

Light scattering on free electrons in semiconductors in the presence of electron-relaxation processes

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The cross section for the scattering of light is obtained for the screened single-particle excitations and the collective plasma modes of a single parabolic band, taking into account the different electron-scattering processes. The dielectric constant in the random-phase approximation is evaluated, including the effect of *energy-dependent* electron-relaxation processes. Thereby, the conservation of the local electron number is observed. We find that with an increasing scattering rate the line shape of the single-particle spectrum changes from a *Gaussian* to a *Lorentzian* form and the shape of the *plasma-mode* spectrum is *shifted* and *broadened* by the collisions. The details of the line shapes, as a function of the frequency ω and the temperature T , are determined by the energy dependence of the relaxation rate. The general theoretical results are used for a comparison with experimental data on GaAs.

I. INTRODUCTION

The scattering of light from a solid-state plasma provides certain information on the excitation spectrum of the electron gas. The spectrum consists of the two parts: the single-particle spectrum and the collective part. Since the single-particle scattering is caused by the individual electrons (Compton scattering) this portion of the spectrum mirrors the Maxwellian velocity distribution in the case of a nondegenerate electron gas. In addition, there is the collective-mode scattering due to plasmon excitations in the electron gas. At room temperature and for electron concentrations of the order of 10^{16} cm^{-3} and less, the single-particle spectrum overlaps with the collective plasmon excitations. Hence the plasma modes are Landau damped.

In this paper we address ourselves to the question: How is the frequency-resolved scattering cross section affected by the electron-scattering processes in the solid that bring about a relaxation of the electron distribution to local equilibrium? In particular, we consider the scattering of electrons by charged impurities and by polar optical modes, since these are the two dominant scattering processes for several of the III-V compounds at room temperature. In fact, recent experimental results on GaAs obtained by Abramsohn, Tsen, and Bray¹ ex-

hibit a more Lorentzian behavior of the scattered light spectrum on the samples with high electron (and hence impurity) concentrations. This behavior replaces the Gaussian line shape to be expected in the absence of relaxation processes, observed first on samples with small electron concentrations by Mooradian.^{2,3}

In Sec. II we present the theory of light scattering in the presence of relaxation processes, emphasizing the importance that the local electron number must be conserved. In the brief Sec. III we give the formulas for the scattering times $\tau(p)$ for those mechanisms which are considered to bring about the relaxation of the electron distribution to local equilibrium. In Secs. IV and V we present some theoretical results for the energy and temperature dependencies of the scattering cross section pertaining to *n*-type GaAs. Finally, we compare our results with the experimental findings by the above authors for the scattering of light by the conduction-electron system in GaAs at $T=300$ K.

II. THEORY: LIGHT SCATTERING BY AN ELECTRON GAS IN THE RELAXATION-TIME APPROXIMATION

The cross section for the scattering of light by density fluctuations of the electrons is directly pro-

portional to the imaginary part of the inverse dielectric function. For Stokes scattering the cross section for scattering into the frequency interval $d\omega$ is given by

$$\frac{d\sigma}{d\omega} = c[n(\omega) + 1] \text{Im}\epsilon^{-1}(q, \omega), \quad (1)$$

where q is the momentum transferred to the electrons by the scattered light, $n(\omega)$ is the Bose function, and the constant c is proportional to $(e^2/m^*c^2)^2$.^{4,5}

When taking into account the scattering of electrons by impurities and phonons, we must use the proper dielectric function. It is well known that the

effect of collisions generally is not accounted for by merely replacing ω by $\omega + i/\tau$ in the collisionless dielectric Lindhard function of the random-phase approximation (RPA).⁶ This approximation fails to conserve the local electron number, as pointed out by Mermin.⁷ The straightforward way of correction is by employing a relaxation-time approximation in which the collisions relax the electron-density distribution towards the local equilibrium density, instead of the uniform equilibrium value. To obtain the proper RPA dielectric function, we start from the conventional Boltzmann equation which is an equation of motion for $f(\vec{p}, \vec{R}, t)$, the average density of particles with momentum \vec{p} at the space-time point \vec{R}, t :

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p} \cdot \vec{\nabla}_{\vec{R}}}{m^*} \right] f(\vec{p}, \vec{R}, t) - \frac{1}{i} \int d^3r \int \frac{d^3p'}{(2\pi)^3} e^{i(\vec{p}' - \vec{p}) \cdot \vec{r}} \left[U_{\text{eff}} \left[\vec{R} + \frac{\vec{r}}{2}, t \right] - U_{\text{eff}} \left[\vec{R} - \frac{\vec{r}}{2}, t \right] \right] f(\vec{p}', \vec{R}, t) = \left[\frac{\partial f}{\partial t} \right]_{\text{coll}} = - \frac{f(\vec{p}, \vec{R}, t) - \bar{f}(\vec{p}, \vec{R}, t)}{\tau(\vec{p})}. \quad (2)$$

Here the relaxation-time approximation is used for the collision term; U_{eff} is defined below, Eq. (4), and $\hbar = 1$.

It is assumed that the density-distribution function relaxes towards the local density distribution,

$$\bar{f}(p, R, t) = \frac{1}{\exp\{\beta[\epsilon(\vec{p}) - \mu] - \beta\delta\mu(\vec{R})\} + 1}, \quad (3)$$

where $\delta\mu(\vec{R})$ is the local deviation of the uniform chemical potential; $\beta = 1/k_B T$, and we take $k_B = 1$.

The effective potential depends on the distribution function f ,

$$U_{\text{eff}}(\vec{R}, t) = U(\vec{R}, t) + \int d^3R' v(\vec{R} - \vec{R}') \int \frac{d^3p}{(2\pi)^3} [f(\vec{p}, \vec{R}', t) - f_0(\vec{p})], \quad (4)$$

where

$$f_0(\vec{p}) = \frac{1}{\exp\{\beta[\epsilon(\vec{p}) - \mu] + 1\}} \quad (5)$$

is the equilibrium density distribution, $v(\vec{R})$ is the Coulomb potential, and U is the external potential. Furthermore, it is assumed that the relaxation time $\tau(\vec{p})$ is isotropic and depends only on the particle energy $\epsilon(p)$.

