Dipole radiation in the presence of a rough surface. Conversion of a surface-polariton field into radiation

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The characteristics of the radiation produced by a dipole, located near the rough surface of a material medium, are examined. The field distribution is calculated at any point outside the medium for arbitrary orientation of the dipole moment, so that one may obtain the electromagnetic Green's function in the presence of surface roughness. The medium can have either local or a nonlocal dielectric function and the results are valid to first order in roughness. The surface roughness converts the surface polariton field, created even in the absence of roughness, into radiation and thus leads to well-defined resonances in the far-field radiation pattern. Numerical results for the case of metallic as well as dielectric gratings are given. The effect of the nonlocality of the dielectric function on the resonances in the radiation is shown to be significant in certain cases. For metallic gratings the dominant effect of the nonlocality is to shift the position of the angular resonances.

I. INTRODUCTION

An excited molecule placed close to a material medium, such as a metal, is known to couple resonantly[1-4] with the surface polariton modes of the material medium provided that the molecular frequencies are such that the surface modes are excited. For example, for a molecule placed close to the surface of a semi-infinite medium, the condition for the excitation of surface polaritons is that the real part of the dielectric function at the molecular frequencies is less than -1. Under such conditions most of the radiation from excited systems is in the form of surface polaritons. Reference 1 deals with this problem extensively for various types of material media including the effect of the spatially dispersive nature of the medium. In order to detect this excitation of nonradiative surface polaritons, one has to convert the surface polaritons into radiative modes. Several schemes[5-7] have been used to achieve this. One approach is to study the spontaneous emission by an excited system in the presence of surface roughness such as a grating structure. The grating structure converts the nonradiative modes into radiative ones. This geometry should be contrasted with that of, for example, a sphere[8] where the surface modes are radiative in nature and the excitation of surface polaritons is thus directly observable. The object of the present study is to calculate the radiation characteristics of a dipole located close to the rough surface and to find out how the excitation of surface polaritons is reflected in the properties of the far-field radiation. In Sec. II we treat the problem assuming a local dielectric function $\epsilon(\omega)$ and calculate the fields radiated for arbitrary orientation of the dipole. In Sec. III we incorporate the effects of spatial dispersion of the dielectric and examine how the resonant coupling between the dipole and the surface polaritons is affected by the k dependence of the dielectric function of the medium. Numerical results for the case of a grating are given in Sec. IV and the structure of the resonances in the far-field radiation pattern is studied in detail.

II. FIELD DISTRIBUTION PRODUCED BY A RADIATING DIPOLE NEAR THE ROUGH SURFACE OF A MEDIUM CHARACTERIZED BY LOCAL DIELECTRIC FUNCTION $\epsilon(\omega)$

The question of the electromagnetic field distribution produced by an electromagnetic wave incident on a rough surface has been investigated, in detail, by a variety of methods.[9-11] Let us assume that the rough surface is described by $z=-hf(x,y)$ where the medium, with dielectric function $\epsilon(\omega)$, occupies the space $z+hf(x,y)>0$. The region outside the medium is assumed to be vacuum. The dipole is taken to be located at a point $\vec{r}_0$ outside the medium. The field produced by a dipole, oscillating at frequency ω, is given by

$$E_i^{(d)}(\vec{r},\omega)=\sum_i \left\{k_0^2\delta_{ij}+\frac{\partial^2}{\partial x_i \partial x_j}\right\}\frac{\exp(ik_0|\vec{r}-\vec{r}_0|)}{|\vec{r}-\vec{r}_0|}p_j, \quad k_0=\omega/c \tag{2.1}$$

which with the use of the angular spectrum representation can be written in the form

$$E^{(d)}(\vec{r},\omega)=\begin{cases}\dfrac{i}{2\pi}\displaystyle\int\int \frac{d^2\kappa}{W_0}[k_0^2\vec{p}-\vec{K}_0'(\vec{K}_0\cdot\vec{p})]\exp[i\vec{\kappa}\cdot(\vec{r}-\vec{r}_0)+iW_0(z-z_0)], & z>z_0\\ \dfrac{i}{2\pi}\displaystyle\int\int \frac{d^2\kappa}{W_0}[k_0^2\vec{p}-\vec{K}_0(\vec{K}_0'\cdot\vec{p})]\exp[i\vec{\kappa}\cdot(\vec{r}-\vec{r}_0)-iW_0(z-z_0)], & z<z_0\end{cases} \tag{2.2}$$

where

$$W_0^2=k_0^2-\kappa^2, \quad \vec{K}_0=(\vec{\kappa},W_0), \quad \vec{K}_0'=(\vec{\kappa},-W_0) \tag{2.3}$$

and $\vec{\kappa}$ is the two-dimensional vector representing the component of the momentum vector parallel to surface $z=0$. The integration in (2.2) is over the entire two-dimensional plane. W_0 is real for $\kappa<k_0$ and pure imaginary for $\kappa>k_0$. The field produced by a dipole is thus expressed as a superposition of plane waves with propagation vector $\vec{K}_0$ ($\vec{K}_0'$) for $z>z_0$ (for $z<z_0$). The waves corresponding to $\kappa<k_0$ are homogeneous whereas the ones for $\kappa>k_0$ are evanascent in nature. The dipole produces fields which are both homogeneous and evanascent in character. For each plane wave, whether homogeneous or evanascent, it is possible to introduce its s and p components. Introducing the unit vectors (perpendicular to the direction of propagation $\vec{k}$)

$$S_s^{\gtrless}(\vec{\kappa})=\frac{\hat{z}\times\vec{\kappa}}{\kappa}, \quad S_p^{>}(\vec{\kappa})=\frac{(\hat{z}\times\vec{\kappa})\times\vec{K}_0}{\kappa k_0}, \quad S_p^{<}(\vec{\kappa})=\frac{(\hat{z}\times\vec{\kappa})\times\vec{K}_0'}{\kappa k_0}, \tag{2.4}$$

we can rewrite the dipole field as

$$E^{(d)}(\vec{r},\omega)=\begin{cases}\displaystyle\int\int d^2\kappa[\vec{S}_s^{>}(\vec{\kappa})\varepsilon_s^{>}(\vec{\kappa})+\vec{S}_p^{>}(\vec{\kappa})\varepsilon_p^{>}(\vec{\kappa})]e^{i\vec{K}_0\cdot\vec{r}}, & z>z_0\\ \displaystyle\int\int d^2\kappa[\vec{S}_s^{<}(\vec{\kappa})\varepsilon_s^{<}(\vec{\kappa})+\vec{S}_p^{<}(\vec{\kappa})\varepsilon_p^{<}(\vec{\kappa})]e^{i\vec{K}_0'\cdot\vec{r}}, & z<z_0.\end{cases} \tag{2.5}$$

