Combined effect of disorder and interaction on the conductance of a one-dimensional fermion system

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It is shown that the conductance of an interacting one-dimensional system may be expressed in the form of a Landauer formula. An insulator–ideal-conductor transition is found as the interaction is varied at zero temperature.

Since the introduction of a scaling theory by Abrahams *et al.*[1] much progress has been made in understanding the properties of disordered electronic systems. In general, it is difficult to treat simultaneously electronic interactions and disorder. However, it is known that interactions play an essential role in the behavior of disordered systems in two and three dimensions.[2] In one dimension this is true *a fortiori* since neither interactions nor disorder can be simply treated in perturbation theory. In this Communication we discuss the effect of interactions on the conductance of a disordered system of electrons moving in one dimension under conditions in which a Peierls transition does not occur. First we derive from linear-response theory an exact expression for the conductance of a finite (length L) disordered system of interacting electrons at temperature T. The conductance has the form of the Landauer formula but the derivation is different from that given by Langreth and Abrahams[3] for the noninteracting case. Then we evaluate this formula in the weak scattering limit and find an insulator–ideal-conductor transition as the interaction between the electrons varies at zero temperature. For the case of spinless electrons, this transition was found previously in the frequency-dependent conductivity.[4] We propose that the unusual high conductivity of one-dimensional metals which do not show a Peierls transition reflects this enhanced conductivity.

We consider an infinite system of electrons in one dimension. The electron-electron interaction is parametrized by its $q \simeq 2k_F$ and $q \simeq 0$ matrix elements, g_1 and g_2 (see Sólyom's review article[5]). In a finite region the electrons are exposed to random impurity potentials $\zeta_\mu(x)$ which scatter the electrons with momentum transfer $2k_F$ (forward scattering does not affect the conductivity). The potentials couple to the $2k_F$ densities as follows

$$H_{\text{imp}} = \int_0^L dx \sum_{\mu=0}^{3} \sum_{s,s'} \psi_{1s}^\dagger(x)(\sigma^\mu)_{ss'} \times \psi_{2s'}(x)\zeta_\mu(x) + \text{H.c.} \quad , \tag{1}$$

where $\psi_{1,2s}^\dagger(x)$ denotes the creation operator for electrons with spin s and momentum $+k_F$, $-k_F$. σ^μ, $\mu=1,2,3$ are the Pauli matrices and $\sigma^0=1$. The $\zeta_\mu(x)$ are taken to be Gaussian random functions with white-noise correlations which are given by the concentration of the impurities times the square of their strength. To derive the formula for the conductance we drive a current $J(x,\omega)$ through the system. The resulting electric field $E(k,\omega)$ is given by

$$E(x,\omega) = \int dx' \rho(x,x';\omega) J(x',\omega) \quad , \tag{2}$$

where the resistivity ρ is defined as the analytic continuation $is \to \omega + i0$ of the inverse of the conductivity $\sigma(x,x';\omega \to is)$. The free part $[\zeta_\mu(x)=0]$ of σ is easily calculated from the density-density response function at small wave vector[5] and we write

$$\sigma(x,x';\omega) = (e^2/\pi)\gamma \exp(i\omega|x-x'|/v) + \sigma_{\text{imp}}(x,x';\omega) \quad , \tag{3}$$

where e is the electronic charge,

$$\gamma = [(2\pi v_F - 2g_2 + g_1)/(2\pi v_F + 2g_2 - g_1)]^{1/2} \quad ,$$

$v = v_F 2\gamma/(\gamma^2+1)$, and v_F is the Fermi velocity. The impurity part of the conductivity is a response function involving the generalized forces[6] $F(x) = [\hat{J}(x), H_{\text{imp}}]$ and need not be specified for the moment. We split ρ also into a free part and a part R coming from the impurities

$$\rho(x,x';\omega) = \left(\frac{e^2}{\pi}\gamma\right)^{-1} \left(\delta(x-x')\frac{\omega^2 + v^2\delta_{x'}^2}{2i\omega v} + R(x,x';\omega)\right) \quad . \tag{4}$$

