Observation of a new thermal wave in a planar superfluid helium layer

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(Received 6 July 1982)

The existence of a new thermal wave in He II has been experimentally confirmed in a channel formed between two plane-parallel glass plates. The thermal wave is the limiting case of second sound in narrow channels under the condition that the channel width is small compared to the penetration depth of a viscous wave. It is a strongly attenuated temperature or entropy wave, which is coupled to pressure oscillations, and it shows dispersion. The phase velocity of this wave mode was determined in dependence on the frequency, the channel width, and the temperature. At temperatures between 1.1 K and the λ point, the results are in very good agreement with a thermohydrodynamic theory. Below 1.1 K the phase velocity is affected by mean-free-path effects of the elementary excitations, and deviations from this theory occur. They can be described by a simple kinetic theory.

I. INTRODUCTION

As a result of its two-fluid nature, He II is capable of supporting several types of wave motion, e.g., first sound, second sound, third sound, and fourth sound.[1] Besides these well-known sound modes, recently several new wave modes have been observed: fifth sound,[2–5] superfluid two-phase sound,[6,7] surface-tension sound,[8] a U-tube sound mode,[9] gravity waves,[9] surface-second sound (in ^{3}He-^{4}He mixtures),[10,11] and crystallization waves.[12–14] Concerning these wave modes, we will only mention here that all of them are connected with the presence of a free surface of liquid or solid helium, and that fifth sound[2–5] is a temperature or entropy wave in a capillary partially filled with liquid helium. Fifth sound can be excited in He films under pressure release conditions, if the evaporation-condensation effects at the liquid-vapor interface are limited.

In this paper we report the detection of a new wave mode in He II, the thermal wave. It was theoretically predicted[15–20] to exist in capillaries completely filled with He II. In capillaries the nature of the wave motion is determined by the ratio $\delta = 2d/\lambda_v$ of the width of the capillary $2d$ to the penetration depth $\lambda_v = (2\eta/\omega\rho_n)^{1/2}$ of the viscous wave. In the limit of very wide capillaries and high frequencies ($\delta \gg 1$), the ordinary first- and second-sound waves propagate. However, if $\delta \ll 1$, i.e., if the normal fluid is nearly completely clamped by friction with the walls, first sound transforms into fourth sound[21–23] and second sound into a strongly attenuated thermal diffusion wave, which has been called "no-sound,"[23] "thermal-wave,"[15,16,24] or "fifth-wave mode."[17–20] In order to avoid confusion with the recently observed fifth sound,[2–5] we will refer to it only as a thermal wave. This wave is primarily a temperature or entropy wave which, however, in contrast to a classical temperature wave, is strongly coupled to pressure oscillations. In the intermediate range ($\delta \approx 1$) the normal fluid is only partially clamped and the velocities of first and second sound depend on the thickness of the capillaries and show dispersion. This range is theoretically thoroughly treated in Refs. 16 and 25–27 and confirmed by experiment in Refs. 24 and 28–30. With regard to thermal waves, it was shown by Pollack and Pellam[24] that the velocity of second sound decreases continuously with decreasing capillary width, whereas the attenuation increases. It was presumed that second sound transforms into a thermal diffusion wave in the limit of very narrow capillaries ($\delta \ll 1$), but with the experimental method[24] employed it was impossible to observe this wave mode directly.

In this paper we will give an unambiguous proof that the thermal wave exists for $\delta \ll 1$. We will determine its phase velocity in dependence on temperature, frequency, and capillary thickness. Section II contains a short review of the theory of wave propagation in narrow capillaries. In order to approximate the theoretical situation as far as possible, we have developed a special experimental arrangement, which is described in Sec. III. In Sec. IV the results will be presented and discussed. Finally, in Sec. V a short summary of the results will be given.

