Magnetoresistance in Si metal-oxide-semiconductor field-effect transistors: Evidence of weak localization and correlation

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We have studied the magnetoresistance in the two-dimensional electron gas of silicon metal-oxide-semiconductor field-effect transistor inversion layers at temperatures down to 50 mK. Both low- and high-mobility samples have been studied and clear evidence for localization as well as correlation effects is observed in the region of resistance $R_\square \ll 10$ k$\Omega/\square$. From detailed fits of the data at low magnetic fields a localization parameter α as well as the temperature dependence of the inelastic scattering rate is extracted. From the data we conclude that the dominant inelastic scattering mechanism at these temperatures is electron-electron scattering in the dirty limit. At higher fields correlation effects dominate and the associated parameters are determined.

INTRODUCTION

The nature of electronic conduction in two dimensions poses a fascinating problem in the high-conductivity limit. Whether a truly "metallic" regime exists in two dimensions has been investigated both theoretically and experimentally. Mott's concept of a minimum metallic conductivity[1] suggests a metal-insulator transition at a resistance $R_\square = 10\text{k}\Omega/\square$. More recent scaling arguments[2] based on localization concepts developed by Thouless[3] imply that there is no metallic behavior in any regime in two dimensions. Rather, it is shown that there is, in the vicinity of $R_\square = 10\text{k}\Omega/\square$, a smooth and continuous transition from exponential localization to rather weak logarithmic behavior.

Measurements of the temperature dependence of $R_\square$ in several two-dimensional (2D) systems[4–6] seemed to verify this prediction, but it was pointed out that quantitatively similar results would be expected if many-body correlation effects were considered,[7] independent of any localization considerations. To distinguish these two effects it was shown that specifically different dependencies of the conductivity on parallel and perpendicular magnetic fields would be expected.

In this paper we report our measurements of the magnetoconductance of the 2D electron gas in Si metal-oxide-semiconductor field-effect transistors (MOSFET's). Several devices have been studied and we will report results from high- and low-mobility ($\geq 25\,000$ and ≤ 1000, respectively) devices where a fairly complete set of measurements have been performed. As we will show, these measurements can be quantitatively interpreted in terms of current ideas of electron localization and scattering in the dirty limit. It was shown by Uren *et al.*[8] that both localization and correlation effects can be observed in the transport properties of similar devices. We show here that these contributions can be separately measured by a combination of parallel and perpendicular magnetic field measurements in the low- and high-field limit. Detailed fits of the temperature and magnetic field dependencies yield a measure of the relative influence of these contributions. Unequivocal evidence for both localization and Coulomb correlation effects is obtained and from detailed fits of the temperature and magnetic field dependences a picture of the relative influence of these contributions is obtained.

THEORETICAL BACKGROUND

Following the concepts of Thouless,[3] Abrahams *et al.*[2] developed a single-parameter scaling theory of transport which showed that in the high-conductivity limit, the scaling length dependence of the conductivity of a two-dimensional electron gas is given by

$$\sigma(L) = \sigma(L_0) - \frac{\alpha e^2}{\pi^2 \hbar} \ln(L/L_0) \; . \tag{1}$$

Here α is a constant of order 1.

It was shown that the temperature dependence of the conductivity can be derived from (1) via the temperature dependence of the inelastic scattering rate

$$\frac{1}{\tau_i} \propto T^p , \quad (2)$$

where p depends on the nature of the dominant inelastic scattering process. This results in a conductance given by

$$\sigma(T)=\sigma(T_0)+\frac{\alpha p}{2}\frac{e^2}{\pi^2\hbar}\ln\left[\frac{T}{T_0}\right] . \quad (3)$$

Physically, as the temperature increases inelastic scattering shortens the length scale L over which one probes the system and the second term in (1) becomes smaller thus enhancing the conductance.

Several conductance measurements on various two-dimensional systems have been performed[4-6] and in the appropriate regime this $\ln T$ dependence has been observed. Determinations of the parameter αp have shown some variation from system to system but on the MOSFET's reported in this from $\sigma(T)$ measurements we determined that $\alpha p = 1.04 \pm 0.1$.