We obtain from the condition that ρ equals $(\sigma)^{-1}$ the following integral equation for R

$$R(x,x';\omega) = f(x,x';\omega) + \int dx_1\, dx_2 f(x,x_1;\omega) \times \exp(i\omega|x_1-x_2|/v) R(x_2,x';\omega) \quad . \tag{5}$$

The kernel f is calculated from σ_{imp},

$$f(x,x';\omega)=\left(\frac{e^2}{\pi}\gamma\right)^{-1}\frac{1}{4i\omega v^2}\{\langle\langle F(x);F(x')\rangle\rangle+\langle[\hat{J}(x),F(x')]\rangle\}\ , \quad (6)$$

where double brackets denote response functions.[6] Now we make the current in (2) spatially homogeneous and then time independent. A current I gives rise to a voltage drop [use (4)]

$$U=\left(\frac{e^2}{\pi}\gamma\right)^{-1}\int dx \lim_{\omega\to 0}\int dx' R(x,x';\omega)I\ . \quad (7)$$

The solution R of (5) then gives the resistance which in turn determines the conductance. A further simplification arises from the zero-frequency limit. Because f vanishes if x or x' leaves the impurity region the integrals in (5) are finite. If the zero-frequency limit of $\int dx' f(x,x';\omega)$ exists (which is the case for $T\neq 0$, see below) we are allowed to set $\omega=0$ in (5) and arrive at our main result for the dimensionless conductance

$$g=\gamma g_0/(1-g_0)\ , \quad (8)$$

where

$$g_0=1-\int dx \lim_{\omega\to 0}\int dx' f(x,x';\omega)\ .$$

Apart from the prefactor γ, g has the form of the Landauer formula where g_0 plays the role of the transmission coefficient. It depends on the actual impurity configuration. We stress that we arrive at the exact formula (8) without use of a multichannel description of our interacting and temperature-dependent system. In the noninteracting case our result (8) [with (6) and (3)] is identical with that of Langreth and Abrahams.[3]

The calculation of the weak scattering limit of g_0 is now straightforward. First we note that the generalized force $F(x)$ is given by $2k_F$ density or spin-density operators only. Working to lowest order in the impurities we then take the $2k_F$ response functions of the pure system. In this approximation we have for g_0

$$g_0\cong 1+2\pi\gamma\sum_{\mu=0}^{3}\int_0^L dx\,dx'\,\zeta_\mu(x)\zeta_\mu^*(x')\lim_{\omega\to 0}\frac{1}{i\omega}[N_\mu(x-x';\omega)-N_\mu(x-x';0)]\ . \quad (9)$$

N_0 is the $2k_F$ charge-density response function, $N_1=N_2=N_3=\chi$ is the $2k_F$ spin-density response function of pure system. Averaging now $\ln g_0$, as should be done,[7] we obtain to lowest order (the averaged g_0 is again denoted by g_0)

$$g_0\simeq 1+2\pi\gamma\sum_{\mu=0}^{3}\frac{L}{l_\mu}\lim_{\omega\to 0}\frac{v^2}{i\omega}[N_\mu(0;\omega)-N_\mu(0;0)]\ , \quad (10)$$

where we have introduced the mean free paths l_μ belonging to the different scattering processes. Confining our attention to the region of the interaction $g_1\geqslant 0$, where the scaling theory of Ref. 5 is valid, we put in for N_μ the well-known power-law behavior as ω and T go to zero.[5] This yields

$$g_0\simeq 1-(L/\bar{l})(T/E_F)^{\gamma-1}c(g_1,g_2),\quad T\ll E_F\ , \quad (11)$$

where $c>0$ is a result which follows from the spectral representation of N_μ. $\bar{l}$ is the geometric mean of the mean free paths l_μ. To this order, impurity potential scattering (ζ_0) gives the same contribution as spin-flip scattering (ζ_1,ζ_2,ζ_3), since the charge and spin-density response functions diverge with the same power.