II. THEORY

The theory of wave propagation in narrow capillaries filled with He II has been studied in detail in Refs. 18–20 and 31, and we will review here shortly the main results concerning the transport of thermal energy. The theory is based on the complete linearized set of the Khalatnikov equations[32] including all dissipative processes connected with the coefficients of first and second viscosity and the thermal conductivity. In order to solve these equations in the case of wave motion in channels, the following boundary conditions at the walls have been taken into account: The tangential component of the velocity of the normal fluid of liquid-He II, and the normal components of the total mass current and the heat flux all vanish at the walls (adiabatic walls). The propagation of waves was considered in a channel bounded by two infinite plane-parallel plates separated by a distance $2d$. The calculations were considerably simplified by averaging the Khalatnikov equations over the thickness of the channel. Then the variables no longer depend on the coordinate perpendicular to the walls and the original problem transforms into a one-dimensional one. Solutions were sought in the form of plane waves. It was found that in the case of very narrow channels ($\delta \ll 1$), two wave modes may exist, the fourth sound and the thermal wave (fifth-wave mode). Fourth sound is essentially a weakly attenuated pressure wave in the superfluid component of He II. The superfluid moves freely, whereas the motion of the normal fluid is almost completely locked by friction with the walls. The field quantities of this wave mode obey a wave equation. Therefore, it is justified to call it a sound wave.

If the motion of the normal fluid is neglected ($\vec{v}_n = 0$), only fourth sound propagates in narrow capillaries. However, considering that the normal fluid is practically immobilized only in the unphysical limit of a vanishingly small capillary thickness ($2d \to 0$), then besides fourth sound another wave mode, the thermal wave, also exists. The dispersion relation of the thermal wave is given by[18–20,31]

$$v_{\mathrm{th}}^2 = \left[\frac{\omega}{k}\right]_{\mathrm{th}}^2 = \frac{i\omega\rho_n u_1^2 u_2^2}{u_4^2\gamma}\left[\frac{d^2}{3\eta} + \frac{\kappa}{\rho^2\sigma^2 T}\right] . \tag{1}$$

Here the following symbols have been introduced: ω is the angular frequency; k is the complex wave number; ρ_n,ρ_s are the densities of the normal fluid and superfluid components; ρ is the total density; u_1,u_2,u_4 are the phase velocities of the first-, second- and fourth-sound modes, respectively,

$$u_4^2 = (\rho_s \mid \rho)u_1^2 + (\rho_n \mid \rho)u_2^2[1 - (2u_1^2\alpha_p \mid \sigma\gamma)] ,$$

α_p is the isobaric coefficient of thermal expansion; γ is the ratio of the specific heats C_p and C_v at constant pressure and constant volume; η is the coefficient of first viscosity; κ is the thermal conductivity; σ is the specific entropy, and T is the absolute temperature. From the dispersion relation (1), the phase velocity u_{th} and the absorption coefficient α_{th} of the thermal wave can be deduced,

$$u_{\mathrm{th}}^2 = \frac{2\omega\rho_n u_1^2 u_2^2}{u_4^2\gamma}\left[\frac{d^2}{3\eta} + \frac{\kappa}{\rho^2\sigma^2 T}\right] \tag{2}$$

and

$$\alpha_{\mathrm{th}} = \frac{u_4}{u_1 u_2}\left[\frac{\omega\gamma}{2\rho_n}\right]^{1/2}\left[\frac{d^2}{3\eta} + \frac{\kappa}{\rho^2\sigma^2 T}\right]^{-1/2} . \tag{3}$$

The phase velocity of the thermal wave shows dispersion and depends on the channel width.

Since the dispersion relation (1) is purely imaginary, this wave satisfies a differential equation of the type of a heat conduction equation, which for temperature fields can be written in the form

$$\frac{\partial T}{\partial t} = \chi\frac{\partial^2 T}{\partial x^2} . \tag{4}$$

χ is the thermal diffusivity. An oscillatory solution of this equation is given by

$$T = T_0 + T'\exp\left[-\left[\frac{\omega}{2\chi}\right]^{1/2} x\right] \times\exp\left\{i\left[\omega t - \left[\frac{\omega}{2\chi}\right]^{1/2} x\right]\right\} , \tag{5}$$

where T_0 is the equilibrium value of the temperature and T' the temperature amplitude of the thermal wave. From Eq. (1), the thermal diffusivity χ can be calculated to be

$$\chi = \frac{\rho_n u_1^2 u_2^2}{u_4^2\gamma}\left[\frac{d^2}{3\eta} + \frac{\kappa}{\rho^2\sigma^2 T}\right] = \frac{u_1^2}{u_4^2}\left[\frac{\rho_n u_2^2 d^2}{3\eta\gamma} + \frac{\rho_s\kappa}{\rho^2 C_p}\right] . \tag{6}$$

The first term of this relation is due to the laminar motion of the normal fluid. The second term corresponds to the thermal diffusivity of a classical fluid $\chi = \kappa/(\rho C_p)$, if one sets approximately $u_4^2 \approx (\rho_s/\rho)u_1^2$.