In addition to localization effects, Altshuler, Aronov, and Lee[9] have pointed out that many-body effects can also influence the conductivity. They have shown that in the limit $k_F\lambda \gg 1$ where λ is the elastic mean free path, the conductivity can be written

$$\sigma(T)=\sigma(T_0)+(1-F)\frac{e^2}{2\pi^2\hbar}\ln\left[\frac{T}{T_0}\right] , \quad (4)$$

where F is a measure of the screening, i.e., when $2k_F/K \to 0$, F approaches 1 while $F \to 0$ when $2k_F/K$ diverges. Here K is the screening constant. In a well-screened system $[(2k_F/K) \to 0]$ the second term approaches zero and there is no T dependence from this effect. In the other limit, however, $(F=0)$, (4) is identical to (3) for $\alpha p = 1$ and a conductance measurement is not able to differentiate between the two.

In the case of MOSFET's both (3) and (4) should be multiplied by valley degeneracy factors in the correction terms but in the limit where intervalley scattering is strong, these factors cancel out and the result is as stated. Fukuyama[10] has shown that not all the relevant interaction terms were included in the derivation of (4) but if a valley degeneracy of 2 is assumed, the differences between the two results are small.

In the case of MOSFET's, we estimate for an electron density $n \approx 1\times 10^{12}$ cm^{-2}, the value $2k_F/K \approx 0.1 \to 0.2$ depending upon the electronic effective mass chosen and the dielectric constant. At any rate, this low value implies $F \approx 1$ and the T corrections from (4) would be negligible. The similarity between (3) and (4), however, is sufficiently compeling to motivate an alternate estimate of the various parameters (α,p and F) making up these conductance corrections.

Fukuyama,[10] Hikami *et al.*,[11] and Lee and Ramakrishnan[12] have pointed out that the localization and correlation effects depend very differently on applied magnetic field H. For $\perp$ field, it was shown[13] that the localization effects are diminished when the size of the first Landau orbit is comparable to the inelastic diffusion length $(\frac{1}{2}l_i l_e)^{1/2}$. As this is an orbital effect, it is absent in parallel H field while the effect of H on the Coulomb corrections, being simply a Zeeman splitting term, is independent of orientation and occurs at higher fields. Lee and Ramakrishnan[12] show the conductivity is given by

$$\sigma(T,H)=\sigma(T_0,0)+\frac{\alpha p e^2}{2\pi^2\hbar}\ln\left[\frac{T}{T_0}\right]+\frac{\alpha e^2}{2\pi^2\hbar}\left[\psi\left[a+\frac{1}{2}\right]-\psi\left[a'+\frac{1}{2}\right]+\ln\left[\frac{l_i}{l_e}\right]\right]$$
$$+\frac{e^2}{2\pi^2\hbar}(1-F)\ln(T/T_0)-\frac{e^2}{2\pi^2\hbar}\frac{F}{2}G(h) . \quad (5)$$

Here $a=\hbar(c/2eHl_el_i)$, $a'=\hbar(c/2eHl_e^2)$, ψ is the digamma function, $h=\mu gH/kT$, and the function $G(h)$ is

$$G(h)=\int d\omega\frac{d^2}{d\omega^2}\left[\omega\frac{1}{e^{\omega}-1}\ln\left|1-\frac{h^2}{\omega^2}\right|\right] .$$

For $h \ll 1$, $G(h)=0.084h^2$; for $h \gg 1$, $G(h)=\ln(h/1.3)$. The second and third terms on the right-hand side of (5) are the T and H corrections due to localization effects (depending on the strength of α and P), while the correlation effects (dependent upon F) are the remaining two terms.

EXPERIMENTS AND DISCUSSION

Several Si(111) and Si(100) MOSFET's have been studied and we report here the detailed results on the two samples with the extreme mobilities (< 1000 and > 25 000). A preliminary report on some of these measurements has been reported elsewhere.[13] Earlier measurements in the low-field regime have also been reported by Kawaguchi and Kawaji,[14] Wheeler,[15] and by Davies *et al*[16]. Our measurements were performed in a ^{3}He-^{4}He dilution refrigerator on four terminal Si MOSFET devices. These were similar devices to those studied earlier[6] in which the logarithmic temperature dependence of the conductivity was demonstrated. Magnetic fields were applied via a superconducting solenoid capable of 50 kG. The resistances were measured using an ac resistance bridge operating at 500 Hz. The amplitude of the voltage modulation used to measure the resistance was always less than 2 mV/cm with substantially lower fields used at the lower temperatures. This was done in an effort to keep electron heating effects to a minimum. The devices were 1.0-mm long and 0.25-mm wide with potential probes separated by 0.25 mm.