With the ansatz

$$f(\vec{p}, \vec{R}, t) = f_0[\epsilon(p)] + \delta f(\vec{p}, \vec{R}, t), \quad (6)$$

we linearize Eq. (2), assuming that the external field U is small; then the deviation δf from the equilibrium distribution f_0 in the Fourier space is given by

$$\left[\omega - \frac{\vec{p} \cdot \vec{q}}{m^*} \right] \delta f(\vec{p}, \vec{q}, \omega) - \left[f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right] \right] \delta U_{\text{eff}}(\vec{q}, \omega) = -i \frac{\delta f(\vec{p}, \vec{q}, \omega) - \delta \bar{f}(\vec{p}, \vec{q}, \omega)}{\tau(p)}, \quad (7)$$

where

$$\delta U_{\text{eff}}(\vec{q}, \omega) = U(\vec{q}, \omega) + v(q) \int \frac{d^3p}{(2\pi)^3} \delta f(\vec{p}, \vec{q}, \omega). \quad (8)$$

In order to get $\delta \bar{f}$, we expand the local density-distribution function, Eq. (3), with respect to the local deviation $\delta\mu$ of the uniform chemical potential and obtain (for details see Appendix A)

$$\delta \bar{f}(\vec{p}, \vec{q}, \omega) = \frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\frac{\vec{p} \cdot \vec{q}}{m^*}} \delta \mu(\vec{q}). \quad (9)$$

By inserting Eq. (9) into Eq. (7) we get

$$\delta f(\vec{p}, \vec{q}, \omega) = \frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\omega + \frac{i}{\tau(p)} - \frac{\vec{p} \cdot \vec{q}}{m^*}} \left[\delta U_{\text{eff}}(\vec{q}) + \frac{i}{\tau(p)} \frac{m^*}{\vec{p} \cdot \vec{q}} \delta \mu(\vec{q}) \right]. \quad (10)$$

The density change is given by

$$\delta n(\vec{q}, \omega) = 2 \int \frac{d^3 p}{(2\pi)^3} \delta f(\vec{p}, \vec{q}, \omega). \quad (11)$$

By integrating the transport equation (7) over the phase-(\vec{p}) space we obtain

$$\omega \delta n(\vec{q}, \omega) = \vec{q} \cdot \vec{j}(\vec{q}, \omega) - 2i \int \frac{d^3 p}{(2\pi)^3} \frac{\delta f(\vec{p}, \vec{q}, \omega)}{\tau(p)} + 2i \delta \mu(\vec{q}) \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\tau(p)} \frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\frac{\vec{p} \cdot \vec{q}}{m^*}}. \quad (12)$$

In order that the relaxation-time approximation is made to conform to the continuity equation, Eq. (12), we are led to the following equation:

$$2 \int \frac{d^3 p}{(2\pi)^3} \frac{\delta f(\vec{p}, \vec{q}, \omega)}{\tau(p)} = \delta \mu(\vec{q}) 2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\tau(p)} \left[\frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right]}{\frac{\vec{p} \cdot \vec{q}}{m^*}} - \frac{f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\frac{\vec{p} \cdot \vec{q}}{m^*}} \right]. \quad (13)$$

The comparison of Eqs. (10) and (13) leads to

$$\begin{aligned} 2 \int \frac{d^3 p}{(2\pi)^3} \frac{\delta f(\vec{p}, \vec{q}, \omega)}{\tau(p)} &= \delta U_{\text{eff}}(\vec{q}, \omega) 2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\tau(p)} \left[\frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\omega + \frac{i}{\tau(p)} - \frac{\vec{p} \cdot \vec{q}}{m^*}} \right] \\ &\times 2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\tau(p)} \left[\frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\frac{\vec{p} \cdot \vec{q}}{m^*}} \right] \\ &\times \left[2 \int \frac{d^3 p}{(2\pi)^3} \left[\frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\omega \tau(p) + i} \right] \right] \\ &\times \left[\frac{\omega}{\frac{\vec{p} \cdot \vec{q}}{m^*}} - \frac{i}{\tau(p) \left[\omega + \frac{i}{\tau(p)} - \frac{\vec{p} \cdot \vec{q}}{m^*} \right]} \right]^{-1}. \end{aligned} \quad (14)$$

We now define the following integrals containing $\tau(p)$:

$$B(q, \omega) = -v(q)2 \int \frac{d^3p}{(2\pi)^3} \frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\omega + \frac{i}{\tau(p)} - \frac{\vec{p} \cdot \vec{q}}{m^*}}, \quad (15)$$

$$B_\tau(q, \omega) = -v(q)2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\omega \tau(p)} \frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\omega + \frac{i}{\tau(p)} - \frac{\vec{p} \cdot \vec{q}}{m^*}}, \quad (16)$$

$$B_{\tau\omega}(q, \omega) = -v(q)2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\omega \tau(p) + i} \left[\frac{1}{\omega + \frac{i}{\tau(p)} - \frac{\vec{p} \cdot \vec{q}}{m^*}} + \frac{1}{\frac{\vec{p} \cdot \vec{q}}{m^*}} \right] \left[f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right] \right], \quad (17)$$

where $v(q) = 4\pi e^2/q^2 \epsilon_\infty$. From the preceding equations, (10)–(14), and with defining Eqs. (15)–(17), we arrive at the general formula for the density change,

$$\delta n(q, \omega) = \frac{\delta U_{\text{eff}}(q, \omega)}{v(q)} \left[-B(q, \omega) + i \frac{B_\tau(\omega, q) B_{\tau\omega}(q, \omega)}{B_\tau(\omega, q) - B_{\tau\omega}(q, \omega)} \right]. \quad (18)$$

The dielectric function is defined by

$$\epsilon(q, \omega) = 1 - v(q) \frac{\delta n(q, \omega)}{\delta U_{\text{eff}}(q, \omega)}. \quad (19)$$

Hence

$$\epsilon(q, \omega) = 1 + B(q, \omega) - i \frac{B_\tau(q, \omega) B_{\tau\omega}(q, \omega)}{B_\tau(q, \omega) - B_{\tau\omega}(q, \omega)}. \quad (20)$$

This is the final result for ϵ (RPA), taking into account energy-dependent electron-relaxation processes.