In terms of the thermal diffusivity the phase velocity and the attenuation coefficient of the thermal wave can be written in the form

$$u_{\rm th}=(2\omega\chi)^{1/2}\ , \tag{7}$$

$$\alpha_{\rm th}=\left[\frac{\omega}{2\chi}\right]^{1/2}\ . \tag{8}$$

The thermal wave is a very strongly damped wave analogous to a diffusion or thermal wave in a classical fluid. The decay of the amplitude over the distance of one wavelength is calculated to have a constant value of $e^{-2\pi}\approx 1/535$.

From an investigation of the field quantities,[17–20] it turns out that, in contrast to classical thermal waves, the temperature oscillations in the thermal-wave mode are strongly coupled to pressure oscillations according to the relation

$$p'_{\rm th}=\rho\sigma T'_{\rm th}\ , \tag{9}$$

a very characteristic feature of the thermal wave in He II. It is caused by the thermomechanical effect (fountain effect). As in a second-sound wave the superfluid and the normal fluid components move nearly in counterphase in a thermal wave, so that the total mass flux is approximately zero. This fact elucidates that the thermal wave is the limiting case of second sound in narrow channels. It should be noted, however, that in the thermal wave the amplitudes of the velocity fields of both components, the normal fluid, and the superfluid, are very small quantities.

In order to find out the most efficient way to generate the thermal wave, the excitation problem was considered in detail in Refs. 18 and 31. It was shown that, in principle, the thermal wave can be excited by periodically heating the surface of a solid body (e.g., a resistance layer) as well as by vibrating a plane-solid surface (e.g., the diaphragm of a condenser microphone). The calculations lead to the result that the thermal wave is most favorably generated by using a heater, because nearly all of the heat flux entering the liquid is dissipated via this wave mode. Therefore, in this work the thermal generation of the thermal wave is preferred. The ratio of the temperature amplitude $T'_{\rm th}$ to the total heat-flux amplitude q' at a distance x from the heater can be described by the relation

$$\begin{aligned}\frac{T'_{\rm th}}{q'}&=\frac{\rho_s u_1^2}{\rho^2 C u_{\rm th} u_4^2}(1-i)\exp(-\alpha_{\rm th}x)\\ &\approx\frac{1}{\rho C u_{\rm th}}(1-i)\exp(-\alpha_{\rm th}x)\\ &=Z_{\rm th}(1-i)\exp(-\alpha_{\rm th}x)\ ,\end{aligned} \tag{10}$$

where the approximation $u_4^2\approx(\rho_s\,|\,\rho)u_1^2$ has been used. $Z_{\rm th}=1/(\rho C u_{\rm th})$ can be regarded as the thermal impedance of the thermal wave. According to Eq. (10), there is a phase difference of $7\pi/4$ between $T'_{\rm th}$ and q'.

The considerations made so far are only valid in the hydrodynamic range, i.e., if the thickness of the channel $2d$ is large compared to the mean free paths of the phonons ($l_{\rm ph}$) and the rotons (l_r). For the channel widths used in the experiment between $2d=1.2$ and 2.8 μm, the validity range of the hydrodynamic theory is illustrated by the hatched area in Fig. 1. This figure shows a semilog plot of the mean free paths of the elementary excitations (solid lines) and the penetration depth of the viscous wave λ_v (dashed lines) as a function of temperature. The mean free paths were calculated in Ref. 33 according to the theory by Khalatnikov.[34] It can be seen that at high temperatures the validity range is bounded by the penetration depth of the viscous wave. This difficulty can be overcome by using only frequencies below 1 kHz near the λ point. At temperatures below 1 K the mean free paths of the phonons exceed the channel widths. In this region the hydrodynamic theory is invalid, and we will expect deviations from its results as it was already observed in measurements of the effective thermal conductivity of He II in restricted geometries.[33] For $l_{\rm ph}>2d$, kinetic theories[33] have to be considered to describe the experimental results.