A trace of the magnetoresistance in perpendicular field at 0.10 K is shown in Fig. 1 for a low-mobility (111) sample. As a function of H, the resistivity is seen to first decrease rather sharply and then at higher fields increase again. The decrease at low fields is due to the suppression of localization effects [the third term in (5)], while the

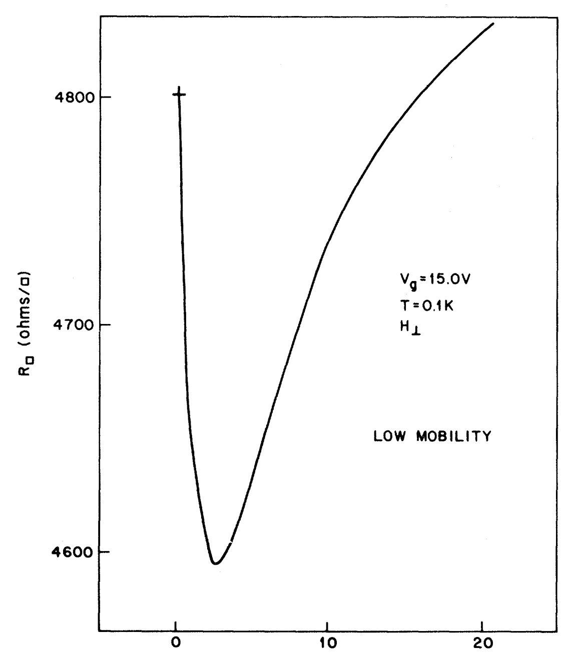

FIG. 1. Magnetoresistance of a Si(111) MOSFET in a perpendicular magnetic field at 0.1 K. Electron density is 1.2×10^{12} cm^{-2}.

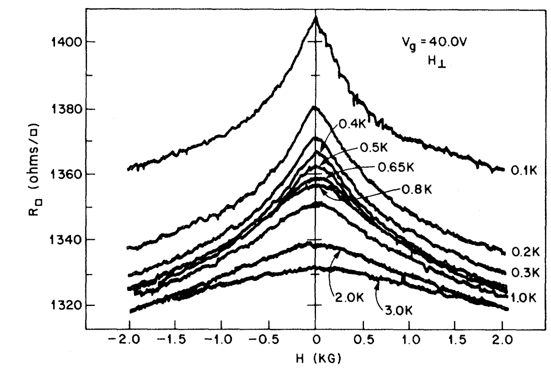

FIG. 2. Low-field magnetoresistance of a Si(111) MOSFET in a perpendicular field for various temperatures. Electron density is 4.52×10^{12} cm^{-2}.

increase is due to the Zeeman term [the last term in (5)]. This rise has a logarithmic dependence on H as predicted in (5) in the limit $\mu gH \gg kT$. This negative magnetoresistance in low fields was first observed by Eisele and Dorda.[17] A more detailed measurement in the low-field region yields the set of data shown in Fig. 2 for an electron density of 4.52×10^{12}cm^{-2}. Here we show the temperature and magnetic field dependence of the resistance due to localization effects. It can be seen that for this low-mobility device ($\mu\approx1000$), these effects persist out to a few kilogauss. If we adopt the interpretation that localization effects should begin to "turn off" when the first Landau orbit becomes comparable in size to the inelastic scattering length we obtain a critical field H_c given by

$$H_c=\frac{\hbar c}{2el_il_e} .$$

For the data at $T=0.1$ K shown in Fig. 2 and the estimate of l_i described below, this corresponds to a magnetic field of $\sim$30 G. Thus as the various Landau orbits become smaller than the inelastic length there is a rapid drop in R beginning at rather low fields. A detailed fit to (5) can be made and in the low-field region only the orbital term contributes. From this fit the parameters α and the inelastic scattering time τ_i can be extracted. The quality of the low-field fit for different values of α is shown in Fig. 3. The curves are fit at $H=0$ and 2.0 kG and τ_i determined. It can be seen clearly that $\alpha=1.0\pm0.05$ yields the best fit resulting in an inelastic scattering time for this temperature and electron density of 3.75×10^{-11} sec. Similar quality fits have been made for the data set in Fig. 2 and for various other electron densities on this particular device and it is found that the best fit oc-

