Theory of coherent propagation of a light wave in semiconductors. I

Gan Zi-zhao
Department of Physics, Beijing University, Beijing, China

Yang Guo-zhen
Institute of Physics, Academia Sinica, Beijing, China
(Received 11 December 1981)

This paper contains the first part of a theory of coherent propagation of a light wave in semiconductors. In the framework of the single-electron band theory and with the neglect of the interaction between electrons, the equations describing the interband transitions induced by coherent light in semiconductors are similar to the Bloch equations for an inhomogeneously broadened two-level system in a light field. Special types of multiphoton processes which are the results of a combination of the interband and intraband transitions are predicted.

I. INTRODUCTION

When an intense coherent light beam enters a semiconductor in which resonant transitions can be induced, several processes will occur: (1) creation and recombination of electron-hole pairs induced by the coherent light, (2) the interaction between the electrons and the holes, (3) the interaction of electrons and holes with phonon and other imperfections in the crystal, and (4) recombination of the electron-hole pairs through spontaneous emission or other recombination processes. If the intensity of the light is sufficiently high, the rate of process (1) is greater than the rates of the other processes. In such a case, the coherence between the excited state of the semiconductor and the light wave becomes important, and a number of phenomena that are typical of coherent propagation will occur. We are already familiar with these phenomena in gaseous media as well as in solids. The observation of effects such as self-induced transparency and the saturation of absorption in interband transition have been reported.[1–4] Some theoretical work has also been reported in the literature.[5–9] The purpose of our work is to develop an adequate theory for dealing with such phenomena.

This paper discusses the first part of our work. Here we investigate the problem only in the framework of the single-electron band theory and neglect the interaction between the electrons. At first sight, this system is equivalent to an inhomogeneously broadened two-level system.[5] But the intense light field induces not only interband transitions, but also intraband transitions of electrons and holes in the conduction and valence bands, respectively. Hence our system is different from an ordinary inhomogeneously broadened two-level system. In our treatment of this problem, we have adopted the "space-translation approximation," i.e., the approximate steady states of the carriers moving within their respective bands under the action of the light field are taken as the base for treating the interband transitions. This is affected by a transformation, which renders the system formally analogous to an inhomogeneously broadened two-level system; at the same time, however, a special type of multiphoton process will be seen to occur.

In the second part of our work, we shall take into account the interaction between electrons.[10] From the point of view of the coherence between the excited state and the light wave, the interaction between electrons can be partitioned into a part that does not change the total momentum of the relevant electron-hole pair and another part that changes the total momentum. In fact, the former is the interaction of the electron with its hole partner which gives rise to the binding of the exciton state, and the latter represents collisions between the electrons and holes which destroys the coherence of the process. When we consider coherent propagation, it is reasonable to focus first on the first type of process, i.e., to assume that the result of light excitation is an exciton state, and to take into account the relaxation processes of these excitons at a later stage. Because the intense light wave can generate a high density of excitons, we must treat the electrons and holes, which make up the excitons, as "fermions." This treatment differs from usual ones that represent low density excitons as "bosons."[11] We have introduced exciton coherent states to describe the process of excitation of excitons by a light wave, and obtained a set of nonlinear equations. These equations can naturally account for the saturation of light absorption of the excitons and for the shift of the exciton line under intense illumination. Within certain approximations, the coherent excitation of discrete exciton lines can be described by a Bloch equation analogous to that for two-level systems in a light field. Hence, the concepts and methods for treating the near-resonant coherent excitation of two-level systems can be used to describe the near-resonant coherent excitation of exciton lines; the corresponding density of the equivalent "two-level atoms" is determined by the properties of the wave function of the exciton state.

In the third part of our work, we shall analyze the coherent propagation of a light pulse in this system.[10] To take into account the propagation of an electromagnetic wave and of the excited state in the system, we have derived Maxwell-Bloch equations describing this process. After some approximations, we can cast these equations in a standard form which can be solved by the "inverse scattering method." From that, we obtained the theoretical form describing the shaping of self-induced transparency pulses in the system.