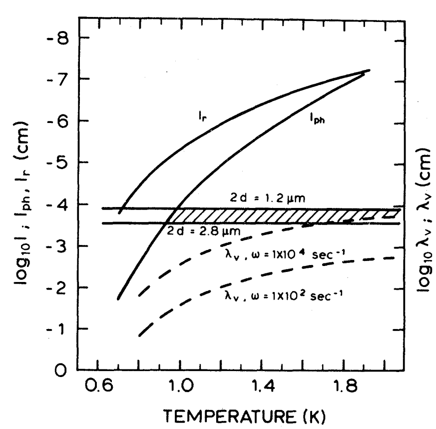

FIG. 1. Mean free paths of the phonons $l_{\rm ph}$ and the rotons l_r (solid lines) and the penetration depth of a viscous wave λ_v (dashed lines) as a function of temperature. The straight lines limit the range of channel widths (1.2–2.8 μm) investigated in the experiment. The hatched area indicates the validity range of the phenomenological theory.

III. EXPERIMENTAL METHOD

Because of the high damping and dispersion, it is very difficult to detect the thermal wave. The known acoustic methods, as e.g., the pulse method or the resonance method, cannot be applied. Therefore, it was necessary to develop another method, which is analogous to the periodic temperature method first proposed by Ångström[35] for a measurement of the thermal diffusivity of solids. With this method the phase velocity is determined from the phase difference of the thermal wave at two points of the sample separated by a known distance.

In order to approximate the theoretical situation as far as possible, a well-defined geometry was chosen. A schematic diagram of the experimental arrangement is shown in Fig. 2. It is in some respects similar to the set-up used by Jelatis *et al.*[4] to observe fifth sound in superfluid helium. The channel is formed by two optically flat plane-parallel glass plates. Seven values of the plate separations were chosen in the range between 1.2−2.8 μm. The narrow slit between the plates was adjusted by silver distance strips deposited on the glass plates. The plates were held in place by means of screws that are not shown in the figure.

The separation between the plates was determined by two methods. The first one consisted of measuring the capacitance of a parallel plate capacitor formed by the circular metallized area deposited on each plate. From the capacitance the plate separation can be calculated. The second method used optical interference fringes of monochromatic light, a technique that also allowed to monitor the plane parallelity of the glass plates over the area of interest at the beginning of each experiment. Both methods gave the same values of the slit width within an experimental accuracy of ± 0.05 μm.

The thermal waves were generated and detected by thin parallel aluminum strips[36] evaporated onto the glass plates. The approximate dimension of each strip was (50−200)-Å thick, 0.2-mm wide, and 7.5- (transmitter) or 10-mm (receiver) long. In order to avoid direct thermal cross talk, the heater was evaporated onto the upper and the three receiver strips onto the lower plate. The separations between the heater (h) and the first bolometer (b_1), and between the bolometers (b_i, $i=1,2,3$), respectively, are $h-b_1$: 1.4 mm, b_1-b_2: 1.04 mm, and b_2-b_3: 1.94 mm. The bolometers were operated in their superconducting-transition regions with a proper amount of dc current as determined by V-T characteristics. The transition temperature of aluminum can be raised well above its bulk value ($T_c=1.196$ K) by reducing the thickness of the film and doping it with impurities.[37] Also, the transition temperature could be lowered by varying the dc bias currents through the strips or applying an external magnetic field. This field was provided by a superconducting solenoid, which generated a field of 4.9 kG at a current of 10 A. All these parameters were properly adjusted to obtain the maximum sensitivity of the bolometers. In general, it was necessary to produce 3 to 4 samples of different aluminum film thickness to cover the total temperature range between 0.8−2.3 K.