For $\tau(p) = \text{const}$, Eq. (20) reduces to Mermin's formula,

$$\epsilon(q, \omega) = 1 + \frac{\left[1 + \frac{i}{\tau\omega} \right] B(q, \omega)}{1 + \frac{i}{\tau\omega} \frac{B(q, \omega)}{B(q, 0)}}, \quad (21)$$

where $B(q, \omega)$ is given by Eq. (15) and

$$B(q, 0) = v(q)2 \int \frac{d^3p}{(2\pi)^3} \frac{f_0 \left[\vec{p} - \frac{\vec{q}}{2} \right] - f_0 \left[\vec{p} + \frac{\vec{q}}{2} \right]}{\frac{\vec{p} \cdot \vec{q}}{m^*}} \quad (22)$$

For $\tau(p) \rightarrow \infty$, Eq. (20) simplifies to Lindhard's formula

$$\epsilon(q, \omega) = 1 + B(q, \omega). \quad (23)$$

The light-scattering cross section is determined by the imaginary part of the inverse dielectric function, Eq. (1). From Eq. (20) we get

$$\text{Im}\epsilon^{-1}(q,\omega) = \frac{-B'' + \frac{B'_{\tau\omega} |B_{\tau}|^2 - B'_{\tau} |B_{\tau\omega}|^2}{|B_{\tau} - B_{\tau\omega}|^2}}{\left[1 + B' + \frac{B''_{\tau\omega} |B_{\tau}|^2 - B''_{\tau} |B_{\tau\omega}|^2}{|B_{\tau} - B_{\tau\omega}|^2}\right]^2 + \left[-B'' + \frac{B'_{\tau\omega} |B_{\tau}|^2 - B'_{\tau} |B_{\tau\omega}|^2}{|B_{\tau} - B_{\tau\omega}|^2}\right]^2}, \quad (24)$$

where B' and B'' are the real and imaginary parts of B , respectively. For the special case $\tau = \text{const}$, Eq. (24) becomes

$$\text{Im}\epsilon^{-1}(q,\omega) = - \frac{-B''(q,\omega_+) + \frac{1}{\omega\tau B(q,0)} [|B(q,\omega_+)|^2 - B(q,0)B'(q,\omega_+)]}{[1 + B'(q,\omega_+)]^2 + B''^2(q,\omega_+) - 2B''(q,\omega_+) \frac{1 + B(q,0)}{\omega\tau B(q,0)} + \frac{|B(q,\omega_+)|^2 [1 + B(q,0)]^2}{[\omega\tau B(q,0)]^2}}, \quad (25)$$

where $\omega_+ \equiv \omega + i/\tau$. Equation (24) yields the general form of $d\sigma/d\omega$, Eq. (1), valid for a degenerate or a non-degenerate electron gas with energy-dependent τ .

In the case of semiconductors we are primarily interested in those electron concentrations and temperature regimes where the electron gas is nondegenerate. The Maxwellian distribution is given by

$$f_0(\vec{p}) = \frac{(4\pi)^{3/2} n}{2(2m^*T)^{3/2}} \exp\left[-\frac{p^2}{2m^*T}\right], \quad (26)$$

where n and m^* are the density and the effective mass of the electrons. For this distribution we can calculate the function B in closed form for the special case $\tau = \text{const}$; the details are given in Appendix B. The result is

$$B(q,\omega,\tau) = -i\pi C \left[w \left[\frac{m^*\omega + \frac{q^2}{2}}{q(2m^*T)^{1/2}} + \frac{im^*}{q(2m^*T)^{1/2}\tau} \right] - w \left[\frac{m^*\omega - \frac{q^2}{2}}{q(2m^*T)^{1/2}} + \frac{im^*}{q(2m^*T)^{1/2}\tau} \right] \right], \quad (27)$$

where

$$C = e^2(4\pi)^{3/2} nm^* / 2\pi\epsilon_{\infty} q^3 (2m^*T)^{1/2}.$$

Here ϵ_{∞} is the background dielectric constant. The function w is in terms of the error function given by⁸

$$w(z) = e^{-z^2} \text{erf}(-iz). \quad (28)$$

In the limit $\tau \rightarrow \infty$, we have

$$B'' = 2\pi C \exp\left[-\frac{\omega^2 m^*}{2q^2 T} - \frac{q^2}{8m^*T}\right] \sinh\left[\frac{\omega}{2T}\right]. \quad (29)$$

For small electron densities, the quantities $B', B'' \ll 1$ and can be neglected in the denominator of Eq. (25). With the relation

$$[n(\omega) + 1] 2 \sinh\left[\frac{\beta\omega}{2}\right] = \exp\left[\frac{\beta\omega}{2}\right], \quad (30)$$

where $n(\omega) = (e^{\beta\omega} - 1)^{-1}$, we arrive at the cross section,

$$\frac{d\sigma}{d\omega} = \text{const} \times \pi C \exp\left[-\frac{\left[\omega - \frac{q^2}{2m^*}\right]^2}{4T \frac{q^2}{2m^*}}\right]. \quad (31)$$

Hence $d\sigma/d\omega$ mirrors a Gaussian distribution displaced from $\omega = 0$ by $q^2/2m^*$, the Compton shift.