II. THE INTERACTION BETWEEN THE CARRIER AND ELECTROMAGNETIC WAVE

We investigate the interaction between the semiconductor and a light wave in the self-consistent single-electron approximation. The Schrödinger equation for the electron is

$$i\hbar\frac{\partial\psi}{\partial t}=\left[\frac{1}{2m}\left[\vec{P}+\frac{e}{c}\vec{A}\right]^2+V\right]\psi\ , \tag{1}$$

where V is the periodic potential for an electron moving in the crystal lattice, and $\vec{A}$ is the vector potential for the light wave, which is assumed to be a plane wave,

$$\vec{A}=\vec{A}_0\sin(\omega t-\vec{q}\cdot\vec{r}+\varphi)\ . \tag{2}$$

As $\vec{A}$ is weaker than V, we can take the Bloch wave functions of the periodic potential as the base to treat this problem. For the sake of simplicity, we consider a simple two-band model:

$$E_c(\vec{k})=\tfrac{1}{2}E_g+\frac{\hbar^2}{2m_e}\vec{k}^2\ ,$$
$$E_v(\vec{k})=-\tfrac{1}{2}E_g-\frac{\hbar^2}{2m_h}\vec{k}^2\ , \tag{3}$$

where the indices c and v refer, respectively, to the conduction and valence band, m_e, m_h are the electrons' and holes' effective masses, and E_g is the forbidden gap width. In the dipole approximation, and neglecting spatial variation of the light field, we can write (1) in the formalism of second quantization as follows:

$$i\hbar\frac{\partial\psi}{\partial t}=(H_0+H_1+H_2)\psi\ , \tag{4}$$

where

$$H_0=\sum_{\vec{k}}E_c(\vec{k})a^\dagger_{c\vec{k}}a_{c\vec{k}}+E_v(\vec{k})\times(a^\dagger_{v\vec{k}}a_{v\vec{k}}-1)\ . \tag{5}$$

$a_{c\vec{k}}, a^\dagger_{c\vec{k}}, a_{v\vec{k}}, a^\dagger_{v\vec{k}}$ are the annihilation and creation operators for electrons in the conduction and valence band, respectively. With a small change of notations, we introduce the annihilation and creation operators for electrons and holes as follows:

$$a^\dagger_{\vec{k}}=a^\dagger_{c\vec{k}},\ a_{\vec{k}}=a_{c\vec{k}}$$

and

$$b_{-\vec{k}}=a^\dagger_{v\vec{k}},\ b^\dagger_{-\vec{k}}=a_{v\vec{k}}\ .$$

Thus, H_0 becomes

$$H_0=\sum_{\vec{k}}E_c(\vec{k})a^\dagger_{\vec{k}}a_{\vec{k}}-E_v(\vec{k})b^\dagger_{\vec{k}}b_{-\vec{k}}\ . \tag{6}$$

H_1 in (4) represents the interaction with the light wave responsible for interband effects:

$$H_1=\sum_{\vec{k}}\frac{e}{mc}\vec{A}\cdot\vec{P}_{cv}(\vec{k})\times(a^\dagger_{\vec{k}}b^\dagger_{-\vec{k}}+b_{-\vec{k}}a_{\vec{k}})\ , \tag{7}$$

where $\vec{P}_{cv}(\vec{k})$ is the interband matrix element of the operator $\vec{P}$. By a suitable choice of the phases of the Bloch wave functions, $\vec{A}\cdot\vec{P}_{cv}(\vec{k})$ can be made real. The two terms in (7) correspond to the generation and recombination of electron-hole pairs, respectively.