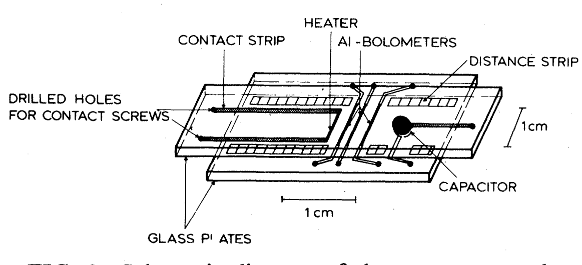

FIG. 2. Schematic diagram of the apparatus used to detect the thermal wave.

The thermal wave was excited by feeding a cw sine wave of current into the heater. Since the heat developed is proportional to the square of current, this produced a thermal wave of double frequency. The passage of the thermal wave over the three bolometers was detected as voltage variations across the strips. Since the thermal waves are very rapidly decaying in amplitude as they travel along the lamina between the glass plates, in general the two bolometers with the smallest distances to the heater were used as receivers. The received signals were amplified and the phase difference between them determined by a phasemeter (Eurelco, Model 400 BN). From the phase difference $\Delta\varphi$, the angular frequency ω and the separation Δx between the two receiving strips, the thermal diffusivity can be calculated according to the relation

$$\chi=\omega\Delta x^2/(2\Delta\varphi^2)\ . \tag{11}$$

The sine generator was controlled by a graphic computer (Tektronix, Model 4051) and measurements were taken with the frequency increasing in steps of 15 Hz in the frequency range between 40 and 800 Hz. To fulfill the condition $\lambda_v \gg 2d$, in the thicker channels the upper frequency limit was chosen as 400 Hz in the vicinity of the λ point. A digital signal proportional to the phase difference was fed into the computer, which was programmed to plot $\Delta\varphi$ vs $\sqrt{\omega}$. About 100 measurements of the phase difference were taken at each frequency step and the mean value calculated to minimize the error. The maximum deviation from the mean value was about $\pm 5\%$. The plot $\Delta\varphi$ vs $\sqrt{\omega}$ yields a straight line, the slope of which was determined by a least-squares fit. From the slope according to Eq. (11), the thermal diffusivity was obtained, and according to Eq. (7), the phase velocity was obtained.

It should be noted that the experimental set-up used here has several advantages compared with other ones usually employed to measure sound modes in restricted geometries, such as, e.g., powder samples. These are the well-defined geometry of the plane-parallel slit between the glass plates, which avoids the use of scattering correction factors from superleak material,[23] and the excitation and detection of the thermal wave directly, *in situ*, in the channel, which avoids matching problems between the transmitter or the receivers and the channel.

The experiments were performed in a standard metal cryostat. In order to reduce the temperature from 1.2 to 0.8 K, a simple helium-cooled charcoal adsorption pump[38] was employed. The thermal-wave probe was located in a high-vacuum-tight stainless-steel can immersed in the main helium bath. It could be filled with liquid helium from the outside through a cold-needle valve. For the purpose of background correction and comparison, it was useful to determine the thermal diffusivity of the glass plates at low temperatures. Therefore, at first, measurements without helium in the can were made by using one of the three Al strips on the lower plate as transmitter and the remaining two as receivers. The thermal diffusivity of the glass plates turned out to be nearly independent of temperature in the range investigated below the λ point. A value of χ of 3.5 cm^2/sec was found in good consistency with data given in the literature.[39] The temperature was determined to an accuracy of ± 3 mK by using a calibrated carbon resistor. A temperature controller served to stabilize the temperature to about ± 1 mK during a measuring cycle.

IV. RESULTS AND DISCUSSION

According to the relation (7) the phase velocity of the thermal wave u_{th} shows dispersion and is proportional to the square root of the angular frequency ω. Thus one should expect a linear behavior in a plot of u_{th} vs $\sqrt{\omega}$. This is indeed the case as can be seen from Fig. 3, where the experimental data are represented at a few selected temperatures between

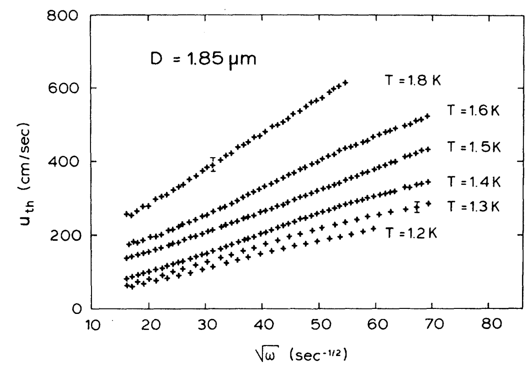

FIG. 3. Phase velocity of the thermal wave u_{th} as a function of the square root of the angular frequency ω for a plate separation of $D=1.85$ μm and at various temperatures between 1.2 and 1.8 K.