The Gaussian distribution is changed when electron collisions become important. A decrease of τ leads to an increase of the imaginary part in the argument of the function w in Eq. (27),

$$T_1^{-1} \equiv \frac{m^*}{q(2m^*T)^{1/2}\tau}. \quad (32)$$

By introducing the mean free path l

$$l = v_T \tau, \quad (33)$$

where v_T is the thermal velocity

$$v_T = \left[\frac{2T}{m^*}\right]^{1/2}, \quad (34)$$

we can express Eq. (32) as

$$T_1^{-1} = \frac{1}{lq}. \quad (35)$$

In the limit of small values for $\omega\tau$ [$\equiv ql(c/v_T)$, where c is the velocity of light in the medium], Eq. (25) becomes

$$\text{Im}\epsilon^{-1} = \frac{(\omega\tau)B_0(|B|^2 - B'B_0)}{|B|^2(1+B_0)^2 + (\omega\tau)^2[(1+B')^2 + B''^2] - 2B''(\omega\tau)(1+B_0)}, \quad (36)$$

with $B_0 = B(q, 0)$. Since B'' is an odd function of ω , the cross section for small frequencies is given by

$$\begin{aligned} \frac{d\sigma}{d\omega} &= \frac{\tau B_0(B'^2 - B'B_0)}{\omega^2 \left[\tau^2 B_0^2 (1+B')^2 - 2\frac{B''}{\omega} \tau (1+B_0) + \left(\frac{B''}{\omega}\right)^2 (1+B_0) \right] + B'^2 (1+B_0)^2} \\ &= \frac{a}{b^2 \omega^2 + c^2}, \end{aligned} \quad (37)$$

where a , b , and c are constants. The Gaussian line shape of Eq. (31) changes into the Lorentzian shape, Eq. (37), for small $\omega\tau$.

For sufficiently high electron densities and/or sufficiently low temperatures, collective plasma modes emerge from the single-particle spectrum. In the limit $\tau \rightarrow \infty$, the plasma peak is determined by

$$[1 + B'(q, \omega)] = 0, \quad (38)$$

which results in

$$\omega^2 = \frac{1}{2} \omega_p^2 \left[1 + \left(1 + \frac{12Tq^2}{m^* \omega_p^2} \right)^{1/2} \right], \quad (39)$$

with ω_p being the free-electron plasma frequency,

$$\omega_p^2 = \frac{4\pi n e^2}{\epsilon_\infty m^*}. \quad (40)$$

However, for small relaxation-time values, the plasma peak becomes less pronounced. It even may disappear altogether due to lifetime smearing.

For the general case of an energy-dependent τ , the integrals in Eqs. (15)–(17) cannot be evaluated analytically. However, in our case where the relaxation time is isotropic, the angle integration can be done analytically (cf. Appendix B). The final expressions for the real and the imaginary parts of B , B_τ , and $B_{\tau\omega}$ are to be calculated numerically by taking into account the known energy dependence of the relaxation time. With Eq. (24), the cross section for Stokes scattering is readily obtained.

III. RELAXATION TIMES

The energy dependence of the relaxation time is determined by the scattering mechanism. Let us

briefly review the relevant mechanisms and their energy dependences, with respect to our evaluation of $\epsilon[\omega, q, \tau(p)]$.

A. Ionized impurity scattering

For a homogeneous distribution of N_I ionized impurities (donors plus acceptors), Brooks and Herring have evaluated^{9,10} the relaxation time due to the scattering by the screened Coulomb potentials,

$$\frac{1}{\tau_{\text{imp}}} = \frac{\pi N_I e^4}{\sqrt{2} \epsilon_\infty^2 (m^*)^{1/2} \epsilon^{3/2}} \left[\ln(1 + b\epsilon) - \frac{b\epsilon}{1 + b\epsilon} \right], \quad (41)$$

where ϵ is the electron energy and the parameter $b = 2\epsilon_\infty m^* T / \pi e^2 n'$. For an uncompensated n -type semiconductor, both N_I and n' can be set equal to the electron concentration n . For a partially compensated n -type semiconductor, however, the number of ionized acceptors must be taken into account, leading to a density n' (which enters the above Brooks-Herring formula) larger than the average electron density n . In this case, N_I and n' are given by

$$N_I = 2N_A + n \quad (42)$$

and

$$n' = n + (n + N_A) \left[1 - \frac{nN_A}{N_D} \right], \quad (43)$$

where N_A is the ionized-acceptor concentration and N_D is the total (neutral plus ionized) donor concentration.¹¹

B. Polar optical scattering

In polar semiconductors, the conduction electrons can interact strongly with the displacement polari-

$$\frac{1}{\tau_{po}} = \alpha \omega_0 \left(\frac{\omega_0}{\epsilon} \right)^{1/2} n(\omega_0) \ln \left[\frac{\left(1 + \frac{\omega_0}{\epsilon} \right)^{1/2} + 1}{\left(1 + \frac{\omega_0}{\epsilon} \right)^{1/2} - 1} \right] + e^{\beta \omega_0} \ln \left[\frac{1 + \left(1 - \frac{\omega_0}{\epsilon} \right)^{1/2}}{1 - \left(1 - \frac{\omega_0}{\epsilon} \right)^{1/2}} \right], \quad (44)$$

where the second term corresponds to phonon emission and occurs for $\epsilon > \omega_0$ only.¹³ The constant α is Fröhlich's parameter; cf. Ref. 13. For the case of GaAs, $\alpha = 0.067$ and $\omega_0 = 417$ K.