H_2 in (4) represents that part of the interaction which is responsible for intraband effects:

$$H_2=\sum_{\vec{k}}\left[\frac{e}{c}\vec{A}\cdot\frac{1}{\hbar}\nabla_{\vec{k}}E_c(\vec{k})a^{\dagger}_{\vec{k}}a_{\vec{k}}-\frac{e}{c}\vec{A}\cdot\frac{1}{\hbar}\nabla_{\vec{k}}E_v(\vec{k})b^{\dagger}_{-\vec{k}}b_{-\vec{k}}\right] . \quad (8)$$

If we take the dipole approximation and neglect the spatial variation of $\vec{A}$, the $\vec{A}^2$ term in (1) can be eliminated by introducing a common phase factor which is identical for all states. Hence, this term is not included in (4).

By introducing the canonical transformation

$$S=\prod_{\vec{k}}S_{\vec{k}} ,$$

$$S_{\vec{k}}=\exp\left[-\frac{i}{\hbar}\int^t\frac{e}{c}\vec{A}(t')\cdot\frac{1}{\hbar}\nabla_{\vec{k}}E_c(\vec{k})dt'a^{\dagger}_{\vec{k}}a_{\vec{k}}+\frac{i}{\hbar}\int^t\frac{e}{c}\vec{A}(t')\cdot\frac{1}{\hbar}\nabla_{\vec{k}}E_v(\vec{k})dt'b^{\dagger}_{-\vec{k}}b_{-\vec{k}}\right] . \quad (9)$$

the intraband term can be eliminated from (4), giving thus

$$i\hbar\frac{\partial\psi_s}{\partial t}=H_s\psi_s . \quad (10)$$

In Eq. (10) we have

$$\psi_s=S^{-1}\psi$$

and

$$H_s=\sum_{\vec{k}}[E_c(\vec{k})a^{\dagger}_{\vec{k}}a_{\vec{k}}-E_v(\vec{k})b^{\dagger}_{-\vec{k}}b_{-\vec{k}}]+\sum_{\vec{k}}\frac{e}{mc}\vec{A}\cdot\vec{P}_{cv}(\vec{k})\{\exp[i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]a^{\dagger}_{\vec{k}}b^{\dagger}_{-\vec{k}}+\exp[-i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]b_{-\vec{k}}a_{\vec{k}}\} , \quad (11)$$

where

$$\theta_{\vec{k}}=\frac{1}{\hbar}\int^t\frac{e}{c}\vec{A}(t')\cdot\frac{1}{\hbar}\nabla_{\vec{k}}E_c(\vec{k})dt',\quad \varphi_{-\vec{k}}=\frac{1}{\hbar}\int^t\frac{e}{c}\vec{A}(t')\cdot\left[-\frac{1}{\hbar}\right]\nabla_{\vec{k}}E_v(\vec{k})dt' . \quad (12)$$

For the sake of simplicity, in the following we leave out the subscript s of ψ_s.

The physical meaning of the transformation (9) is easily understood. For the electron, it corresponds to a transformation from a base $e^{i\vec{k}\cdot\vec{r}}U_{c\vec{k}}$ to a new base

$$\exp\left[i\vec{k}\cdot\left[\vec{r}+\frac{e}{m_e c}\right]\int^t\vec{A}(t')dt'\right]U_{c\vec{k}}(\vec{r}) .$$

If we neglect the spatial dependence of $\vec{A}$, it is just the wave function for the steady-state motion of the electron in the conduction band under the action of the electromagnetic wave. For the hole, the situation is similar. Thus we may designate the transformation S as the "space translation approximation."[12]