1.2 and 1.8 K for a plate separation of $2d=D=1.85$ μm. Within experimental accuracy ($\pm 5\%$) at frequencies between 40 and 800 Hz the data can be fitted by straight lines that pass through the origin of the coordinate system. This is a first direct proof for the existence of the new thermal wave mode. At high frequencies and low temperatures slight systematic deviations from the straight lines appear, which, however, are still within experimental accuracy. They are probably due to the fact that the attenuation of the thermal wave rises rapidly with increasing frequency and decreasing temperature [see Eq. (8)]. Consequently, the signal-to-noise ratio deteriorates more, which results in larger systematic errors of the phase difference measurements.

From the slopes of the straight lines, which are determined by a least-squares fit to the data, the thermal diffusivities χ at various temperatures can be found. Figure 4 shows the results for the channel width of $D=1.85$ μm. Between 0.9 and 2 K the thermal diffusivity rises continuously to a maximum at about 2 K, and then decreases again rapidly near the λ point. The solid line indicates the result of the thermohydrodynamic theory according to Eq. (6). In order to calculate χ, well-established experimental values of the thermodynamic quantities ρ, ρ_n, ρ_s, σ, C_p, and η were taken from Ref. 40, u_1, u_2, u_4, and γ from Ref. 41, and κ from Ref. 42. From Fig. 4 it is evident that there is excellent agreement between theory and experiment except for $T<1$ K. It should be noted that no fit parameters have been used. Thus one may conclude that the thermohydrodynamic theory is capable of describing dissipative processes in narrow channels.

According to relation (6), the thermal diffusivity

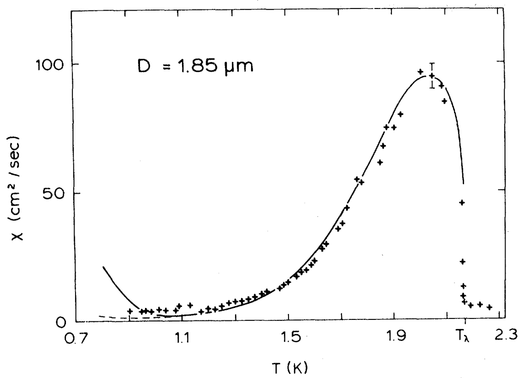

FIG. 4. Thermal diffusivity χ in dependence on temperature for a channel width of $D=1.85$ μm. The solid line represents the result of the thermohydrodynamic theory according to Eq. (6), and the dashed line is calculated on the basis of a kinetic theory (Ref. 33) by taking mean-free-path effects of the elementary excitations into account.

χ contains two contributions: the first one from the laminar flow of the normal fluid and the second one from the diffusive heat-flow mechanism. The experiment fully confirms the first contribution, which dominates at temperatures between 1.1 K and the λ point. Owing to the second contribution, the thermal diffusivity should increase at temperatures below 1 K. This is not observed by the experiment. The data remain nearly constant as can also be seen in Fig. 5. Following our considerations in Sec. II, in this temperature range mean-free-path effects of the elementary excitations become increasingly important, which leads to deviations from the bulk values of the thermal conductivity κ. When $l_{\text{ph}} > D$ and $l_r > D$, the elementary excitations do not interact among each other but are only scattered at the walls of the plane-parallel slit. In this region the phonon and roton gases undergo a Knudsen type of flow in close analogy to the behavior of a rarefied gas. Then the phenomenological theory is no longer valid and must be replaced by kinetic considerations as given in Ref. 33 on the basis of a simple model of Casimir.[43] The dashed line plotted in Fig. 4 shows the result of this theory.[33] Within experimental accuracy good agreement between experiment and theory is obtained as will become somewhat clearer in the next figure. Thus our data lead to the conclusion that at temperatures below 1 K, phonon collisions with the walls mainly determine the thermal diffusivity. The same result was already obtained by steady-state heat flow measurements of He II contained in porous superleaks made of fine-grained powder.[33]