For completeness we mention the relaxation times for acoustic-phonon scattering and for neutral impurity scattering in Ref. 14. For the case of combined scattering mechanisms, the total relaxation time is given by

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i}, \quad (45)$$

where τ_i is a single energy-dependent relaxation time defined for each scattering mechanism. The carrier mobility, i.e., the drift velocity of the carriers, is determined by the mean value of $\tau(\epsilon)$,

$$\mu = \frac{e}{m^*} \langle \tau \rangle, \quad (46)$$

where for a nondegenerate electron gas the average value of τ is given by

$$\bar{\tau} \equiv \langle \tau \rangle = \frac{4}{3\sqrt{\pi}} \beta^{5/2} \int_0^\infty d\epsilon e^{-\beta\epsilon} \epsilon^{3/2} \tau(\epsilon). \quad (47)$$

In some III-V semiconductors, the polar optical scattering is the dominant scattering process for high energies, whereas the ionized impurity scattering is dominant for low energies. Hence we can use the approximation

$$\frac{1}{\tau(\epsilon)} = \frac{1}{\tau_{imp}(\epsilon)} + \frac{1}{\tau_{po}(\epsilon)}. \quad (48)$$

Figure 1 indicates the energy dependence of τ for the parameter values characteristic of GaAs. The density $n = 6 \times 10^{15} \text{ cm}^{-3}$, the temperature $T = 300$ K, and $N_I = n' = n$ in Eq. (41). When only the polar optical scattering process is taken into account, we get a mobility value $\mu = 7560 \text{ cm}^2/\text{V sec}$ (a larger value if $\alpha < 0.067$). For the combined scattering

processes, Eq. (48), the mobility is evaluated to be $6300 \text{ cm}^2/\text{V sec}$ which is in the general range of mobilities quoted for GaAs samples of this carrier concentration.

IV. RESULTS AND COMPARISON WITH EXPERIMENTS

To discuss the effect of electron-relaxation processes on the light scattering cross section, given by Eq. (1), we choose the III-V compound GaAs with a nondegenerate electron gas in the conduction band as an example. The generalization to a degenerate electron gas for GaAs (and for other substances) is straightforward and can be carried out by using the general result for the dielectric function, Eq. (20), and Appendix B.

The parameter values for the electron-relaxation processes are given in Sec. III. The three different electron concentrations, $n = 6 \times 10^{15}$, 2.6×10^{16} , and

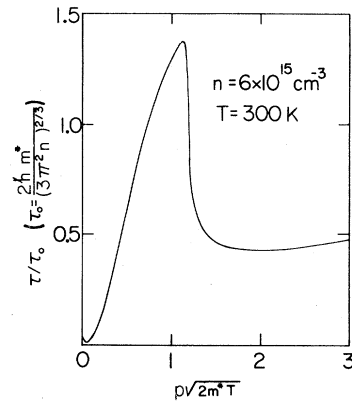


FIG. 1. Energy dependence of the relaxation time τ for the combined processes of impurity and polar optical scattering; $\epsilon = p^2/2m^*$, electron density $n = 6 \times 10^{15} \text{ cm}^{-3}$, temperature $T = 300$ K.

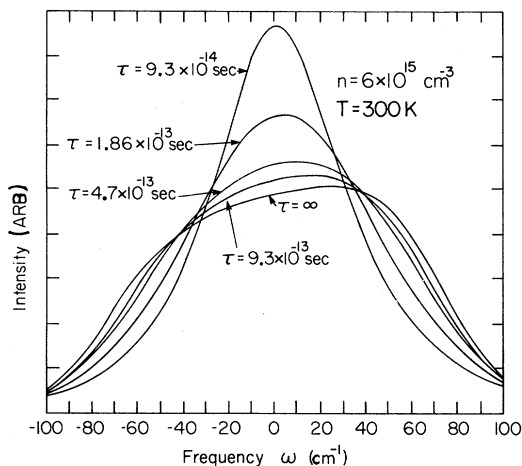


FIG. 2. Intensity of the scattered light for different constant relaxation times; $n = 6 \times 10^{15} \text{ cm}^{-3}$, and $T = 300 \text{ K}$.

$7 \times 10^{16} \text{ cm}^{-3}$, are being considered.

In order to accentuate the qualitative effect of the relaxation processes on the line shape, we first take $\tau = \bar{\tau} = \text{const}$. Figures 2–4 show the results for different values of $\bar{\tau}$. The following two findings are emphasized: (a) The shape of the single-particle spectrum changes from the Gaussian to a Lorentzian behavior as the value for $\bar{\tau}$ decreases, and (b) the shape of the spectrum for collective plasma modes is determined by Landau damping, the strength of which depends on the overlap between the plasma mode and the single-particle spectrum, and by collision processes. As the number of collisions increases, the plasmon spectrum broadens and its center shifts to smaller frequencies. For sufficiently small electron-relaxation times, the plasmon structure is submerged in the single-particle spectrum.

Next, we consider the temperature dependence of

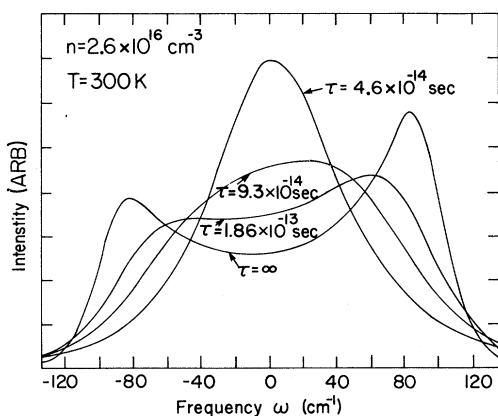


FIG. 3. Same as Fig. 2; here $n = 2.6 \times 10^{16} \text{ cm}^{-3}$.