III. INTRABAND-INTERBAND MULTIPHOTON TRANSITION

The explicit expression of $\exp[i(\theta_{\vec{k}}+\psi_{-\vec{k}})]$ is

$$\exp[i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]=\exp\left[-i\frac{e}{\omega c}\vec{A}_0\cdot\left[\frac{1}{m_e}+\frac{1}{m_h}\right]\vec{k}\cos(\omega t+\theta)\right]$$

and

$$\exp[-i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]=\exp\left[i\frac{e}{\omega c}\vec{A}_0\cdot\left[\frac{1}{m_e}+\frac{1}{m_h}\right]\vec{k}\cos(\omega t+\theta)\right] , \quad (13)$$

where

$$\theta=\varphi-\vec{q}\cdot\vec{r} .$$

If we expand (13), we obtain

$$\exp[-i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]=\exp[i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]^*=\sum_{m=-\infty}^{+\infty} i^m J_m(\eta)\exp[im(\omega t+\theta)] , \quad (14)$$

where

$$\eta=\frac{e}{\omega c}\vec{A}_0\cdot\left[\frac{1}{m_e}+\frac{1}{m_h}\right]\vec{k}$$

and where J_m denotes the m-order Bessel function. Obviously, the interband transition represented by (10) is different from the interband term in (4); the former includes various harmonic terms of the fundamental frequency of the light wave. Hence, provided $E_c(\vec{k})-E_v(\vec{k})\approx nh\omega$ (n is an arbitrary integer), the interband transition can occur. This is a n-quantum transition. This is an intraband-interband multiphoton transition, which is distinct from the usual high-order interband transitions. Some papers have already discussed this type of process.[13–16] Of course, if we take into account the complex structure of the band in the semiconductors, the expressions of this transition will become complicated, but the character of the process will still be represented by (14).

IV. THE BLOCH EQUATIONS FOR INTERBAND TRANSITIONS

We shall assume the wave function of the system to have the following form:

$$\psi=\prod_{\vec{k}}[\alpha_{\vec{k}}(t)+\beta_{\vec{k}}(t)a^\dagger_{\vec{k}}b^\dagger_{-\vec{k}}]\psi_0 , \quad (15)$$

where ψ_0 is the wave function of ground state (the conduction band is empty and the valence band is completely filled); $|\beta_{\vec{k}}|^2$ represents the probability of finding the electron in the state having wave vector $\vec{k}$ in the conduction band and a hole in the state having wave vector $\vec{k}$ in the valence band. In other words, $|\beta_{\vec{k}}|^2$ is the expectation value of the operators $a^\dagger_{\vec{k}}a_{\vec{k}}$ and $b^\dagger_{-\vec{k}}b_{-\vec{k}}$. Similarly, $\alpha^*_{\vec{k}}\beta_{\vec{k}}$ is the expectation value of $b_{-\vec{k}}a_{\vec{k}}$ and $\alpha_{\vec{k}}\beta^*_{\vec{k}}$ of $a^\dagger_{\vec{k}}b^\dagger_{-\vec{k}}$.

On substituting (15) into (10), we obtain the following equations for $\alpha_{\vec{k}},\beta_{\vec{k}}$:

$$\begin{aligned} i\hbar\frac{d}{dt}\alpha_{\vec{k}}&=\frac{e}{mc}\vec{A}\cdot\vec{P}_{cv}(\vec{k})\exp[-i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]\beta_{\vec{k}} ,\\ i\hbar\frac{d}{dt}\beta_{\vec{k}}&=[E_c(\vec{k})-E_v(\vec{k})]\beta_{\vec{k}}\\ &\quad+\frac{e}{mc}\vec{A}\cdot\vec{P}_{cv}(\vec{k})\exp[i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]\alpha_{\vec{k}} . \end{aligned} \quad (16)$$

After introducing the notations

$$\omega_{\vec{k}}=\frac{1}{\hbar}[E_c(\vec{k})-E_v(\vec{k})] ,$$

$$\Omega_{\vec{k}}=\frac{e}{2mc\hbar}\vec{A}_0\cdot\vec{P}_{cv}(\vec{k}) ,$$

and

$$\tilde{\alpha}_{\vec{k}}\equiv\alpha_{\vec{k}},\ \ \tilde{\beta}_{\vec{k}}=\beta_{\vec{k}}e^{i\omega t} ,$$