The theory is based on the assumption of adiabatic walls; this means that the heat exchange between helium and the walls can be neglected. A detailed theoretical study[44] has shown that this assumption is nearly fulfilled in the frequency range investigated under the following conditions: (1) the thermal diffusivity of the wall material must be small compared to that of the helium confined in the plane-parallel slit, (2) the Kapitza boundary resistance between the walls and the helium must be large. As was verified in a subsidiary experiment with an empty sample cell (see Sec. III), the first condition is met over most of the temperature range by choosing glass as wall material. There are only two temperature regions, where the thermal diffusivities of the glass plates and the helium become comparable, below 1.2 K and above the λ point. In the former region the high value of the Kapitza resistance essentially prevents heat exchange with the walls, so that the value of χ measured is mainly that of the helium in the plane-parallel slit. In the latter region $(T > T_\lambda)$ the heat exchange with the walls cannot be completely neglected and the thermal diffusivity attains a value determined by a combination of values of the glass plates and of He I.

In order to study the variation of the thermal diffusivity with the width of the plane-parallel slit, six further plate separations D were investigated. Figure 5 shows the data of the thermal diffusivity as a function of temperature for the slit widths: $D=2.8$, 2.35, 1.7, 1.55, 1.3, and 1.2 μm. Again the solid lines represent the results of the theory according to Eq. (6), and the dashed lines were calculated by taking mean-free-path effects of the elementary excitations[33] into account. In general, the same features can be seen as in Fig. 4. The theory describes the data very well. The thermal diffusivity decreases with decreasing channel thickness above 1.2 K, because the motion of the normal fluid is more and more suppressed by viscous interaction with the walls.

Since at $T \geq 1.2$ K the contribution of the second term of the relation (6) connected with the thermal conductivity κ to the thermal diffusivity is always smaller than 3%, it can be approximately neglected. In this situation it can be inferred from Eq. (2) that the phase velocity of the thermal wave should be proportional to the width of the plane-parallel slit D. In Fig. 6 the phase velocity of the thermal wave is plotted versus the slit width D at four temperatures. Within experimental uncertainty the data can be fitted by straight lines in agreement with the theory. The slopes of the straight lines confirm the