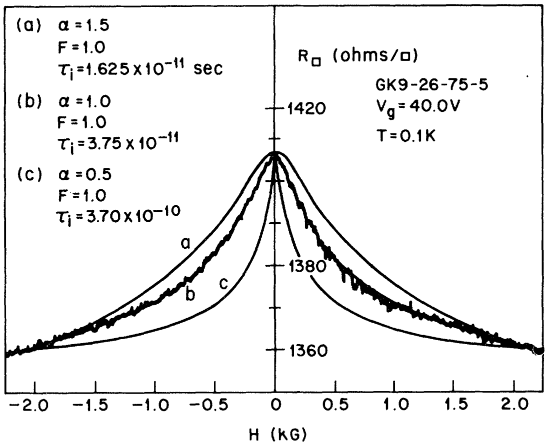

FIG. 3. Same data as in Fig. 2 at 0.1 K with various fits to Eq. (5). The data is fitted at $H=0$ and 2.0 kG and the resultant parameters are listed on the figure. The fit is relatively insensitive to the choice of F.

curs at all temperatures and electron densities studied for a value $\alpha=1.0\pm0.1$. The resultant values of τ_i from these fits are shown in Fig. 4. In agreement with the earlier measurements,[8,16] it is seen that there is a region where $\tau_i \propto T^{-P}$ and $p\approx 1$ down to temperatures of ~ 0.3 K at which point τ_i apparently begins to saturate. This is consistent

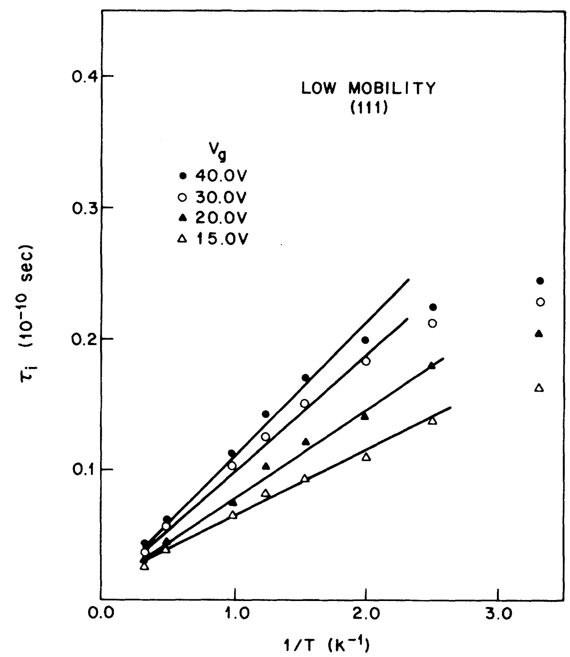

FIG. 4. τ_i obtained from the fitted magnetoresistance of Fig. 2 as a function of $1/T$. Note the approximately linear region ($p\approx 1$) and the fact the slope changes with electron density and thus diffusively. This data is for the low-mobility (111) sample.

with our earlier temperature-dependent resistivity measurements[6] where a deviation from logarithmic behavior was observed at lower temperatures and an apparent saturation resulted. This was interpreted as an electron heating effect resulting from the fact that the phonon relaxation time for electrons at these temperatures becomes so long that the electon gas cannot cool in the finite size of the sample. If electron-electron scattering is the dominant inelastic scattering mechanism (a result indicated by the $1/T$ dependence of τ_i) the electrons can still inelastically scatter amongst themselves but there is no channel for energy loss. Thus the electron gas can cool no further over this distance.