Eq. (16) can be rewritten in the form

$$\begin{aligned} \frac{d}{dt}\tilde{\alpha}_{\vec{k}}&=-\Omega_{\vec{k}}\{\exp[i(\omega-\omega_{\vec{k}})t+i\theta-i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]-\exp[-i(\omega+\omega_{\vec{k}})t-i\theta-i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]\}\beta_{\vec{k}} ,\\ \frac{d\tilde{\beta}_{\vec{k}}}{dt}&=-\Omega_{\vec{k}}\{\exp[i(\omega+\omega_{\vec{k}})t+i\theta+i(\theta_{\vec{k}}+\varphi_{-\vec{k}})]-\exp[-i(\omega-\omega_{\vec{k}})t-i\theta+i(\theta_{\vec{k}}+\varphi_{\vec{k}})]\}\alpha_{\vec{k}} . \end{aligned} \quad (17)$$

With the help of Eq. (14), the right-hand side of (17) can be represented as a sum of various harmonic terms. For example, one has

$$\begin{aligned} \frac{d\tilde{\alpha}_{\vec{k}}}{dt}=-\Omega_{\vec{k}}\sum_{m=-\infty}^{+\infty} i^m J_m(\eta)(&\exp\{i[(m+1)\omega-\omega_{\vec{k}}]t+i(m+1)\theta\}\\ &-\exp\{i[(m-1)\omega+\omega_{\vec{k}}]t+i(m-1)\theta\})\tilde{\beta}_{\vec{k}} . \end{aligned}$$

If one of the harmonic terms satisfies the condition $(m+1)\omega-\omega_{\vec{k}}\approx 0$, the contribution of this term will clearly be the largest, and the rest of the terms can be neglected; this is equivalent to the rotating-wave approximation in magnetic resonance.[17] Physically, this approximation corresponds to taking into account only the resonant $(m+1)$-photon transition. After introducing the notations

$$\Delta\omega_{\vec{k}}=(m+1)\omega-\omega_{\vec{k}}\ ,\quad G_m=i^m J_m(\eta)\ ,$$

Eq. (17) can be rewritten approximately in the form

$$\begin{aligned}\frac{d}{dt}\tilde{\alpha}_{\vec{k}}&=-\Omega_{\vec{k}}(G_m-G_{m+2})\exp\{i[\Delta\omega_{\vec{k}}t+(m+1)\theta]\}\tilde{\beta}_{\vec{k}}\ ,\\ \frac{d}{dt}\tilde{\beta}_{\vec{k}}&=\Omega_{\vec{k}}(G_m^*-G_{m+2}^*)\exp\{-i[\Delta\omega_{\vec{k}}t+(m+1)\theta]\}\tilde{\alpha}_{\vec{k}}\ .\end{aligned}\tag{18}$$

Equation (18) describes the creation of an electron-hole pair by the absorption of the $(m+1)$ photon and the recombination of an electron-hole pair by the emission of the $(m+1)$ photon. Formally, they are entirely analogous to the equation for a two-level system in a near-resonant light field. We can treat the system in analogy with a spin system in an external field; thus we introduce

$$\begin{aligned}m_{\vec{k},x}&=\tfrac{1}{2}(\tilde{\alpha}_{\vec{k}}\tilde{\beta}^*_{\vec{k}}+\tilde{\alpha}^*_{\vec{k}}\tilde{\beta}_{\vec{k}})\ ,\\ m_{\vec{k},y}&=\tfrac{1}{2}i(\tilde{\alpha}_{\vec{k}}\tilde{\beta}^*_{\vec{k}}-\tilde{\alpha}^*_{\vec{k}}\tilde{\beta}_{\vec{k}})\ ,\\ m_{\vec{k},z}&=\tfrac{1}{2}(|\tilde{\beta}_{\vec{k}}|^2-|\tilde{\alpha}_{\vec{k}}|^2)\ ,\end{aligned}\tag{19a}$$

and

$$\begin{aligned}\mu_{\vec{k}}&=\Omega_{\vec{k}}|G_m-G_{m+2}|\ ,\\ \Phi&=-(m+1)\theta-\Delta\ ,\end{aligned}\tag{19b}$$