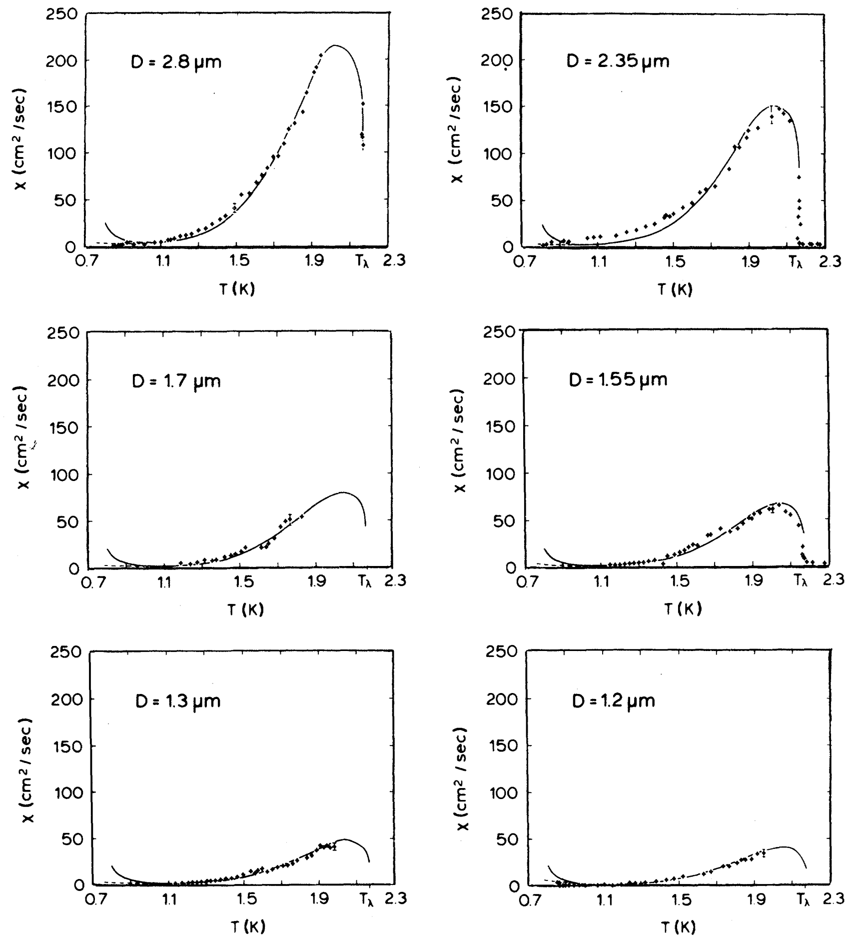

FIG. 5. Thermal diffusivity as a function of temperature at the channel widths D=2.8, 2.35, 1.7, 1.55, 1.3, and 1.2 μm. The solid lines indicate the results of the theory according to Eq. (6), and the dashed lines are calculated by means of a kinetic theory (Ref. 33).

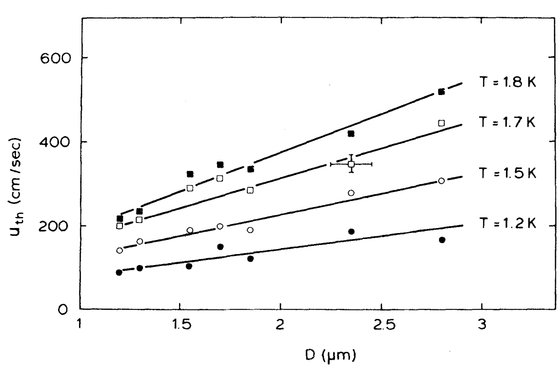

FIG. 6. Phase velocity of the thermal wave u_{th} plotted against the width of the plane-parallel slit D at the temperatures T=1.2, 1.5, 1.7, and 1.8 K. The straight lines are fitted to the data and reveal in consistency with the theory according to Eq. (2) a linear relationship between the phase velocity and the slit width.

data obtained from Eq. (6) by using known values[40,41] of the thermodynamic quantities. This strengthens the theoretical result that at $T > 1.2$ K predominantly the laminar flow of the normal fluid determines the phase velocity of the thermal wave.

V. CONCLUSION

In this paper it has been firmly established that a strongly attenuated wave mode, the thermal wave, exists besides fourth sound in narrow channels ($\delta \ll 1$) filled with He II. It satisfies a differential equation of the type of a diffusion equation, whereas ordinary sound modes obey wave equations. Therefore, it should be more generally denoted as a wave mode rather than as a sound mode.

The thermal-wave mode shows strong dispersion. In order to detect it, we have developed a special method, which consists of measuring the phase differences of the wave at two different points of the channel. The thermal wave was excited thermally and its temperature oscillations were detected by using superconducting bolometers. The experiments were carried out in a well-defined plane-parallel slit between two glass plates separated by a known distance. This simple channel geometry enabled us to compare data of the phase velocity with the theory directly without using correction factors. The phase velocity was determined as a function of the frequency, the channel width, and the temperature. At temperatures between 1.1 K and the λ point very good agreement with the thermohydrodynamic theory was obtained. This is strong proof of the new wave. The deviations from the theory below 1.1 K could be very well described with a kinetic model by taking mean-free-path effects of the elementary excitations into account.

Even if the existence of the thermal wave is proven, several questions still remain open. It would be of interest to study whether this wave mode can be generated mechanically by means of a condenser microphone or electrostrictively by a capacitor, and whether the pressure oscillations, which accompany the wave, can be detected by capacitance measurements. Moreover, the attenuation of this wave mode should be determined, and it seems feasible to extend the experiments to ^{3}He-^{4}He mixtures. Because of its low phase velocity, the thermal-wave mode may be a good tool for precision investigations of persistent currents and of critical velocities in narrow channels filled with He II.

ACKNOWLEDGMENTS

The authors are grateful to Professor G. U. Schubert and Professor K. Knorr for useful discussions.

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