Then, using (2.2) and (2.4), one can show that

$$\varepsilon_p^{>}(\vec{\kappa})=-\left\{\frac{k_0}{\kappa}\right\}\frac{i}{2\pi W_0}[k_0^2p_z-W_0(\vec{K}_0\cdot\vec{p})]\exp(-i\vec{\kappa}\cdot\vec{r}_0-iW_0z_0), \tag{2.6}$$

$$\varepsilon_p^{<}(\vec{\kappa})=-\left\{\frac{k_0}{\kappa}\right\}\frac{i}{2\pi W_0}[k_0^2p_z+W_0(\vec{K}_0'\cdot\vec{p})]\exp(-i\vec{\kappa}\cdot\vec{r}_0+iW_0z_0), \tag{2.7}$$

and

$$\varepsilon_s^{>}(\vec{\kappa})=\frac{i}{2\pi\kappa W_0}[k_0^2(\vec{\kappa}\times\vec{p})_z]\exp(-i\vec{\kappa}\cdot\vec{r}_0-iW_0z_0), \tag{2.8}$$

$$\varepsilon_s^{<}(\vec{\kappa})=\frac{i}{2\pi\kappa W_0}[k_0^2(\vec{\kappa}\times\vec{p})_z]\exp(-i\vec{\kappa}\cdot\vec{r}_0+iW_0z_0). \tag{2.9}$$

Having expressed the dipole field in terms of a superposition of plane waves and the plane waves in terms of their respective s and p components, it would now be easier to use the results from the problem of electromagnetic scattering from a rough surface to find the fields produced by a dipole placed near a rough surface. This is because one can study the interaction of individual plane waves in (2.5) and then superpose the result to obtain the total field distribution. One can only do a perturbation theory in powers of the surface roughness parameter h. The total field produced in the region $z<z_0$ can be written [in analogy to (2.5)] as

$$E(\vec{r},\omega)=\int\int d^2\kappa\{\vec{S}_s^{<}(\vec{\kappa})[\varepsilon_s^{<}(\vec{\kappa})+\varepsilon_s^{(0)}(\vec{\kappa})+h\varepsilon_s^{(1)}(\vec{\kappa})+\cdots] + \vec{S}_p^{<}(\vec{\kappa})[\varepsilon_p^{<}(\vec{\kappa})+\varepsilon_p^{(0)}(\vec{\kappa})+h\varepsilon_p^{(1)}(\vec{\kappa})+\cdots]\}e^{i\vec{K}_0'\cdot\vec{r}}, \quad z<z_0 \tag{2.10}$$

In this section, as well as in Sec. III, we will only be concerned with dipolar fields outside the medium, and the expressions for first-order fields inside the medium will not be given. The zeroth-order fields $\mathcal{E}_s^{(0)}(\vec{\kappa}),\mathcal{E}_p^{(0)}(\vec{\kappa})$, for $z<0$, are related to $\mathcal{E}_s^{>}(\vec{\kappa}),\mathcal{E}_p^{>}(\vec{\kappa})$ by Fresnel relations

$$\mathcal{E}_p^{(0)}(\vec{\kappa})=\frac{W_0\epsilon-W}{W_0\epsilon+W}\mathcal{E}_p^{>}(\vec{\kappa}) , \tag{2.11}$$

$$\mathcal{E}_s^{(0)}(\vec{\kappa})=\frac{W_0-W}{W_0+W}\mathcal{E}_s^{>}(\vec{\kappa}) , \tag{2.12}$$

where

$$W^2=k_0^2\epsilon(\omega)-\kappa^2 . \tag{2.13}$$

Note that W gives the z component of the propagation vector of a plane wave inside the medium and that W, W_0 depend on $\vec{\kappa}$.

Note the presence of the denominator in (2.11) which can vanish (assuming real ϵ) provided the following conditions are satisfied:

$$\epsilon(\omega)<-1, \quad \kappa>\omega/c . \tag{2.14}$$

The vanishing of this denominator can be shown to be equivalent to the result

$$W_0\epsilon+W=0 \Longrightarrow D(\kappa,\omega)\equiv\kappa^2-\frac{\omega^2}{c^2}\frac{\epsilon(\omega)}{\epsilon(\omega)+1}=0 , \tag{2.15}$$

which is the well-known surface polariton dispersion relation. Since the dipolar field also consists of evanescent waves $[\kappa>(\omega/c)]$, hence, surface polaritons get excited. This excitation leads to large widths of the dipolar frequencies and thus a very large decay rate as discussed in detail in Ref. 1.

The first-order fields have been calculated earlier for a plane wave incident on a rough surface. If the incident field is a superposition of plane waves with propagation vectors $\vec{K}_0^{(0)}=\vec{\kappa}^{(0)}, W_0^{(0)}$, then the first-order fields are given by [cf. Ref. 11, Eqs. (4.12) and (4.13)]

$$\mathcal{E}_s^{(1)}(\vec{\kappa})=\int\frac{ik_0^2(\epsilon-1)F(\vec{\kappa}-\vec{\kappa}^{(0)})}{(W+W_0)}\left[\frac{\vec{\kappa}\cdot\vec{\kappa}^{(0)}}{\kappa\kappa^{(0)}}\frac{2W_0^{(0)}}{(W^{(0)}+W_0^{(0)})}\mathcal{E}_s^{(i)}(\vec{\kappa}^{(0)}) + \frac{W^{(0)}}{k_0}\left[\frac{\vec{\kappa}\times\vec{\kappa}^{(0)}}{\kappa\kappa^{(0)}}\right]\frac{2W_0^{(0)}}{W_0^{(0)}\epsilon+W^{(0)}}\mathcal{E}_p^{(i)}(\vec{\kappa}^{(0)})\right]d^2\kappa^{(0)} , \tag{2.16}$$

$$\mathcal{E}_p^{(1)}(\vec{\kappa})=\int\frac{-i(\epsilon-1)F(\vec{\kappa}-\vec{\kappa}^{(0)})}{(W_0\epsilon+W)}\left\{k_0W\frac{\vec{\kappa}^{(0)}\times\vec{\kappa}}{\kappa\kappa^{(0)}}\left[\frac{2W_0^{(0)}}{W^{(0)}+W_0^{(0)}}\right]\mathcal{E}_s^{(i)}(\vec{\kappa}^{(0)}) + \left[WW^{(0)}\left[\frac{\vec{\kappa}\cdot\vec{\kappa}^{(0)}}{\kappa\kappa^{(0)}}\right]-\epsilon\kappa\kappa^{(0)}\right]\frac{2W_0^{(0)}}{W_0^{(0)}\epsilon+W^{(0)}}\mathcal{E}_p^{(i)}(\vec{\kappa}^{(0)})\right\}d^2\kappa^{(0)} , \tag{2.17}$$

and $F(\vec{\kappa})$, the Fourier transform of the surface roughness $f(x,y)$, is given by

$$F(\vec{\kappa})=\frac{1}{4\pi^2}\int dx\,dy\,f(x,y)e^{i\vec{\kappa}\cdot\vec{r}} . \tag{2.18}$$