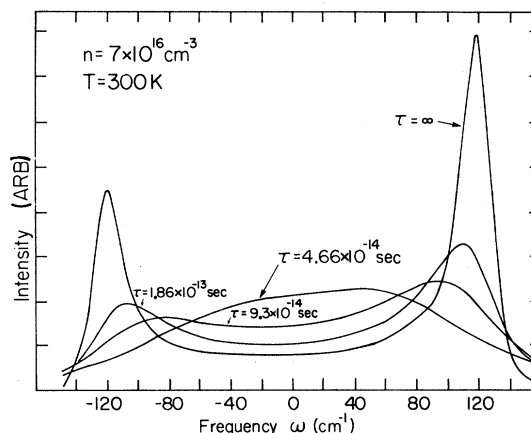


FIG. 4. Same as Fig. 2; here $n = 7 \times 10^{16} \text{ cm}^{-3}$.

the line shape. Here, the energy dependence of the relaxation time must be taken into account for the following reason: The relaxation time $\tau(\epsilon)$ is governed at low temperatures by ionized impurity scattering which is strongly energy dependent [Eq. (41)] whereas at high temperatures it is determined by polar optical scattering with a weak energy dependence [Eq. (44)]. We use the combined scattering rate for impurity and polar optical scattering processes [Eq. (48)]. Figures 5–7 are the line-shape results for the three different cases: $\tau = \bar{\tau} = \infty$, $\tau = \bar{\tau}(T)$, and $\tau = \tau(\epsilon, T)$. $n = 6 \times 10^{15} \text{ cm}^{-3}$ and $\bar{\tau}$ is evaluated from Eq. (47). We can see that for all temperatures the plasmon peak is most pronounced when $\tau = \infty$, e.g., for Landau damping alone. As for the two other cases, $\bar{\tau}(T)$ and $\tau(\epsilon, T)$, there is a substantial difference between the

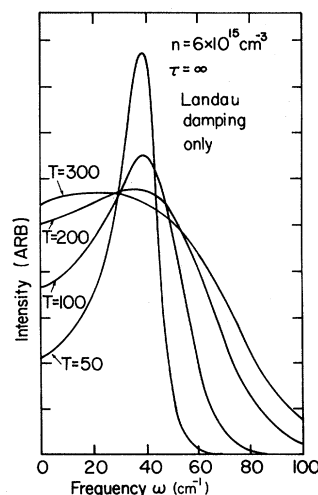
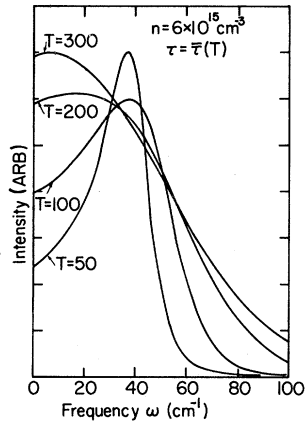
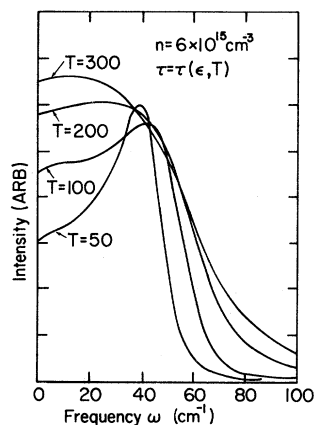
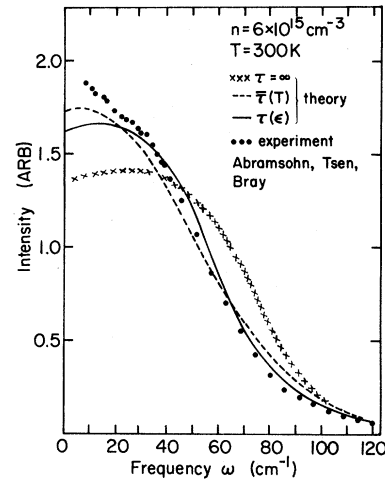


FIG. 5. Temperature dependence of the scattered light intensity; $n = 6 \times 10^{15} \text{ cm}^{-3}$; here $\tau = \infty$.

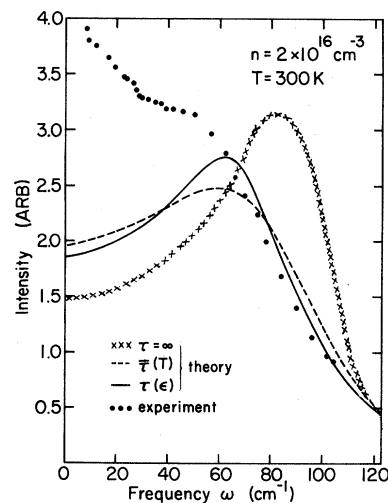
FIG. 6. Same as Fig. 5; here $\tau = \bar{\tau}(T)$.

two spectra at low temperatures and at low frequencies, due to the dominance of the energy-dependent impurity scattering. Our results are in qualitative agreement with the temperature dependence of the scattering cross section observed by Mooradian³ on a *n*-GaAs sample with $n = 10^{16} \text{ cm}^{-3}$ (cf. Fig. V.4 in Ref. 3). In calculating the line shapes we assume nondegeneracy, the Fermi temperature $T_F \approx 17 \text{ K}$.

Let us now proceed to compare theory and experiment. The detailed experiments by Abramsohn, Tsen, and Bray¹ are on GaAs at $T = 300 \text{ K}$, for $q = 2.91 \times 10^5 \text{ cm}^{-1}$, and for the same three n values as above. For the comparison we use the relaxation-time formula for combined impurity and polar optical scattering, Eq. (48); cf. Sec. III. In addition we consider the limit $\bar{\tau} = \infty$ for Landau damping alone. The results are shown in Figs. 8–10. For the cases $\tau = \bar{\tau}(T)$ [Eq. (47)] and $\tau = \tau(\epsilon, T)$ [Eq. (48)], there is fair agreement at the higher frequencies, while for $\bar{\tau} = \infty$ the theory completely fails to reproduce the experimental data.

FIG. 7. Same as Fig. 5; here $\tau = \tau(\epsilon, T)$.FIG. 8. Theoretical spectrum for the light scattering in *n*-GaAs for $\tau = \infty$, $\tau = \bar{\tau}(T)$, and $\tau = \tau(\epsilon, T)$. Experimental data, ●, by Abramsohn, Tsen, and Bray (Ref. 1). Here $n = 6 \times 10^{15} \text{ cm}^{-3}$.

