in the localized states of the conducting medium,[1] treats them together in a theoretical synthesis,[2,3] and, through the diffusion-noise theory, accounts for and contains the approaches that, as a $1/f^{\gamma}$ noise cause, postulate the carrier-mobility fluctuation.[4] The island model has been here elaborated further both for itself and in order to make several comparisons between its theoretical results and the experimental data possible and easy.

For these purposes the distribution function of the conductance logarithm of the localized states has been determined through the island model itself, the thermally activated hopping model of the emission coefficient, the central-limit theorem, the rule for computing the distribution of the sum of uncorrelated normal random variables, and finally by taking into account that island sets characterized by different emission processes may exist in the conducting medium. It has been shown that the distribution D so achieved is temperature independent only in the case of pure tunnel emission from islands with a single energy level, whereas in all the other cases it becomes a complex temperature function through the mean and the variance of the Gaussian distribution of the single emission processes.

On the other hand, according to the island model, the distribution D, computed as a function of the frequency logarithm, is also proportional to the frequency-noise spectrum product. Therefore, it has been stressed that a valuable strategy to check experimentally the proposed distribution model and chiefly, in general, to check the island theory of the flicker noise, as well as to explore the physical information enclosed by it, just consists in the analysis of the product fS_V vs $\ln(f/f_r)$ over wide temperature and frequency ranges. It is shown that this analysis is made ready and reliable by the use of a digital signal analyzer.

A detailed study of such measurements on thick-film resistors, as well as of the noise spectrum and of its local frequency exponent, has been presented. By means of these experiments it has been shown that at low temperatures tunnel emission intervenes, whereas at the mean and high temperatures two thermally activated processes prevail.

The new method of measuring the frequency-spectrum product versus the frequency logarithm allowed us to evaluate, from noise measures, the mean and the variance of the activation energy, as well as the other parameters of the distribution function, for one of the two localized state sets involved in the thermally activated emissions. Such a distribution and the whole model then yield theoretical results for the noise spectrum and its frequency exponent, which are fully confirmed, independently, by their experimental plots versus the temperature.

The proposed model of the distribution function, in general, allows one to account for the complex temperature dependence of the flicker-noise parameters that has been observed, other than in thick-film resistors, in several other conducting media. On the other hand, the theoretical and experimental results, which from the cross comparison in many different ways appear completely self-consistent, have provided experimental evidence of deep localized states in thick-film resistors. It should be pointed out that this evidence is remarkable since previously we knew only that these levels were necessary to explain the electrical transport in thick-film resistors and we had only qualitative information,[17] while in the study of flicker noise it seems that we have a possible technique to obtain quantitative data on these states.

In conclusion, through new theoretical contributions and measurement methods also, we seem to have brought experimental support to the island model of the flicker noise, which in our opinion, according to such a model, and to reliable computations of the local frequency exponent—which are now possible and easy by means of the digital analysers and which have been performed by ourselves on semiconductor, composition, metal tin-film, and ink thick-film resistors and on field-effect, metal-oxide–semiconductor, and bipolar junction transistors—and to the frequency and temperature dependence of the exponent itself, should not exist physically in the perfect $1/f$ form.

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