Infrared Faraday rotation in compensated nonpolar semiconductors

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The infrared Faraday rotation due to hot carriers in a compensated nonpolar semiconductor in the presence of shallow attractive traps is investigated. The calculations are carried out for a parabolic law of dispersion and a scalar effective mass. Numerical calculations are made for *n*-Ge at 4.2 K. The results are useful for the estimation of the degree of compensation in the material and the capture parameter of the trapping centers which control the lifetime of the nonequilibrium carriers.

In a semiconductor containing trapping centers, some of the hot-carrier effects such as the galvanomagnetic phenomena and the Faraday rotation of an electromagnetic wave traversing the material are affected not only by the heating of the free carriers but also by the change of free-carrier concentration. The galvanomagnetic characteristics of a covalent semiconductor in the presence of shallow attractive traps and under the condition when the free carrier loses momentum and energy, respectively, by scattering from a system of randomly oriented dipoles and acoustic phonons via deformation potential coupling have been studied elsewhere.[1] The purpose of the present report is to study the applied field dependence of the angle of Faraday rotation of an infrared signal traversing a semiconductor under the same conditions.

For the simple model of band structure, nonquantizing magnetic fields and low frequency corresponding to the infrared region of the spectrum, e.g., $\omega_c\tau \ll 1$ and $\omega\tau \ll 1$, the angle of Faraday rotation (θ) of the plane of polarization of a linearly polarized electromagnetic signal is proportional to $n\langle\tau^2\rangle$; ω_c being the cyclotron frequency, τ the dominant momentum relaxation time, ω the frequency of the electromagnetic wave, n the nonequilibrium concentration of the free carriers, and the angular bracket means an average over the energy distribution function.[2,3]

If the lifetime of the free carriers is much longer than all other relevant characteristic times, the stationary energy distribution of the carriers is set up only by the prevalent quasielastic relaxation processes. Moreover, the form of the distribution depends upon the band structure of the material.[2]

The energy of the free carriers in a covalent semiconductor at lattice temperature around 4.2 K considerably exceeds that of the lattice atoms when an external field as low as a few V/cm is applied.[4,5] At such low lattice temperatures neither optical phonon scattering in Ge nor intervalley scattering in Si need be considered since the characteristic temperatures of both the phonons are much higher.[6] Moreover, if the concentration of the free carriers does not exceed $10^{18}/\text{cm}^3$, the carrier-carrier collisions plays an insignificant role and hence can also be neglected. Thus the nonequilibrium distribution function of the free carriers in a covalent semiconductor at a lattice temperature $T=4.2$ K is established by the dominant processes of energy relaxation due to scattering by acoustic phonons and momentum relaxation due to neutral and ionized impurity atoms if $k_BT \gg m^*s^2\langle x\rangle$ and $l_a \gg l_I(x)$ where k_B is the Boltzmann constant, m^* the isotropic effective mass, s the acoustic velocity, and $x=\epsilon/k_BT$, the normalized energy. l_a and l_I are the mean free paths due to deformation potential scattering by acoustic phonons and the dominant impurity scattering mechanisms, respectively.

It is known from experimental observations[7] that in a compensated semiconductor, ionic pairs in the form of dipoles are formed and as a result, they are removed as Coulomb centers. Using the first Born approximation, the calculation of the isotropic momentum free path l_D yields[8,9]

$$l_D=\frac{3\hbar^2k_Dk_BT}{4\pi m^*e^4C_0N_D^{1/3}}x\equiv l_D^0x \quad , \tag{1}$$

where $\hbar=h/2\pi$, h being the Planck's constant, k_D is the dielectric constant, e is the electronic charge, and l_D^0 is the energy-independent factor of the free path. $C_0(=N_A/N_D)$ is the compensation ratio for an n-type sample, N_A and N_D being the concentrations of acceptor and donor impurities, respectively.

The dipole scattering can be made to dominate over that due to neutral impurity centers by suitably adjusting the degree of compensation.[9]

Though none of the covalent semiconductors possesses a parabolic band structure, one can assume a parabolic dispersion law in the region of liquid-He temperature and for a field of a few V/cm.[1] For a carrier system in the presence of a weak field defined as $E<(6m^*s^2k_BT/e^2l_D^{02})^{1/2}$ and a magnetic field

$H < c(2m^*k_BT)^{1/2}/el_D^0$, the isotropic part of the distribution function is given by[10]

$$f_0(x) \sim \left[1+\frac{\alpha_D l_a}{l_D^0}x\right]\exp(-x)\,. \tag{2}$$

On the other hand, if the electric field is so strong as to satisfy the condition $E > (6m^*k_BTs^2/e^2l_al_D^0)^{1/2}$, one can obtain an analytical expression for $f_0(x)$ in the presence of a magnetic field $H < C(2m^*k_BT)^{1/2}/el_D^0\langle x\rangle$ as[10]

$$f_0(x) \sim \left[1-\frac{\eta_D\sin^2\beta}{2\alpha_D l_a/l_D^0}x^2\right]\exp\left[-\frac{x}{\alpha_D l_a/l_D^0}\right]\,, \tag{3}$$

where c is the velocity of light, α_D and η_D are the squares of the normalized electric and magnetic fields and are given by $\alpha_D=(el_D^0E)^2/(6m^*k_BT)$ and $\eta_D=(el_D^0H)^2/(2m^*C^2k_BT)$, respectively; β is the angle between the magnetic field ($\vec{H}$) and the heating electric field ($\vec{E}$).