The shapes of the experimental spectra fit somewhat better to the curves for $\tau(\epsilon, T)$ than to those for $\bar{\tau}(T)$ (Figs. 8 and 9). We expect the $\tau(\epsilon, T)$ curves to be in even better agreement at $T < 300 \text{ K}$, when the energy-dependent impurity scattering dominates. Therefore, it would be of great interest to have the corresponding experimental spectra at lower temperatures, in particular for nondegenerate semiconductors. At low frequencies and for increasing electron densities, the disagreement between our theory and the experimental findings is quite obvious, as shown in Figs. 9 and 10. It appears that the involved light scattering processes cannot be described on the basis of quasifree elec-

FIG. 9. Same as Fig. 8; here $n = 2.6 \times 10^{16} \text{ cm}^{-3}$.

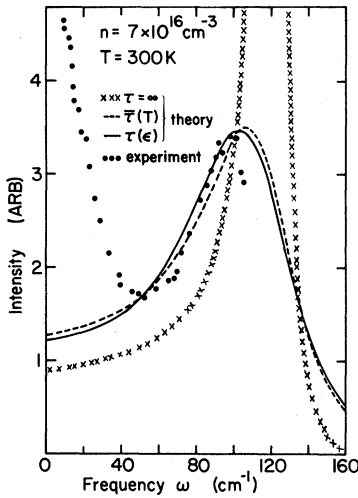


FIG. 10. Same as Fig. 8; here $n = 7 \times 10^{16} \text{ cm}^{-3}$.

trons. Instead, the effect of the electron band structure on the scattering cross section has to be taken into account. The scattering of light by electron-energy fluctuations, first suggested by Wolff,¹⁵ can be considered as a possible explanation for part of this effect.^{1,16} When all of the band-structure effects can be expressed in terms of the constant effective mass of a single parabolic band, Eq. (1) should accurately account for the light scattering by the conduction electrons.

V. SUMMARY

The energy-resolved cross section for the scattering of light by quasifree conduction electrons in a single parabolic band, Eq. (1), is given in terms of

the RPA dielectric function, Eq. (20). This function accounts for the energy-dependent electron-relaxation processes involved in relaxing the electron density to its local equilibrium distribution. Because of its general form, it applies to the nondegenerate as well as to the degenerate state of the electron gas. Electron-relaxation processes have a pronounced effect on the spectrum of the scattered light. In a nondegenerate electron gas the shape of the spectrum is changed from a Gaussian to a Lorentzian behavior as the relaxation time increases, and the peaks of the collective plasma modes are shifted and lifetime broadened as the collision rate increases. Within the considered range of electron densities the collective plasma modes can be damped out completely by the single-particle excitation spectrum, provided the relaxation times and the temperatures are sufficiently small and high, respectively. Relaxation processes play a decisive role for the understanding of experimental findings as the comparison with experimental data on GaAs indicates. At high frequencies, when band-structure effects can be ignored, the comparison results in almost quantitative agreement.

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APPENDIX A: LOCAL DENSITY DISTRIBUTION \bar{f}

The local density-distribution function in Fourier space is obtained from the density-matrix operator,

$$\bar{f}(\vec{p}, \vec{q}, t) = \left\langle \vec{p} - \frac{\vec{q}}{2} \left| \frac{1}{\exp[\beta(H_0 - \mu) - \beta\delta\mu] + 1} \right| \vec{p} + \frac{\vec{q}}{2} \right\rangle. \quad (\text{A1})$$

In order to get $\delta\bar{f}$ [Eq. (9)], we expand \bar{f} with respect to the local deviation $\delta\mu$ of the uniform chemical potential. Using the identity¹⁷

$$e^{A+B} = e^A \exp \left[\int_0^1 dt e^{-At} B e^{At} \right], \quad (\text{A2})$$

where A and B do not commute (in general), we obtain in first order in $\delta\mu$,

$$\begin{aligned}
\bar{f}(\bar{p}, \bar{q}, t) &= \left\langle \bar{p} - \frac{\bar{q}}{2} \left| \left[1 + \beta \frac{\exp[\beta(H_0 - \mu)]}{\exp[\beta(H_0 - \mu)] + 1} \int_0^1 dt \exp[-\beta(H_0 - \mu)t] \delta\mu \exp[\beta(H_0 - \mu)t] \right] \right. \right. \\
&\quad \left. \left. \times \frac{1}{\exp[\beta(H_0 - \mu)] + 1} \left| \bar{p} + \frac{\bar{q}}{2} \right\rangle \right. \\
&= \left\langle \bar{p} - \frac{\bar{q}}{2} \left| \frac{1}{\exp[\beta(H_0 - \mu)] + 1} \left| \bar{p} + \frac{\bar{q}}{2} \right\rangle \right. \right. \\
&\quad \left. \left. - \frac{e^{\beta\epsilon_-}}{e^{\beta\epsilon_-} + 1} \frac{\exp[-\beta(\epsilon_- - \epsilon_+)] - 1}{\epsilon_- - \epsilon_+} \frac{1}{e^{\beta\epsilon_+} + 1} \left\langle \bar{p} - \frac{\bar{q}}{2} \left| \delta\mu \left| \bar{p} + \frac{\bar{q}}{2} \right\rangle \right. \right. \right. \\
&\quad \left. \left. \equiv f_0(\bar{p}) \delta_{\bar{q}, 0} - \frac{f_0 \left[\bar{p} - \frac{\bar{q}}{2} \right] - f_0 \left[\bar{p} + \frac{\bar{q}}{2} \right]}{\epsilon_- - \epsilon_+} \delta\mu(\bar{q}), \right. \right. \tag{A3}
\end{aligned}$$

where $\epsilon_{\pm} = \epsilon[\bar{p} \pm (\bar{q}/2)] - \mu$. Hence, we have

$$\delta\bar{f} = \frac{f_0 \left[\bar{p} - \frac{\bar{q}}{2} \right] - f_0 \left[\bar{p} + \frac{\bar{q}}{2} \right]}{\frac{\bar{p} \cdot \bar{q}}{m^*}} \delta\mu(q). \tag{A4}$$