where Δ is defined by

$$G_m-G_{m+2}=|G_m-G_{m+2}|e^{i\Delta}\ .$$

With the use of (19), Eq. (18) can be written in the form

$$\frac{d}{dt}\vec{m}_{\vec{k}}=\vec{m}_{\vec{k}}\times\vec{h}_{\vec{k}}\ ,\tag{20}$$

where

$$\vec{m}_{\vec{k}}=(m_{\vec{k},x},m_{\vec{k},y},m_{\vec{k},z})$$

and the components of $\vec{h}_{\vec{k}}$ are given by

$$\begin{aligned}h_{\vec{k},x}&=2\mu_{\vec{k}}\sin(\Delta\omega_{\vec{k}}t-\Phi)\ ,\\ h_{\vec{k},y}&=2\mu_{\vec{k}}\cos(\Delta\omega_{\vec{k}}t-\Phi)\ ,\\ h_{\vec{k},z}&=0\ .\end{aligned}\tag{21}$$

Equation (20) is formally identical to the equation of motion for a magnetic moment precessing in an external field which is rotating with an angular frequency $-\Delta\omega_{\vec{k}}+(d\Phi/dt)$. If the amplitude A_0 and the phase θ of light wave are independent of time, we can transform to a rotating coordinate system with frequency $-\Delta\omega_{\vec{k}}$. In this case, (20) can be transformed into the precession equation for a constant field. The solution in such a case is well known.[17,18] The frequency of precession (Rabi frequency) is

$$\omega_k=[(\Delta\omega_{\vec{k}})^2+4\mu^2_{\vec{k}}]^{1/2}\ .\tag{22}$$

This just represents the frequency of the interband transition back and forth between the two bands. If the intensity of the light is of the order of 10 $MW/cm^2\sim100\ MW/cm^2$, typical values of $2|\mu_{\vec{E}}|$ are $10^{12}\sim10^{13}\ sec^{-1}$. Hence, as emphasized in the introduction, when the intensity of light is high enough, the rate of the coherent excitation process ($\sim\omega_k$) can become much larger than the rates of all the other main processes, and the coherent propagation phenomena will become apparent. If the incident light is a coherent laser pulse, some transient phenomena (self-induced transparency, photon echos, etc.) can occur. Even if the incident light is not very intense, but the frequency of the light is near resonant with the interband transition, and if the relaxation time of the system is longer, some coherent propagating phenomena can also occur. This problem will be discussed in the third paper of this series.

V. RELAXATION TIME

We introduce the definitions

$$\begin{aligned}m_{\vec{k},x}&=\tfrac{1}{2}(\langle a^\dagger_{\vec{k}}b^\dagger_{-\vec{k}}\rangle e^{-i\omega_{\vec{k}}t}+\langle b_{-\vec{k}}a_{\vec{k}}\rangle e^{i\omega_{\vec{k}}t})\ ,\\ m_{\vec{k},y}&=\tfrac{1}{2}i(\langle a^\dagger_{\vec{k}}b^\dagger_{-\vec{k}}\rangle e^{-i\omega_{\vec{k}}t}-\langle b_{-\vec{k}}a_{\vec{k}}\rangle e^{i\omega_{\vec{k}}t})\ ,\\ m_{\vec{k},z}&=\tfrac{1}{2}(\langle a^\dagger_{\vec{k}}a_{\vec{k}}\rangle+\langle b^\dagger_{-\vec{k}}b_{-\vec{k}}\rangle-1)\ ,\end{aligned}\tag{23}$$

where $\langle\ \rangle$ represents the expectation value of an operator. From the Hamiltonian (11), we can derive Eq. (20) directly from the equations of motion of the operators $a^\dagger_{\vec{k}}a_{\vec{k}}$, $b^\dagger_{-\vec{k}}b_{-\vec{k}}$, and $b_{-\vec{k}}a_{\vec{k}}$, without using the form of the wave function (15).