It should be kept in mind that $W^{(0)}, W_0^{(0)}$ are functions of $\vec{\kappa}^{(0)}$ and obtained from the corresponding W by $\vec{\kappa}\rightarrow\vec{\kappa}^{(0)}$. The first-order fields that appear in (2.10) can be obtained from (2.16) and (2.17) by the replacement

$$\mathcal{E}_s^{(i)}(\vec{\kappa}^{(0)})\rightarrow\mathcal{E}_s^{(>)}(\vec{\kappa}^{(0)}), \quad \mathcal{E}_p^{(i)}(\vec{\kappa}^{(0)})\rightarrow\mathcal{E}_p^{(>)}(\vec{\kappa}^{(0)}) . \tag{2.19}$$

We have thus obtained the field distribution produced by a dipole (with arbitrary orientation) to first order in the surface roughness parameter. The function $f(x,y)$ is quite arbitrary. The self fields which are important for lifetime[1] studies can be obtained from (2.10) by letting $\vec{r}\rightarrow\vec{r}_0$. The far-zone behavior of the fields can be obtained from the asymptotic expansion of the angular spectrum[12]

$$E(\vec{r},\omega)\sim -2\pi i k_0 |\cos\theta| \frac{e^{ik_0 r}}{r}\{\vec{S}_s^{<}(\vec{\kappa})[\varepsilon_s^{(<)}(\vec{\kappa})+\varepsilon_s^{(0)}(\vec{\kappa})+h\varepsilon_s^{(1)}(\vec{\kappa})] + \vec{S}_p^{<}(\vec{\kappa})[\varepsilon_p^{<}(\vec{\kappa})+\varepsilon_p^{(0)}(\vec{\kappa})+h\varepsilon_p^{(1)}(\vec{\kappa})]\}\ , \tag{2.20}$$

with

$$\vec{\kappa}=(k_0\sin\theta\,\cos\phi, k_0\sin\theta\,\sin\phi),\quad \frac{\pi}{2}<\theta<\pi\ . \tag{2.21}$$

Here the angles (θ,ϕ) characterize the direction of observation. The power radiated per unit solid angle is

$$\frac{dP}{d\Omega}=\frac{\pi c}{2}k_0^2\cos^2\theta(|\varepsilon_s^{(<)}+\varepsilon_s^{(0)}+h\varepsilon_s^{(1)}|^2+|\varepsilon_p^{(<)}+\varepsilon_p^{(0)}+h\varepsilon_p^{(1)}|^2)\ . \tag{2.22}$$

Note that only the homogeneous waves contribute to the far-zone behavior.

Let us now examine the structure of (2.16) and (2.17) for $|\kappa|<k_0$. The field for $|\vec{\kappa}|<k_0$ is obtained from a linear superposition of the incident field amplitudes at all possible momentum ($\vec{\kappa}$) values. It is therefore possible that the denominator $W_0^{(0)}\epsilon+W^{(0)}$ under the integral in (2.16) and (2.17) can vanish for certain $\kappa^{(0)}$ [satisfying (2.15)], if $\mathrm{Re}\epsilon(\omega)<-1$. Thus the resonantly excited surface polaritons get converted into radiation (homogeneous waves) by surface roughness. Therefore, the dipolar radiation[13] in the presence of surface roughness would have important contributions from the excitation of surface polaritons inside the medium. The radiation characteristics depend on the orientation of the dipole. For a dipole oriented along the z axis, the s component of the field $\varepsilon^{(>)}$ is zero as seen from (2.8) and (2.9) and the p components acquire simpler form

$$\varepsilon_p^{(>)}(\vec{\kappa})=-\frac{k_0 i\kappa p}{2\pi W_0}\exp(-i\vec{\kappa}\cdot\vec{r}_0-iW_0 z_0)\ , \tag{2.23}$$

$$\varepsilon_p^{(<)}(\vec{\kappa})=-\frac{k_0 i\kappa p}{2\pi W_0}\exp(-i\vec{\kappa}\cdot\vec{r}_0+iW_0 z_0)\ . \tag{2.24}$$

On substituting (2.23) and (2.24) in (2.16) and (2.17), we obtain the s and p components of the first-order dipolar fields (in the domain $z<0$):

$$\varepsilon_s^{(1)}(\vec{\kappa})=\frac{2ik_0(\epsilon-1)}{(W+W_0)}\int d^2\kappa^{(0)}F(\vec{\kappa}-\vec{\kappa}^{(0)})\left[\frac{\vec{\kappa}\times\vec{\kappa}^{(0)}}{\kappa\kappa^{(0)}}\right]\frac{W^{(0)}W_0^{(0)}}{(W_0^{(0)}\epsilon+W^{(0)})} \times\exp(-i\vec{\kappa}^{(0)}\cdot\vec{r}_0-iW_0^{(0)}z_0)\left[-\frac{i\kappa^{(0)}k_0 p}{2\pi W_0^0}\right]\ , \tag{2.25}$$

$$\varepsilon_p^{(1)}(\vec{\kappa})=\frac{-2i(\epsilon-1)}{(W_0\epsilon+W)}\int d^2\kappa^{(0)}F(\kappa-\kappa^{(0)})\left[-\frac{i\kappa^{(0)}k_0 p}{2\pi W_0^{(0)}}\right]\exp(-i\vec{\kappa}^{(0)}\cdot\vec{r}_0-iW_0^{(0)}z_0) \times\frac{W_0^{(0)}}{(W_0^0\epsilon+W^{(0)})}\left[WW^{(0)}\frac{(\vec{\kappa}\cdot\vec{\kappa}^{(0)})}{\kappa\kappa^{(0)}}-\epsilon\kappa\kappa^{(0)}\right]\ , \tag{2.26}$$

where

$$\begin{aligned} W^2&=k_0^2\epsilon-\kappa^2\ ,\\ W^{(0)2}&=k_0^2\epsilon-(\kappa^0)^2\ ,\\ W_0^{02}&=k_0^2-(\kappa^{(0)})^2\ ,\\ W_0^2&=k_0^2-\kappa^2\ . \end{aligned} \tag{2.27}$$

In Sec. IV, we will examine the numerical evaluation of (2.26) for certain models of the material medium.