The problem of calculating the carrier concentration in the presence of the attractive traps and at low lattice temperatures actually leads to calculating the thermal recombination and the impact ionization coefficients.[11] The recombination coefficient for shallow attractive traps is calculated with the help of the Lax cascade theory.[12] The ionization coefficient is calculated by assuming that the effective ionization cross section is independent of carrier energy, i.e., $\sigma_I(x)=\sigma_I^0$. Such an assumption is indeed justified when the distribution function is a rapidly varying function of energy and the impact ionization coefficient is, in general, mainly determined by the distribution function of the hot carriers, and is not too sensitive to the actual form of the energy dependence of the ionization cross section.[11] Moreover, it should be mentioned here that such assumptions are known to give results of the low-temperature breakdown field which are in satisfactory agreement with experimental data. Thus one can obtain

$$\frac{\theta}{\theta_0}=1+\frac{\alpha_D l_a}{l_D^0}\left[1-f\left(\frac{\delta}{\nu}\right)\right] \tag{4}$$

and

$$\frac{\theta}{\theta_0}=\frac{N_D(1-C_0)}{n_0 b}\left\{1-5ab^{-2}-\frac{F_2f_1}{(1-C_0)}\left[a\left(F_1-\frac{35}{4}\right)+a^2b^{-2}\left(\frac{75}{4}-\frac{5}{4}F_1\right)-a^2\left(F_1\frac{f_2}{f_1}-\frac{35}{4}\frac{f_2}{f_1}\right)-a^3b^{-4}\left(\frac{75}{4}F_1\right)\right.\right.$$
$$\left.\left.-a^3b^{-2}\left(\frac{45}{2}\frac{f_2}{f_1}-5\frac{f_2}{f_1}F_1\right)-ab^2\left(\frac{f_2}{f_1}\right)+a^4b^{-4}\left(\frac{75}{4}F_1\frac{f_2}{f_1}\right)+b^2\right]\right\} \tag{5}$$

in the presence of weak and strong fields, respectively. Here

$$f\left(\frac{\delta}{\nu}\right)=\frac{1+\frac{3}{2}\exp(\delta/\nu)E_i(-\delta/\nu)+(\delta/\nu)\exp(\delta/\nu)E_i(-\delta/\nu)}{-\exp(\delta/\nu)E_i(-\delta/\nu)}\,,$$

$$F_1=\frac{zW_{3/2,2}(z)}{W_{1/2,1}(z)}\,,\quad F_2=\frac{2\times4^5}{3\nu^4}\frac{\sigma_1}{\sigma_I^0}\frac{\exp(z/2)}{z^{1/2}W_{1/2,1}(z)}\,,$$

$$f_1=-\exp(b\delta/\nu)E_i(-b\delta/\nu)\,,$$

$$f_2=-\left(\frac{\delta}{\nu}\right)^2\exp\left(\frac{b\delta}{\nu}\right)E_i\left(-\frac{b\delta}{\nu}\right)+\tfrac{15}{4}b^{-2}\exp\left(\frac{b\delta}{\nu}\right)E_i\left(-\frac{b\delta}{\nu}\right)+\sum_{m=1}^{2}(m-1)!\left(-\frac{\delta}{\nu}\right)^{2-m}b^{-m}\,,$$

$$b=1\Big/\left(\frac{\alpha_D l_a}{l_D^0}\right)\,,\quad z=\frac{bI}{k_BT}\,,\quad a=\frac{b\eta_D\sin^2\beta}{2}\,,\quad \nu=\frac{2k_BT}{m^*s^2}\,.$$

The subscript "0" stands for the condition of thermodynamic equilibrium. As introduced by Lax[12], δ plays the role of a binding energy, and σ_1 is a constant with the dimension of a cross section. I is the ionization energy. $E_i(x)$ and $W(z)$ are the integral exponential[13] and the Whittakar functions,[14] respectively.

Equations (4) and (5) give the applied field dependence of (θ) only in the fixed field regime where the applied field is identical with the heating field, i.e., $\vec{E}_x=\vec{E}$ and β is identical with the angle χ between the magnetic field $\vec{H}$ and the applied electric field $\vec{E}_x$ since the Hall field is zero in this case. In the fixed current regime $\vec{E}\cong\vec{E}_x$ for weakly heated system, but in the presence of a strong heating field one can show that $\vec{E}=\vec{E}_x(1+\tan^2\theta_H)^{1/2}$ and $\beta=\cos^{-1}(\cos\theta_H\cos x)$, where θ_H is the Hall angle.