It is interesting to compare these results to those for a high mobility (l_e much longer) sample. A comparable set of low-field resistance fits are shown in Fig. 5 for a (100) sample with a mobility $>20\,000$. Because of the higher mobility H_c is now estimated to be of the order of a few gauss and so the necessary range of magnetic field is now up to 50 G rather than 2 kG. Again, the best fit is achieved for a value $\alpha=1.0\pm0.1$. Again the fits at these low fields are insensitive to the choice of F and so α can be independently determined. The values obtained for τ_i as a function of T are shown in Fig. 6. As for the low-mobility sample, τ_i varies approximately as $1/T$ over the same temperature range that the logarithmic temperature dependence of the conductivity is observed. At lower temperatures, τ_i saturates as does the resistivity. It is worth noting that the saturation temperature in the high-mobility case is consistently higher than that of the low-mobility one. This is consistent with the explanation offered previously as the energy relaxation length $L_E=(\frac{1}{2}l_e l_{\rm ph})^{1/2}$ will increase as l_e increases. The saturation point occurs when $L_E=L_0$ the sample dimension, i.e. $L_0=(\frac{1}{2}l_e l_{\rm ph})^{1/2}$. For different samples, with different mobilities, this occurs at different temperatures given by

$$\tfrac{1}{2}l_e l_{\rm ph}(T)=L_0^2\ .$$

If $l_{\rm ph}\propto T^{-3}$, and a mobility ratio of ~ 20 one would expect an increase in the saturation temperature by a factor of ~ 2.7, consistent with the results. These hot electron finite-size effects will ultimately limit the temperature to which the conductivity can be studied.

We reemphasize that this explanation relies on the assumption that electron cooling only occurs via electron-phonon scattering. Inelastic electron-electron scattering is substantially more frequent in

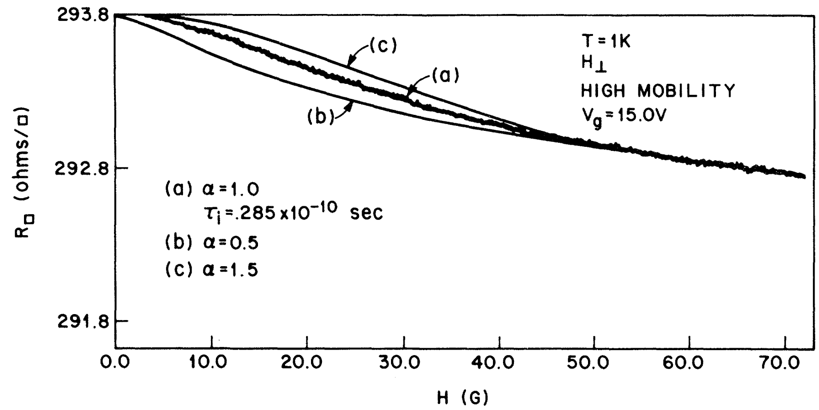

FIG. 5. Magnetoresistance for a high-mobility (100) Si MOSFET at $T=0.1$ K in a perpendicular magnetic field. The solid lines are fits to Eq. (5) for various choices of α fitted at $H=0$ and 50 G. The electron density is 1.03×10^{12} cm^{-2}.

this temperature range but that only allows the electron gas to reach its own thermodynamic equilibrium. In order for the gas to cool, inelastic scattering to some other excitations (phonons) must occur and the rates for this process are substantially smaller.

From all of these magnetoresistance fits we conclude that in the electron-density range studied $\alpha=1.0\pm0.1$ and $p=1.0$. Recall from earlier temperature-dependent resistance measurements[6] it was concluded that the product $\alpha p=1$. These results are then consistent and are strong evidence

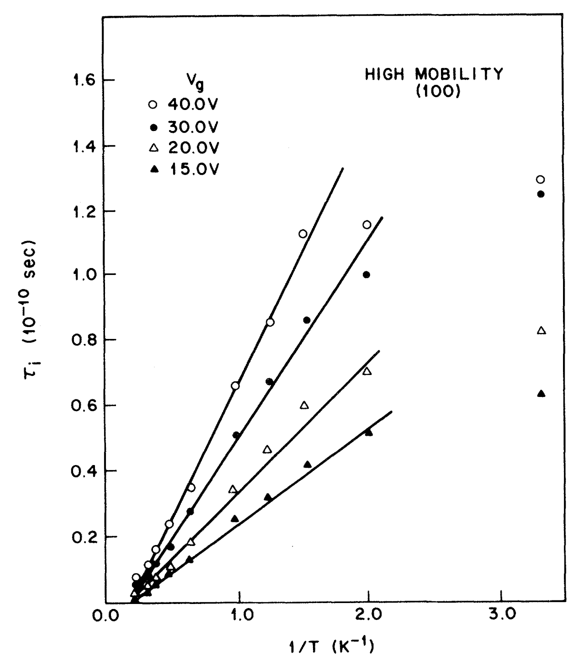

FIG. 6. τ_i obtained from the fitted magnetoresistance of the high-mobility sample as a function of $1/T$. Note the approximately linear region ($p\approx1$).