Finally, it should be noted that the calculations given in this section determine the Green's function G_{ij} for the electromagnetic problem in the presence of a rough surface:

$$G_{ij}(\vec{r},\vec{r}_0,\omega)=\frac{\partial E_i(\vec{r},\omega)}{\partial[p_j(\omega^2/c^2)]} \tag{2.28}$$

with $\vec{E}$ given by (2.10). The far-zone behavior of such a Green's function is obtained from (2.20). The above analysis then shows the type of resonant structure that $G^{(1)}$ can have, even when $\vec{r}$ is in the far zone. Such Green's functions are quite important in the theory of surface-enhanced Raman scattering.[14]

III. EFFECTS OF THE NONLOCALITY OF THE DIELECTRIC FUNCTION $\epsilon(\vec{k},\omega)$ ON THE RADIATION EMITTED BY A DIPOLE NEAR A ROUGH SURFACE

In this section we examine how the nonlocal nature of the dielectric function affects the radiation properties of the dipole near a rough surface. When the dielectric function depends on $\vec{k}$, then the electromagnetic problem is an involved one and has no unique solution because of the difficulties associated with the question of additional boundary conditions.[15–22] Let us write the k dependence of the dielectric function as

$$\epsilon(\vec{k},\omega)=\epsilon_0+\frac{\chi}{k^2-\mu^2}\rightarrow\epsilon_t\equiv\epsilon_0-\frac{\chi}{\mu^2} \tag{3.1}$$

where

$$\mu^2=\eta(\omega^2-\omega_0^2+i\omega\Gamma)\ . \tag{3.2}$$

The parameters η, ω_0, χ, etc. depend on the medium. For example, if a metallic medium is treated in hydrodynamic approximation, then the longitudinal dielectric function is well approximated by (3.1)

$$\begin{gathered}\epsilon_0=1,\ \ \omega_0=0,\ \ \chi=\omega_p^2/\beta\ ,\\ \eta=1/\beta,\ \ \beta=\tfrac{3}{5}v_F^2\ ,\end{gathered} \tag{3.3}$$

where v_F represents the Fermi velocity. The transverse dielectric function ϵ_t can be taken to be independent of k. On the other hand, for an excitonic medium in effective-mass approximation, ϵ_0 is the high-frequency dielectric constant, ω_0 is the excitonic frequency, and

$$\chi=4\pi\alpha\omega_0^2\eta,\ \ \eta=\frac{m_e^*}{\hbar\omega_0}\ . \tag{3.4}$$

Since the results are sensitive to the additional boundary conditions, we assume that the electric induction and the electric field for an excitonic medium occupying domain V are related by

$$\vec{D}(\vec{r},\omega)=\epsilon_0\vec{E}(\vec{r},\omega)+\frac{\chi}{4\pi}\int_V\frac{\exp(i\mu\,|\vec{r}-\vec{r}\,'|)}{|\vec{r}-\vec{r}\,'|}\vec{E}(\vec{r}\,',\omega)\times d^3r'\ . \tag{3.5}$$

For the metallic medium, we will use the additional boundary condition that the normal component of the current at the boundary is zero. This is the additional boundary condition most extensively used for a metallic medium. Ruppin[23] and Dasgupta and Fuchs[22] have used it to study the Mie scattering and the polarizability of the sphere when the hydrodynamic dispersion of the metal is taken into account. Using these boundary conditions, we solve Maxwell's equations to obtain the field distribution in the two cases.

A. Spatially dispersive dielectric medium

The most general solution of the integral equation (3.5) is given in Ref. 20, for arbitrary domain V. The explicit form of the perturbative solution for the case of an electromagnetic field incident on a rough surface, i.e., for the case when V represents the domain $Z+hf(x,y)>0$, has been obtained in Ref. 21. Such a solution can be used, in analogy to our treatment in Sec. II, to obtain the field distribution produced by a radiating dipole in the neighborhood of the rough surface of a spatially dispersive medium. The fields in the domain $z<z_0$ would be given by (2.10) but now the zeroth-order and first-order contributions like $\varepsilon_s^{(0)},\varepsilon_s^{(1)},\varepsilon_p^{(0)},\varepsilon_p^{(1)}$ would be different. The zeroth-order terms can be written in terms of the well-known[19] Fresnel relations for a spatially dispersive medium ($z<0$):

$$\varepsilon_s^{(0)}(\vec{\kappa})=\frac{1}{2W_0}[(W_0-W_1)\varepsilon_{1s}^{(0)}(\vec{\kappa})+(W_0-W_2)\varepsilon_{2s}^{(0)}(\vec{\kappa})]\ , \tag{3.6}$$

$$\varepsilon_{1s}^{(0)}(\vec{\kappa})=2W_0\varepsilon_s^{(>)}(\vec{\kappa})\left[(W_1+W_0)-\frac{\alpha_1}{\alpha_2}(W_2+W_0)\right]^{-1},\ \ \varepsilon_{2s}^{(0)}=\varepsilon_{1s}^{(0)}\,|_{1\rightleftarrows2}\ , \tag{3.7}$$

$$\varepsilon_p^{(0)}(\vec{\kappa})=-\varepsilon_p^{(>)}(\vec{\kappa})(M+W_0P)/(M-W_0P)\ , \tag{3.8}$$

$$M=\left[\kappa^2k_0^2+\frac{\alpha_l}{\alpha_1}\frac{\kappa^2+W_2W_l}{\omega_2-\omega_1}k_0^2W_1+1\rightleftarrows 2\right], \tag{3.9}$$

$$P=-\left[\frac{\alpha_l}{\alpha_1}\left[\frac{\kappa^2+W_2W_l}{W_2-W_1}\right]k_1^2+1\rightleftarrows 2\right], \tag{3.10}$$

where $\varepsilon^{\gtrless}$ are the dipolar fields given by (2.6)–(2.9). The various W's and α's are functions of κ and are defined by

$$\alpha_i=(W_i-W_\mu)^{-1},\ i=1,2,l,\ W_i^2=k_i^2-\kappa^2, k_i^2=k_0^2\epsilon,\ i=1,2\ ,$$
$$W_l^2=k_l^2-\kappa^2,\ \epsilon(k_l,\omega)=0,\ W_0^2=k_0^2-\kappa^2,\ W_\mu^2=\mu^2-\kappa^2\ . \tag{3.11}$$

The wave vectors of the two transverse waves inside the medium are represented by $\vec{k}_1,\vec{k}_2$ and that of the longitudinal wave by $\vec{k}_l$. Note again that the vanishing of the denominator $M-W_0P$ gives the dispersion relation for surface polaritons inside the spatially dispersive medium. The first-order fields also involve the zeroth-order fields *inside* the medium. The s components of the zeroth-order transverse fields in the medium are given by (3.7). The p components (inside the medium) are given by

$$\varepsilon_{1p}^{(0)}(\vec{\kappa})=\left[\frac{\alpha_l}{\alpha_1}\right]\left[-\frac{k_1}{\kappa}\right]\frac{\kappa^2+W_2W_l}{W_1-W_2}\varepsilon_l^{(0)}(\vec{\kappa}),\ \ \varepsilon_{2p}^{(0)}(\vec{\kappa})=\varepsilon_{1p}^{(0)}(\vec{\kappa})\,|_{1\rightleftarrows 2}\ . \tag{3.12}$$

The longitudinal field inside the medium is given by

$$\varepsilon_l^{(0)}(\vec{\kappa})=2W_0\kappa k_0(M-W_0P)^{-1}\varepsilon_p^{(>)}(\vec{\kappa})\ . \tag{3.13}$$