Now we must consider the processes that were mentioned in the Introduction and that are not included in Eq. (11). They are the interaction between electrons (holes) and phonons, and crystal imperfections, the collisions between electrons and holes, the spontaneous emission and other recombination processes, etc. Of course, in this case, the equations of motion of the operators become very complicated. But because these processes (except for the interaction between an electron and its hole partner) have certain random and incoherent character, their effect is to induce relaxation for the coherent excitation of the system. Therefore, we can understand the brackets $\langle\ \rangle$ in (23) as ensemble averages on a statistical ensemble. Under some very general assumption about the statistical properties of these processes, the relaxation can be characterized by a suitable relaxation time. Phenomenologically, Eq. (20) can be modified as follows:

$$\frac{d}{dt}\vec{m}_{\vec{k}}=\vec{m}_{\vec{k}}\times\vec{h}_{\vec{k}}-\hat{R}_{\vec{k}}(\vec{m}_{\vec{k}}-\vec{m}_{\vec{k},0})\ , \qquad (24)$$

where $\vec{m}_{\vec{k},0}$ is the thermal equilibrium value of $\vec{m}_{\vec{k}}$,

$$\vec{m}_{\vec{k},0}=\begin{bmatrix}0\\0\\-\frac{1}{2}\end{bmatrix}\ , \qquad (25)$$

$\hat{R}_{\vec{k}}$ is the so-called "relaxation time matrix",

$$\hat{R}_{\vec{k}}=\begin{bmatrix}1/\tau_{t\vec{k}} & 0 & 0\\ 0 & 1/\tau_{t\vec{k}} & 0\\ 0 & 0 & 1/\tau_{l\vec{k}}\end{bmatrix}\ , \qquad (26)$$

$\tau_{t,\vec{k}}$ is called the transverse relaxation time, and $\tau_{l,\vec{k}}$ is called the longitudinal relaxation time.

The physical meaning of the relaxation time can be explained as follows: First we introduce the wave vector of the light wave into the preceeding discussion. More precisely, we introduce the operators $a^\dagger_{\vec{k}+\vec{q}}b^\dagger_{-\vec{k}}$ and $b_{-\vec{k}}a_{\vec{k}+\vec{q}}$ into the Hamiltonian (11) instead of $a^\dagger_{\vec{k}}b^\dagger_{-\vec{k}}$ and $b_{-\vec{k}}a_{\vec{k}}$, and we modify the wave function (15) as

$$\psi=\prod_{\vec{k}}(\alpha_{\vec{k}}+\beta_{\vec{k}}a^\dagger_{\vec{k}+\vec{q}}b^\dagger_{-\vec{k}})\psi_0\ . \qquad (27)$$

The frequency ω_k in (16) must be modified as follows:

$$\omega_{\vec{k}}=\frac{1}{\hbar}[E_c(\vec{k}+\vec{q})-E_v(\vec{k})]\ , \qquad (28)$$

and the same modifications must be applied to (20)−(23). Obviously, after these changes, the preceding description is also correct.

Thus, we can understand the wave function (27) to be the electron-hole polarization wave, which is excited by a light field in the semiconductor. Specifically, an electron suffers a change from the state of the valence band with wave vector $\vec{k}$ to the state described by the wave function $\alpha_{\vec{k}}\psi_{v,\vec{k}}+\beta_{\vec{k}}\psi_{v,\vec{k}+\vec{q}}$. The functions $\alpha_{\vec{k}}\beta^*_{\vec{k}}$ and $\alpha^*_{\vec{k}}\beta_{\vec{k}}$ [or $(m_{\vec{k},x}-im_{\vec{k},y})e^{i\omega_{\vec{k}}t}$ and $(m_{\vec{k},x}+im_{\vec{k},y})\times e^{-i\omega_{\vec{k}}t}$] represent the coherence properties of the interaction and the average of this function on a statistical ensemble describes the polarization contribution by the electron-hole pair. The function $|\beta_k|^2$ (or $m_{\vec{k},z}$) represents the occupation probability of the conduction-band electron with wave vector $\vec{k}+\vec{q}$ (or valence-band hole with wave vector $-\vec{k}$). Hence, the transverse relaxation time represents the dephasing time of the polarization wave. The longitudinal relaxation time represents the lifetime of the electron-hole pair in the state $(\vec{k}+\vec{q},-\vec{k})$. Of course, this relaxation process not only includes the recombination of electron-hole pairs, but also the scattering from one to another state. The value of $\tau_{t,\vec{k}}$ may be smaller than $\tau_{l,\vec{k}}$, but the difference between them will not be very large.