that these are localization corrections in the conductivity and that Coulomb corrections are inoperative in this regime. Kawaguchi and Kawaji[14] have studied the magnetoconductance at higher temperatures and find p at these temperatures becomes larger than our measured value of 1.

If electron-electron scattering is the dominant inelastic scattering mechanism, simple arguments suggest a scattering rate $1/\tau_{ee}$ of order $v_F K(k_B T/E_F)^2$, where K is the screening constant and v_F the Fermi velocity. Following an argument by Schmidt,[18] it was shown by Abrahams, Anderson, Lee, and Ramakrishnan[19] that in the dirty limit the electron-electron scattering rate is given by

$$\frac{1}{\tau_{ee}}=\frac{e^2}{\epsilon\hbar^2 DK}kT\left|\ln\left(\frac{T}{T_1}\right)\right| , \qquad (6)$$

where T_1 is given by

$$kT_1=\frac{\hbar^3 D^3 K^4 \epsilon^2}{e^4} .$$

Here ϵ is the dielectric constant. This result implies that the electron-electron scattering rate varies as $T\ln T$. The $\ln(T/T_1)$ contribution is so weakly dependent on T that this prediction is consistent with our results. Equation (6) also predicts that $1/\tau_{ee}$ also scales inversely with the diffusivity. It can be seen from Figs. 4 and 6 that τ_i does vary with electron density and so with diffusivity.

We estimate for the sample studied, that T_1 varies from 10^6 to 10^9 K. This large value of T_1 results in a predicted enhancement of the scattering rate by over an order of magnitude due to these logarithmic divergences. These experimental results give evidence that this strong enhancement is

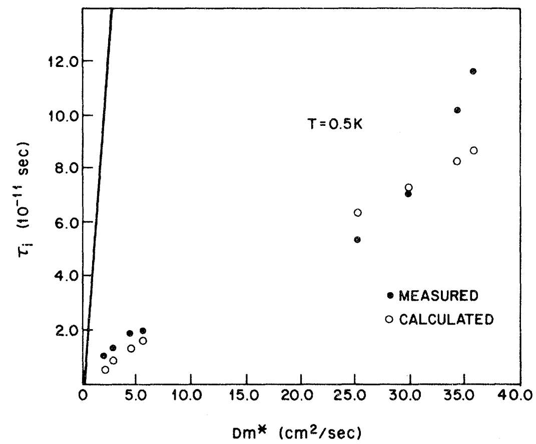

FIG. 7. τ_i vs Dm^* for the data of Figs. 4 and 6 at 0.5 K (solid circles). The open circles are calculated from Eq. (6). The straight line is the value calculated from Eq. (6) ignoring the $\ln(T/T_i)$ enhancement.

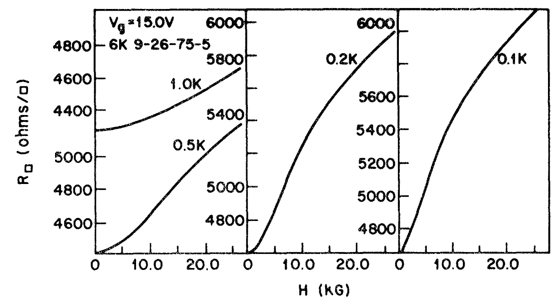

FIG. 8. High-field magnetoresistance on the Si(111) device in parallel H field. Electron density is 1.2×10^{12} cm^{-2}. As the temperature increases, note the change from $\ln H$ to H^2 behavior.

operative as the scattering rates shown in Figs. 4 and 6 are surprisingly strong. To better illustrate this, we show in Fig. 7 the measured τ_i vs Dm^* for the various data at 0.5 K from Figs. 4 and 6. The data is shown as the closed circles and the calculated value of τ_{ee} from Eq. (6) is shown as the open circles. The solid line on the left-hand side of the figure is the value of τ_{ee} estimated if the logarithmic enhancement is ignored. From this figure, it is seen very clearly that the predicted magnitude of the enhancement of the scattering rate due to this logarithmic contribution is observed.