We now present expressions for the first-order dipolar fields in the case of a dipole in the vicinity of a rough surface. The results given below are generalizations of the results obtained in Ref. 21. The p component of the first-order field, for $z<0$, is given by

$$\varepsilon_p^{(1)}(\vec{\kappa})=-\frac{1}{2W_0\kappa k_0(M-W_0P)}\left\{M\left[-2W_0(\vec{\kappa}\cdot\vec{X})+\kappa^2B+\left[-\frac{2W_0k_1^2(\vec{\kappa}\cdot\vec{Y}+W_2Y_z)}{\alpha_1(W_2-W_1)}+1\rightleftarrows 2\right]\right]\right.$$
$$\left.+\,W_0P\left[\kappa^2C+\left[-\frac{2W_1k_0^2(\vec{\kappa}\cdot\vec{Y}+W_2Y_z)}{\alpha_1(W_2-W_1)}+1\rightleftarrows 2\right]\right]\right\} \tag{3.14}$$

and the s component $(z<0)$ by

$$\varepsilon_s^{(1)}(\vec{\kappa})=\frac{1}{2W_0}\left[(W_0-W_1)\varepsilon_{1s}^{(1)}(\vec{\kappa})+(W_0-W_2)\varepsilon_{2s}^{(1)}(\vec{\kappa})-\frac{(\vec{\kappa}\times\vec{X})z}{\kappa}\right]. \tag{3.15}$$

The various functions $\vec{X},\vec{Y},\varepsilon_{1s}^{(1)},A,B,C$ are given by

$$\varepsilon_{1s}^{(1)}=\left[(W_1+W_0)-\frac{\alpha_1}{\alpha_2}(W_2+W_0)\right]^{-1}\left[\frac{W_2+W_0}{\alpha_2}\frac{(\vec{\kappa}\times\vec{Y})z}{\kappa}-\frac{(\vec{\kappa}\times\vec{X})z}{\kappa}\right],$$

$$\varepsilon_{2s}^{(1)}=\varepsilon_{1s}^{(1)}(1\rightleftarrows 2)\ , \tag{3.16}$$

$$\vec{Y}=-i\int d^2\kappa' F(\vec{\kappa}-\vec{\kappa}')\sum_{i=1,2,l}\vec{\varepsilon}_i^{(0)}(\vec{\kappa}')\ , \tag{3.17}$$

$$\vec{X}=-i\int d^2\kappa' F(\vec{\kappa}-\vec{\kappa}')\left[\sum_{i=1,2,l}(k_i^2-k_0^2)\vec{\varepsilon}_i^{(0)}(\vec{\kappa}')+k_l^2\varepsilon_l^{(0)}(\vec{\kappa}')\{\vec{\kappa}-\vec{\kappa}'+\hat{z}[W_0-W_l(\kappa')]\}\right], \tag{3.18}$$

$$C=-2i\int d^2\kappa' F(\kappa-\kappa')\left[\sum_{i=1,2,l}(k_i^2-k_0^2)\varepsilon_{iz}^{(0)}(\kappa')-k_l^2\varepsilon_l^{(0)}(\vec{\kappa}')W_l(\kappa')\right],$$

$$B=-2iW_0\int d^2\kappa' F(\kappa-\kappa')k_l^2\varepsilon_l^{(0)}(\vec{\kappa}')\ . \tag{3.19}$$

The zeroth-order transmitted fields that occur in (3.16)–(3.19) are given by (3.7), (3.12), and (3.13). Thus the complete expression for the first-order dipolar field is known in terms of $\varepsilon_p^{(>)}$. The Green's function for the problem at hand can then be obtained using (2.28). The far-zone behavior of the fields is obtained using (2.20). The zeroth-order fields (p components) involve denominators,

$$M - W_0 P = k_0 \kappa^2 + \left[\left[\frac{\alpha_l}{\alpha_1} \right] \frac{(k_0^2 W_1 + W_0 k_1^2)(\vec{K}_2 \cdot \vec{K}_l)}{(W_2 - W_1)} + 1 \rightleftarrows 2 \right] , \tag{3.20}$$

the vanishing of which results in the surface polariton dispersion relation[24] in a spatially dispersive medium. Since the first-order fields involve a superposition of the zeroth-order fields, with a weight factor that depends on surface roughness, it is clear that the excitation of surface polaritons would have a dramatic effect on the first-order fields. The case of a dipole oriented along the z axis is much simpler since the zeroth-order fields have no s components. In the next section the field (3.14) will be computed for various directions of observation.

B. Metallic medium

The electromagnetic fields outside and inside the medium can be written in the form

$$\begin{aligned} \vec{E} = & \int\int d^2\kappa \exp(i\vec{\kappa}\cdot\vec{r} + iW_t z)[\vec{\varepsilon}_t^{(0)}(\vec{\kappa}) + h\vec{\varepsilon}_t^{(1)}(\vec{\kappa}) + \cdots] \\ & + \int\int d^2\kappa \exp(i\vec{\kappa}\cdot\vec{r} + iW_l z)\vec{K}_l[\varepsilon_l^{(0)}(\vec{\kappa}) + h\varepsilon_l^{(1)}(\vec{\kappa}) + \cdots], \quad z > -hf , \end{aligned} \tag{3.21}$$

$$\vec{E} = \int\int d^2\kappa \exp(i\kappa\cdot\vec{r} - iW_0 z)[\vec{\varepsilon}_R^{(0)}(\vec{\kappa}) + h\vec{\varepsilon}_R^{(1)}(\vec{\kappa}) + \cdots], \quad z < -hf , \tag{3.22}$$

$$\vec{E}^{(i)} = \int\int d^2\kappa \exp(i\vec{\kappa}\cdot\vec{r} + iW_0 z)\vec{\varepsilon}^{(i)}(\vec{\kappa}) , \tag{3.23}$$

where $\vec{\varepsilon}^{(i)}$ is the incident field produced by an oscillating dipole at $z = z_0$ on the medium and

$$W_t^2 = \frac{\omega^2}{c^2}\epsilon_t(\omega) - \kappa^2, \quad \epsilon_t = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma} . \tag{3.24}$$

The boundary conditions that are to be used at $z = -hf$ are (i) tangential component of the electric field continuous, (ii) tangential component of the magnetic field continuous, and (iii) normal component of the current zero. The last boundary condition when combined with the continuity of the normal component of the electric induction leads to the continuity of the normal component of the electric field. It can be shown that the matching of these boundary conditions leads to the following equations:

$$\begin{aligned} & \vec{\varepsilon}_R^{(0)}(\vec{\kappa}) + \vec{\varepsilon}^{(i)}(\vec{\kappa}) = \varepsilon_t^{(0)}(\vec{\kappa}) + \vec{K}_l \varepsilon_l^{(0)}(\vec{\kappa}) , \\ & \vec{\varepsilon}_t^{(1)}(\vec{\kappa}) + \vec{K}_l \varepsilon_l^{(1)}(\vec{\kappa}) - \vec{\varepsilon}_R^{(1)}(\vec{\kappa}) = \vec{x}(\vec{\kappa}) , \\ & \hat{z}\times[\vec{K}_t \times \vec{\varepsilon}_t^{(0)}(\vec{\kappa})] = \hat{z}\times[\vec{K}_0 \times \vec{\varepsilon}^{(i)}(\vec{\kappa})] + \hat{z}\times[\vec{K}_0' \times \vec{\varepsilon}_R^{(0)}(\vec{\kappa})] , \\ & \hat{z}\times[\vec{K}_t \times \vec{\varepsilon}_t^{(1)}(\vec{\kappa})] - \hat{z}\times[\vec{K}_0' \times \vec{\varepsilon}_R^{(1)}(\vec{\kappa})] = \vec{y}(\vec{\kappa}) , \end{aligned} \tag{3.25}$$