From Eq. (4) we see that the angle of Faraday ro-

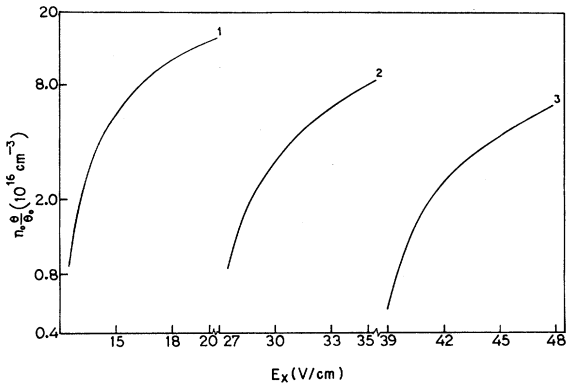

FIG. 1. Dependence of the angle of Faraday rotation upon the applied electric field under the fixed field condition for different values of the compensation ratio when $\chi = 45°$. Curves 1–3 are for $C_0 = 0.01$, 0.05, and 0.1, respectively.

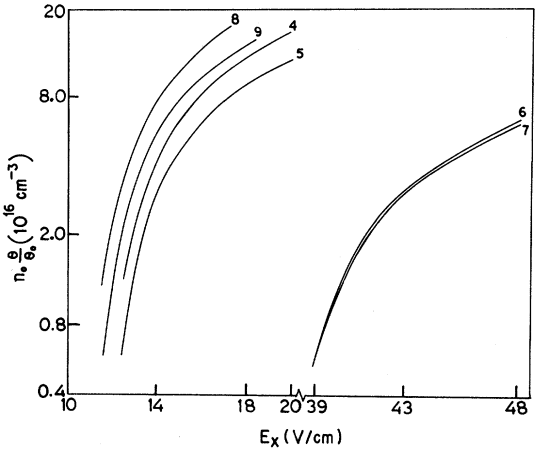

FIG. 2. Dependence of the angle of Faraday rotation upon applied electric field for different values of χ and C_0 and for different experimental regimes. Curves 4–7 are drawn for the fixed field regime; 4 and 5 for $C_0 = 0.01$ and $\chi = 20°$ and 70°, respectively; 6 and 7 for $C_0 = 0.1$ and $\chi = 20°$ and 70°, respectively. Curves 8 and 9 are drawn for the fixed current regime at $C_0 = 0.01$ and for $\chi = 20°$ and 70°, respectively.

tation in the presence of weak electric fields increases with $\vec{E}_x$ at very slow rate. At a fixed value of applied electric field, if the compensation ratio C_0 is increased, the angle of rotation increases linearly. However, the capture parameter σ_1/σ_l^0 has no influence on (θ).

For the strongly heated system, the dependence of the angle Faraday rotation on the applied field is very involved. Moreover, θ depends on the ionization energy I in a very complicated manner. However, for shallow trap satisfying the condition $I \ll (\alpha_D l_a k_B T)/l_D^0$, θ becomes independent of I. The angle of Faraday rotation is numerically calculated for a sample of n-Ge at 4.2 K. The trapping centers are assumed to be shallow enough as to satisfy the above inequality and the parameters used in the computation are

$m^*/m_0 = 0.12$; $S = 5.4 \times 10^5$ cm/sec; $k_D = 15.8$; $l_a = 6.72 \times 10^{-4}$ cm; $\delta = 8.0$;

$\sigma_1 = 2.13 \times 10^{-9}$ cm^2; $\sigma_l^0 = 1.05 \times 10^{-14}$ cm^2; $N_D = 10^{16}$/cm^3; $H = 500$ Oe.

It is to be mentioned here that the effect of magnetic quantization on the carrier system of interest here need not be considered for magnetic fields less than a few thousands of Oe.

From Fig. 1 it is seen that the angle of rotation in the fixed field regime at first increases very fast with $\vec{E}_x$, then in the region of concentration saturation the rate of increase slows down and approaches the quadratic relation $\theta \sim E_x^2$, where the effect of trapping is no longer felt and the rotation of the angle of polarization of the infrared signal is due mainly to the heating of the free carriers. The field at which θ approaches the quadratic relation is higher the higher the degree of compensation. From Fig. 2 it is seen that the nature of variation of θ with E_x in the fixed current regime is nearly identical with that of the fixed field regime. The θ-E_x curve in the fixed current regime always lies above those in the fixed field regime since the heating field $\vec{E}$ in the fixed current regime is always greater than the applied field. A weak dependence of θ upon χ is seen here at higher values of the applied electric field and at lower values of the degree of compensation. The rate of change $\partial\theta/\partial E_x$ at any field is determined by the degree of compensation and by the trapping parameter σ_1/σ_l^0.

In the absence of any experimental data[2,6,15] our results could not be checked. However, the results are useful, since from a comparison of the results with those of an experiment performed within the limits of the approximations made here, one can estimate the degree of compensation and also the capture parameter σ_1/σ_l^0 of the traps which control the lifetime of the carriers.

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