VI. CONCLUSION

It follows from our analysis that the interband transition in semiconductors may be described by a set of equations, which are formally identical to the Bloch equations for a two-level system, as long as the interaction between the electrons is neglected. In this case, the various wave vectors $\vec{k}$ correspond to the inhomogeneously broadening of two-level systems. We have also introduced the relaxation times phenomenologically. Hence, the known techniques which have been developed for the interaction between a two-level system and a near-resonant light wave in laser spectroscopy can be applied. In papers II and III of this series, we shall discuss the interaction between electrons and the coherent propagation of a light pulse.

ACKNOWLEDGMENT

The authors would like to thank Professor Kun Huang and Professor L. M. Narducci for very useful discussions and kind help.

[1]T. L. Gvardzhaladze, A. Z. Grasyuk, and V. A. Kovalenko, Zh. Eksp. Teor. Fiz. 64, 446 (1973) [Sov. Phys.—JETP 37, 227 (1973)].

[2]F. Bryukner, V. S. Dneprovskii, and D. G. Koshchug, Zh. Eksp. Teor. Fiz. Pis'ma Red. 20, 10 (1974) [JETP Lett. 20, 4 (1974)].

[3]J. Shah, R. F. Leheny, and C. Lin, Solid State Commun. 18, 1035 (1976).

[4]H. M. Gibbs, A. C. Gossard, S. L. McCall, A. Passner, and W. Wiegmann, Solid State Commun. 30, 271 (1979).

[5]T. A. Poluektov and Yu. M. Popov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 9, 542 (1969) [JETP Lett. 9, 330 (1969)].

[6]H. Haken, Z. Phys. 262, 119 (1973); H. Haken and A. Schenzle, *ibid.* 258, 231 (1973).

[7]H. Haken, in *Proceedings of the International School of Physics, Enrico Fermi, Course LXIV,* edited by N. Bloembergen, (North-Holland, Amsterdam, 1977), p. 350.

[8]E. Hanamura, J. Phys. Soc. Jpn. 37, 1545 (1974); 37, 1553 (1974).

[9]J. Goll and H. Haken, Phys. Rev. A 18, 2241 (1978).

[10]Z. Gan and G. Yang, Phys. Rev. B 26, 6833 (1982), following paper.

[11]J. J. Hopfield, Phys. Rev. 112, 1555 (1958).

[12]F. M. M. Faisal, J. Phys. B 6, L89 (1973); 6, L312 (1973).

[13]L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1945 (1964) [Sov. Phys.—JETP 20, 1307 (1965)].

[14]Yu. I. Balkarei and E. M. Epshtein, Fiz. Tverd. Tela 17, 2312 (1975) [Sov. Phys.—Solid State 17, 1529 (1975)].

[15]V. D. Blazhin, Fiz. Tverd. Tela 17, 2325 (1975) [Sov. Phys.—Solid State 17, 1537 (1975)].

[16]H. D. Jones and H. R. Reiss, Phys. Rev. B 16, 2466 (1977).

[17]A. Abragam, *The Principle of Nuclear Magnetism,* (Clarenden, Oxford, 1961).

[18]M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics,* (Addison-Wesley, Reading, Massachusetts, 1974).