where

$$\vec{K}_0 = (\vec{\kappa}, W_0), \quad \vec{K}_0' = (\vec{\kappa}, -W_0), \quad \vec{K}_t = (\vec{\kappa}, W_t) ,$$

$$\begin{aligned} & \vec{x}(\vec{\kappa}) = i \int d^2\kappa' F(\vec{\kappa} - \vec{\kappa}')\{ W_0(\vec{\kappa}')[\varepsilon_R^{(0)}(\vec{\kappa}') - \varepsilon^{(i)}(\vec{\kappa}')] W_t(\vec{\kappa}')\vec{\varepsilon}_t^{(0)}(\vec{\kappa}') + W_l(\vec{\kappa}')\vec{K}_l(\vec{\kappa}')\varepsilon_l^{(0)}(\vec{\kappa}')\}, \\ & \vec{y}(\vec{\kappa}) = i \int d^2\kappa' F(\vec{\kappa} - \vec{\kappa}')(W_0(\vec{\kappa}')\{\hat{z}\times[\vec{K}_0'(\vec{\kappa}') \times \vec{\varepsilon}_R^{(0)}(\vec{\kappa}')] - \hat{z}\times[\vec{K}_0(\vec{\kappa}') \times \vec{\varepsilon}^{(i)}(\vec{\kappa}')]\} \\ & \qquad + W_t(\vec{\kappa}')\hat{z}\times[\vec{K}_t(\vec{\kappa}') \times \vec{\varepsilon}_t^{(0)}(\vec{\kappa}')] \\ & \qquad + (\vec{\kappa} - \vec{\kappa}')\times[\vec{K}_0'(\vec{\kappa}') \times \vec{\varepsilon}_R^{(0)}(\vec{\kappa}') + \vec{K}_0(\vec{\kappa}') \times \vec{\varepsilon}^{(i)}(\vec{\kappa}') - \vec{K}_t(\vec{\kappa}') \times \vec{\varepsilon}_t^{(0)}(\vec{\kappa}')]) . \end{aligned} \tag{3.26}$$

The above linear equations can be solved in general. Instead of presenting the general solution, we give the result for the first-order fields produced by a dipole oriented along the z axis. As before, for such an orientation of the dipole s components of all the fields are zero. The zeroth-order fields are found to be

$$\begin{aligned}
&\varepsilon_{tz}^{(0)}=\frac{k_0^2}{k_t^2}(\varepsilon_{Rz}^{(0)}+\varepsilon_z^{(i)}) ,\\
&\varepsilon_l^{(0)}=\frac{(W_t+W_0)\varepsilon_{Rz}^{(0)}+(W_t-W_0)\varepsilon_z^{(i)}}{(\kappa^2+W_tW_l)} ,\\
&\varepsilon_{Rz}^{(0)}=D^{-1}W_t(\kappa^2+W_tW_l)\varepsilon_z^{(i)}\left[-k_0^2+\frac{k_t^2(\kappa^2+W_0W_l)}{\kappa^2+W_tW_l}\right] ,\\
&D\equiv(\kappa^2+W_tW_l)(W_0k_t^2+k_0^2W_t)-\kappa^2k_t^2(W_t+W_0) .
\end{aligned} \tag{3.27}$$

The z component of the first-order field $\vec{\varepsilon}_R{}^{(1)}$ is found to be

$$\varepsilon_{Rz}^{(1)}=D^{-1}\{k_t^2[k^2x_z-(\vec{\kappa}\cdot\vec{x})W_l]-(\vec{\kappa}\cdot\vec{y})(\kappa^2+W_tW_l)\}W_t \tag{3.28}$$

with

$$\begin{aligned}
&(\vec{\kappa}\cdot y)\equiv ik_0^2\int d^2\kappa' F(\vec{\kappa}-\vec{\kappa}')\{\vec{\kappa}\cdot[\vec{\varepsilon}_R^{(0)}(\vec{\kappa}')+\vec{\varepsilon}^{(i)}(\kappa')]-\varepsilon_t[\vec{\kappa}\cdot\vec{\varepsilon}_t^{(0)}(\kappa')]\} ,\\
&x_z\equiv i\int d^2\kappa' F(\vec{\kappa}-\vec{\kappa}')\{W_0(\vec{\kappa}')[\varepsilon_{Rz}^{(0)}(\vec{\kappa}')-\varepsilon_z^{(i)}(\vec{\kappa}')]+W_t(\vec{\kappa}')\varepsilon_{tz}^{(0)}(\vec{\kappa}')+W_l^2(\vec{\kappa}')\varepsilon_l^{(0)}(\vec{\kappa}')\} ,\\
&(\vec{\kappa}\cdot\vec{x})\equiv i\int d^2\kappa' F(\vec{\kappa}-\vec{\kappa}')\{W_0(\vec{\kappa}')[\vec{\kappa}\cdot\vec{\varepsilon}_R^{(0)}(\vec{\kappa}')-\vec{\kappa}\cdot\vec{\varepsilon}^{(i)}(\vec{\kappa}')]+W_t(\kappa')[\vec{\kappa}\cdot\vec{\varepsilon}_t^{(0)}(\vec{\kappa}')]\\
&\qquad\qquad +W_l(\vec{\kappa}')(\vec{\kappa}\cdot\vec{\kappa}')\varepsilon_l^{(0)}(\vec{\kappa}')\} .
\end{aligned} \tag{3.29}$$

The dot products that appear in (3.29) can be related to the z component of the field by utilizing the p-polarized nature of the field, e.g.,

$$\vec{\kappa}\cdot\vec{\varepsilon}_t^{(0)}(\vec{\kappa}')=-\frac{(\vec{\kappa}\cdot\vec{\kappa}')W_t(\kappa')}{\kappa'^2}\varepsilon_{tz}^{(0)}(\vec{\kappa}') . \tag{3.30}$$

Expression (3.28) will be used in the next section to study the effect of the hydrodynamic dispersion of the dielectric function associated with a metallic medium. The new surface-plasmon dispersion relation will be given by the vanishing of D [Eq. (3.27)] and is in agreement with the result of Fuchs and Kliewer.[18]

IV. NUMERICAL RESULTS FOR THE RADIATION FROM A DIPOLE IN THE PRESENCE OF A GRATING

To see the effects of the excitation of surface polaritons and the conversion of these two polaritons into radiation because of surface roughness, we have evaluated the radiation in the far zone assuming that the roughness is in the form of a grating, i.e.,

$$f(x,y)=\sin gy,\quad F(\vec{\kappa})=\frac{1}{2i}[\delta(\vec{\kappa}-\vec{g})-\delta(\vec{\kappa}+\vec{g})] . \tag{4.1}$$