At high fields, it is seen that the correlation effects dominate and from the behavior at fields much greater than H_c the contribution from these effects should be measurable. To avoid localization effects the high-field measurements were performed in parallel field where only the last term in (5) is operative. In Fig. 8 we show $R(H)$ for the low-mobility device at a particular electron density. The features anticipated from (5) are indeed observed. At $T=0.1$ K logarithmic behavior for $h \gg 1$ is seen. As the temperature is raised a parabolic region appears at low h until by 1.0 K the predicted h^2 dependence is seen. From this data we can extract a value for the screening parameter F. This is accomplished by taking the low-temperature $R(H)$ data in the high-field regime $(H \rightarrow kT)$ and plotting it on a semilogarithmic plot. The slope of the resultant straight line then yields the parameter F. It is also worth pointing out that in the other regime $(H < kT)$ taken at $T=1$ K, a fit to $R(H) \propto H^2$ yields a similar value for F implying no strong temperature dependence to this parameter. The values obtained for F are shown in Fig. 9 as a function $k_F\lambda$. It is seen that at large values of $k_F\lambda$ the expected value of $F \approx 1$ is obtained but as $k_F\lambda$ is decreased F increases to 3.5. A value of $F > 1$ was totally unexpected and we have no explanation for such a result. The theoretical framework associated with the Coulomb interaction portion of Eq. (5) is perturbative in nature and so strictly requires that $k_F\lambda$ be much greater than 1. It is possible that what is observed here is the breakdown of this perturbation limit as the resistance of the sheet approaches 10 k$\Omega/\square$ and exponential localization. This does not explain why the functional form of the Zeeman term in (5) remains intact in this regime but it could be argued that the $\ln h$ dependences would more generally be expected in this regime. At any rate we believe more work, both theoretical and experimental, is necessary on this point before it is resolved.

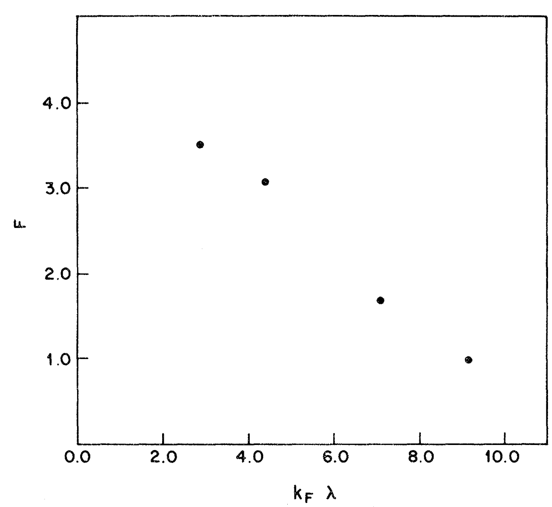

FIG. 9. Parameter F from data similar to that in Fig. 8 as a function of $k_F\lambda$. F is obtained via Eq. (5).

CONCLUSIONS

We have investigated the magnetoresistance of the two-dimensional electron gas in several Si MOSFET's in an attempt to distinguish between weak localization contributions and many-body correlation effects on the transport properties. Clear indications of both contributions are observed. In the low-field regime, localization effects are recognized and from fits of $R(H,T)$ we are able to extract the localization parameter α, and the temperature dependence of the inelastic scattering rate. These parameters are found consistent with the earlier $R(T)$ measurements and indicate that electron-electron scattering in the dirty limit most likely dominates the inelastic scattering processes. We see a strong enhancement of the inelastic scattering rate in agreement with predictions[19] in two dimensions which show a logarithmic divergence and corresponding enhancement of the electron-electron scattering rate. In high fields, measurements indicate that the correlation effects are also present and effective. Some difficulties relating to the absolute strength of these correlation effects remains. It is clear, however, that both localization and correlation effects have been observed and measured.

ACKNOWLEDGMENTS

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