We discuss first the behavior of the system at $T=0$. Because $\int dx\int dx' f(x,x';\omega)$ is no longer finite when we set $\omega=0$ and $T=0$, we cannot directly use (11). But the leading singular behavior as L goes to infinity can be inferred from (11) if we set $T=T_{\min}(=v/L)$ as the smallest temperature for which (11) still holds true. An alternative derivation would be to set $T=0$ from the beginning but to give ω an imaginary part of the order of v/L. So we obtain at $T=0$

$$g_0\simeq 1-(\bar{l}k_F)^{-1}(k_FL)^{2-\gamma}c(g_1,g_2),\quad k_FL\gg 1\ . \quad (12)$$

For $\gamma<2$ the singular term is the leading term when $k_FL\gg 1$ and one scales to a strong coupling region as L becomes large. For $\gamma>2$ the singular term is no longer the leading one. Going through the zero-temperature calculation mentioned above we find a regular term which gives a value of $g_0(L=\infty)<1$. This in turn implies a finite conductance for $L=\infty$ and therefore an infinite conductivity.[8] This result agrees with a previous calculation for the spinless case[4] which gave a conductivity in this range of interactions, $\sigma(\omega)\sim ig_0/\omega$, where g_0 is the value $g_0(L=\infty)$.

If we express our results in terms of the β function[1] we have for $\gamma<2$,

$$\beta(g)=\frac{\partial\ln g}{\partial\ln L}\simeq-(2-\gamma)\frac{g+1}{g},\quad g\to\infty\ . \quad (13)$$

It is negative and the asymptotic value $\beta(\infty)$ depends continuously on the interaction parameter γ. In the noninteracting case, $\gamma=1$, we have $\beta(\infty)=-1$ in agreement with the known result.[1,7] The sign of the leading correction term $(-1/g)$ is negative in agreement with previous work but the value differs from the exact value $(-1/2g)$.[7] For $\gamma < 2$ the scaling behavior of β is the same as in the noninteracting case and we conclude that the system insulates in this range of interactions at $T=0$. For $\gamma > 2$ we have

$$\beta(g) \simeq -\kappa \frac{g-g_\infty}{g_\infty}, \quad \text{as } g \to g_\infty = g(L=\infty) \quad , \qquad (14)$$

where κ is the power of the next leading term in L^{-1} in g. The ideal conducting behavior (see above) manifests itself in an attractive fixed point of β.

Turning to the case $T > 0$ in this region $(\gamma > 2)$, where the weak coupling limit is valid, we find for the conductivity with the aid of (11)

$$\sigma = L\frac{e^2}{\pi} g \simeq \frac{e^2}{\pi} \bar{l} c(g_1,g_2)^{-1}(E_F/T)^{\gamma-1} \quad . \qquad (15)$$

For the spinless case, a corresponding formula was given by Luther and Peschel.[9] The impurity resistivity is strongly suppressed at low temperature. There is experimental evidence of such a suppression in one-dimensional metals which do not have a Peierls transition.[10] An alternative explanation of the anomalous resistivity, proposed by Schulz *et al.*,[11] is a large fluctuation regime below a high mean-field transition temperature $T_{\rm MF}$ to a BCS superconductor. We note, however, that in the model discussed here with $g_1 > 0$ then $T_{\rm MF}$ for a single chain is actually reduced to zero.[5] In the other regime of the interaction $(\gamma < 2)$ g scales to the strong coupling limit and we are *not* allowed to expand with respect to the impurities as is clear from the zero-temperature or noninteracting limits. Only for very small lengths $L \ll \bar{l}$ can we apply (15).

To summarize, at $T=0$ we have a transition from insulating behavior to an ideal conductor as the interaction parameter γ varies. At any finite temperature in the latter regime, however, we find finite conductivity in contrast to conventional superconductivity.

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