We assume for concreteness that the dipole is oriented along the z axis and evaluate the p component of the emitted radiation. The important contribution to the radiation [Eq. (2.22)] comes from $\varepsilon_p^{(1)}$ terms and hence we compute explicitly the quantity defined by

$$S^{(2)}(\theta,\phi)=\cos^2\theta\,|\,\varepsilon_p^{(1)}(k_0\sin\theta\cos\phi,k_0\sin\theta\sin\phi)\,|^2 \tag{4.2}$$

and for the sake of illustration we take $\phi=\pi/4$. On substituting (4.1) in (2.26), we obtain for the p component of the first-order dipolar field in the far zone ($\vec{\kappa}=k_0\sin\theta\cos\phi,k_0\sin\theta\sin\phi$)

$$\varepsilon_p^{(1)}(\vec{\kappa})=-\frac{(\epsilon-1)}{(W_0\epsilon+W)}\left[\left[WW_- \frac{(\vec{\kappa}\cdot\vec{\kappa}_-)}{\kappa\kappa_-}-\epsilon\kappa\kappa_-\right]\frac{W_{0-}\varepsilon_p^{(>)}(\kappa_-)}{(W_{0-}\epsilon+W_-)}-\cdots\right] , \tag{4.3}$$

where the ellipsis represents terms with $-\rightarrow+$ and where $\varepsilon_p^{(>)}(\vec{\kappa}_-)$ is given by (2.23) with $\vec{\kappa}\rightarrow\vec{\kappa}_-$ and where

$$\vec{\kappa}_{\pm}=\vec{\kappa}\pm\vec{g} \Longrightarrow \vec{\kappa}_{\pm}=(k_0\sin\theta\cos\phi, k_0\sin\theta\sin\phi\pm g) ,$$
$$W_{\mp}^2=k_0^2\epsilon-\kappa_{\mp}^2, W_{0\mp}^2=k_0^2-\kappa_{\mp}^2 . \tag{4.4}$$

It is thus clear that the radiation in the direction $\vec{\kappa}$ can arise from the components $\vec{\kappa}_{\pm}$ in the angular spectrum representation of the free space dipolar field. The denominator $W_0\epsilon+W$ in (4.3) cannot vanish since $|\kappa|<\omega/c$, but the denominators $(W_{0\pm}\epsilon+W_{\pm})$ can vanish if

$$\kappa_{\pm}^2=\frac{k_0^2\epsilon(\omega)}{\epsilon(\omega)+1} . \tag{4.5}$$

Of course, since $\kappa_{\pm}$ and ω are real, one really has a resonant structure corresponding to values given by (4.5), i.e., for (θ,ϕ) values around (θ_0,ϕ_0) such that

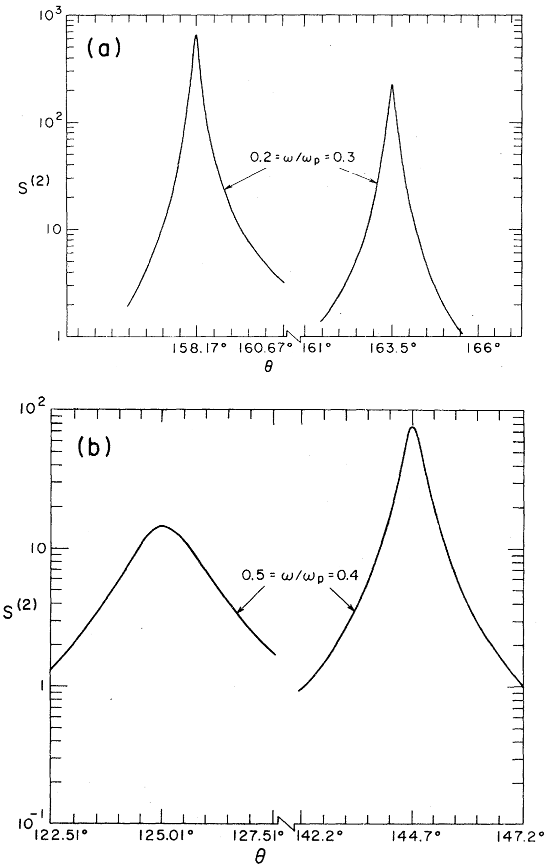

FIG. 1. Signal $S^{(2)}$, in arbitrary units, as a function of the angle θ of observation, in the resonant region, for azimuthal angle 45°; for the case of a dipole located at $(0,0,z_0)$ near a metallic grating. The curves are in the absence of any hydrodynamic dispersion. The dipolar frequencies are as indicated in the figure. The other parameters have been chosen as $\Gamma=10^{-2}\omega_p, \omega_p|z_0|/c=0.1$ and the grating periodicity to be taken to be order of a wavelength $gc/\omega_p=0.25$.

$$k_0^2\sin^2\theta_0+g^2\pm 2gk_0\sin\theta_0\sin\phi_0=k_0^2\frac{\mathrm{Re}\epsilon(\omega)}{\mathrm{Re}\epsilon(\omega)+1} . \tag{4.6}$$

One can similarly write the analog of (4.2) for the case when the spatial dispersion of the dielectric is taken into account.

The results of our numerical computations are shown in Figs. 1–4 for two different types of material medium. We plot $S^{(2)}(\theta,\pi/4)$ as a function of the angle θ of observation for various values of the frequency ω. We only show the resonant region, i.e., the region in which (4.6) is satisfied. In Figs. 1 and 2, we show the behavior for the case of a metal with longitudinal and transverse dielectric functions

$$\epsilon_l(\vec{\kappa},\omega)=1-\frac{\omega_p^2}{(\omega^2+i\omega\Gamma-\beta k^2)} ,$$
$$\beta=\tfrac{3}{5}v_F^2 \tag{4.7}$$
$$\epsilon_t(\vec{\kappa},\omega)=1-\omega_p^2/(\omega^2+i\omega\Gamma) .$$

The parameter β can be rewritten in terms of $\Delta=\hbar\omega_p/E_F$ as

$$\frac{5\beta}{3c^2}=\left[\frac{0.012\,388}{\Delta^2}\right]^2 . \tag{4.8}$$

The dipole is assumed to be at a short distance $(\ll\lambda)$ from the surface. Note that such short distances were used in the experiments[6] of Pockrand *et al.* In Fig. 1 we show the behavior of the dipole radiation in the absence of spatial dispersion. The resonances in $S^{(2)}$ occur in accordance with (4.6). For dipolar frequencies far away from static surface

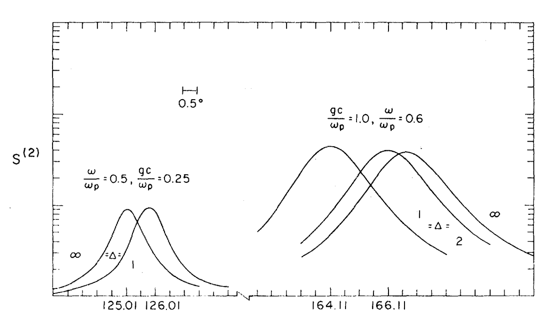

FIG. 2. Same as Fig. 1, but now the hydrodynamic dispersion of the metal is taken into account; the other parameters for various curves are as shown.

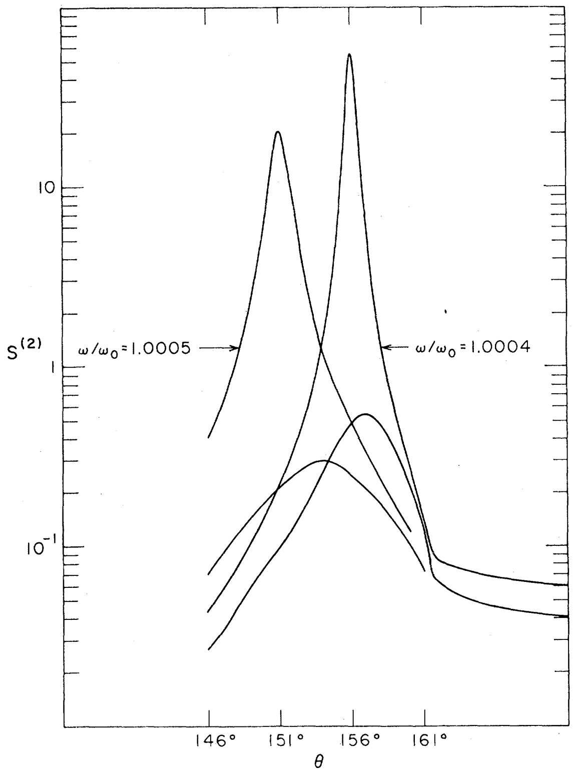

FIG. 3. Same as in Fig. 1, but the material medium is now a dielectric grating [Eq. (4.9)] with parameters $4\pi\alpha = 0.0125$, $\epsilon_0 = 8.3$, $\hbar\Gamma = 5\times10^{-5}$ eV, $\hbar\omega_0 = 2.5524$ eV, $m_e^* c^2/\hbar\omega_0 = 1.8045\times10^5$, $\omega_0 \mid z_0 \mid /c = 0.1$, and $gc/\omega_0 = 0.75$. The upper (lower) curves give the signal in the absence (presence) of spatial dispersion.

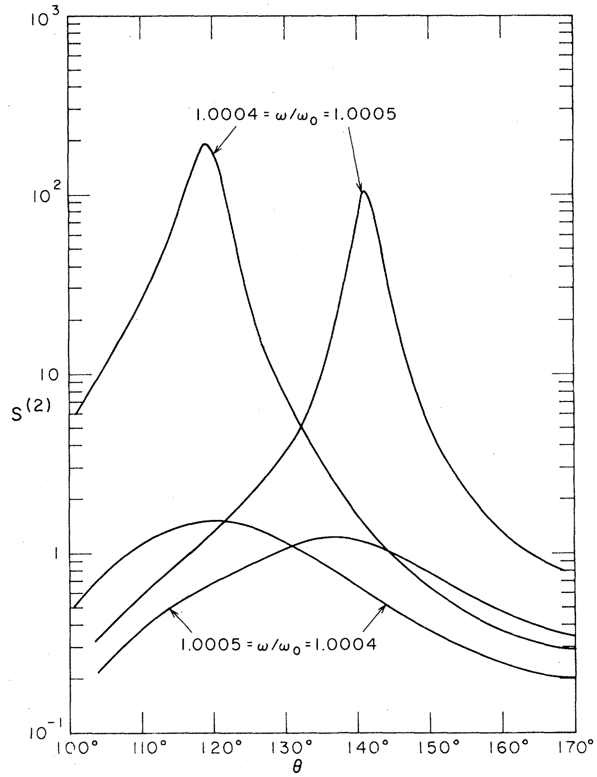

FIG. 4. Same as in Fig. 3, but with a reduced grating periodicity $gc/\omega_0 = 1.5$.

plasmon frequency $\omega_p/\sqrt{2}$, the effect of hydrodynamic dispersion is found to be unimportant and hence is not displayed. However, as one approaches the $\omega_p/\sqrt{2}$, the effect of hydrodynamic dispersion becomes quite pronounced as seen in Fig. 2. The most notable effect of the hydrodynamic dispersion is to shift the position of the peak.

Figures 3 and 4 give the variation of the signal $S^{(2)}$ as a function of θ for the dipolar radiation in the presence of a dielectric grating with the dielectric function

$$\epsilon(\vec{\kappa},\omega) = \epsilon_0 + \frac{4\pi\alpha\omega_0^2}{\left[\dfrac{\hbar\omega_0}{m_e^*}k^2\right] - (\omega^2 - \omega_0^2 + i\omega\Gamma)} . \tag{4.9}$$

Such a dielectric function has been extensively used[15,19,20,22] in the study of the optical properties of materials, such as CdS and ZnSe, near an isolated excitonic resonance. The upper curves give results in the absence of spatial dispersion ($m_e^* \to \infty$), whereas the lower ones are in the presence of spatial dispersion. In the case of a dielectric material with dielectric constant of the form (4.9), the effect of spatial dispersion is quite pronounced in contrast to the case of a metal.[25] A shift in the resonance position due to the spatially dispersive nature of the medium may also be noticed. The grating periodicity is also found to change the width of the resonance as is seen from a comparison of Figs. 3 and 4. This can be understood from the behavior of the denominator ($W_{0\pm}\epsilon + W_\pm$) or equivalently that of $\kappa_\pm^2 - [k_0^2\epsilon/(\epsilon+1)] \equiv D_\pm$. Assuming that (4.6) is satisfied with the upper sign, then writing the roots of (4.6) as α and β, we obtain for the denominator

$$D_+ \equiv (\sin\theta - \alpha)(\sin\theta - \beta) - \left[\frac{\epsilon}{\epsilon+1} - \frac{\text{Re}\epsilon}{\text{Re}\epsilon + 1}\right] . \tag{4.10}$$

The term in large parentheses is proportional to $\text{Im}\epsilon$ or to the damping Γ. Thus in the vicinity of resonance $\sin\theta = \alpha$, (4.10) becomes

$$D_+ \approx (\alpha-\beta)\left[\sin\theta-\alpha - \frac{i}{(\alpha-\beta)}\left[\frac{\epsilon}{\epsilon+1}-\frac{\text{Re}\epsilon}{\text{Re}\epsilon+1}\right]\right]. \tag{4.11}$$

The width of the resonance thus depends on $\alpha-\beta$, which obviously depends on the grating periodicity. This dependence is in addition to its dependence on ω.

Thus in conclusion we have shown how the excitation of the surface polaritons by the dipolar field is reflected in the far-field radiation pattern produced by the dipole in the presence of surface roughness and how the nature of the resonances in the radiation is affected by the nonlocal nature of the dielectric function of the medium.

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