APPENDIX B: INTEGRALS WITH $\tau(p)$

Assuming a parabolic band, $\epsilon(p) = p^2/2m^*$, we must evaluate the following integrals:

$$\begin{aligned}
B_{\mathcal{F}} &= v(q) 2 \int \frac{d^3p}{(2\pi)^3} \mathcal{F}(\omega\tau(p)) \frac{f_0 \left[\bar{p} + \frac{\bar{q}}{2} \right] - f_0 \left[\bar{p} - \frac{\bar{q}}{2} \right]}{\omega + \frac{i}{\tau(p)} - \frac{\bar{p} \cdot \bar{q}}{m^*}} \\
&\equiv v(q) 2 \int \frac{d^3p}{(2\pi)^3} f_0(p) \left[\frac{\mathcal{F} \left[\omega\tau \left(\left| \bar{p} - \frac{\bar{q}}{2} \right| \right) \right]}{\omega + \frac{q^2}{2m^*} + \frac{i}{\tau \left(\left| \bar{p} - \frac{\bar{q}}{2} \right| \right)} - \frac{\bar{p} \cdot \bar{q}}{m^*}} - \frac{\mathcal{F} \left[\omega\tau \left(\left| \bar{p} + \frac{\bar{q}}{2} \right| \right) \right]}{\omega - \frac{q^2}{2m^*} + \frac{i}{\tau \left(\left| \bar{p} + \frac{\bar{q}}{2} \right| \right)} - \frac{\bar{p} \cdot \bar{q}}{m^*}} \right], \tag{B1}
\end{aligned}$$

where $\mathcal{F}(\omega\tau(p))$ stands for the functions 1, $1/\omega\tau(p)$, or $1/[\omega\tau(p) + i]$. The first form of $B_{\mathcal{F}}$ is conveniently used if τ is a function of p , whereas the second form is more suitable if $\tau = \text{const}$.

For constant τ , \mathcal{F} is also a constant, and we can proceed with the angular integrations; the result is

$$\begin{aligned}
B_{\mathcal{F}} = 2v(q)\mathcal{F}(\omega\tau) \int \frac{p^2 dp}{(2\pi)^2} f_0(p) & \left(\frac{m^*}{2pq} \ln \left[\frac{\left[\omega + \frac{q^2}{2m^*} + \frac{pq}{m^*} \right]^2 + \frac{1}{\tau^2}}{\left[\omega + \frac{q^2}{2m^*} - \frac{pq}{m^*} \right]^2 + \frac{1}{\tau^2}} \right] - \frac{m^*}{2pq} \ln \left[\frac{\left[\omega - \frac{q^2}{2m^*} + \frac{pq}{m^*} \right]^2 + \frac{1}{\tau^2}}{\left[\omega - \frac{q^2}{2m^*} - \frac{pq}{m^*} \right]^2 + \frac{1}{\tau^2}} \right] \right. \\
& - i \frac{m^*}{pq} \left\{ \arctan \left[\left[\omega + \frac{q^2}{2m^*} + \frac{pq}{m^*} \right] \tau \right] - \arctan \left[\left[\omega + \frac{q^2}{2m^*} - \frac{pq}{m^*} \right] \tau \right] \right\} \\
& \left. + i \frac{m^*}{pq} \left\{ \arctan \left[\left[\omega - \frac{q^2}{2m^*} + \frac{pq}{m^*} \right] \tau \right] - \arctan \left[\left[\omega - \frac{q^2}{2m^*} - \frac{pq}{m^*} \right] \tau \right] \right\} \right), \tag{B2}
\end{aligned}$$

where f_0 is the Fermi-Dirac distribution.

For a nondegenerate gas, we obtain by partial integration:

$$B_{\mathcal{F}} = Tm^*C'v(q)\mathcal{F}(\omega\tau) \int_{-\infty}^{+\infty} dp \exp \left[-\frac{p^2}{2m^*T} \right] \left(\frac{\omega + \frac{q^2}{2m^*} + \frac{pq}{m^*} - \frac{i}{\tau}}{\left[\omega + \frac{q^2}{2m^*} + \frac{pq}{m^*} \right]^2 + \frac{1}{\tau^2}} - \frac{\omega - \frac{q^2}{2m^*} + \frac{pq}{m^*} - \frac{i}{\tau}}{\left[\omega - \frac{q^2}{2m^*} + \frac{pq}{m^*} \right]^2 + \frac{1}{\tau^2}} \right), \tag{B3}$$

where $C' = (4\pi)^{3/2}n/(2\pi)^2(2m^*T)^{3/2}$. The final integrations yield

$$\text{Re}B_1 = \pi C \left[\text{Im}w \left[\frac{m^*\omega + \frac{q^2}{2}}{q(2m^*T)^{1/2}} + i \frac{m^*/\tau}{q(2m^*/T)^{1/2}} \right] - \text{Im}w \left[\frac{m^*\omega - \frac{q^2}{2}}{q(2m^*T)^{1/2}} + i \frac{m^*/\tau}{q(2m^*T)^{1/2}} \right] \right], \tag{B4}$$

$$\text{Im}B_1 = -\pi C \left[\text{Re}w \left[\frac{m^*\omega + \frac{q^2}{2}}{q(2m^*T)^{1/2}} + i \frac{m^*/\tau}{q(2m^*T)^{1/2}} \right] - \text{Re}w \left[\frac{m^*\omega - \frac{q^2}{2}}{q(2m^*T)^{1/2}} + i \frac{m^*/\tau}{q(2m^*T)^{1/2}} \right] \right], \tag{B5}$$

where $C = C'v(q)m^*(2m^*T)^{1/2}/2q$; $B_1 \equiv B$ [Eq. (15)].

For an energy-dependent τ , $\tau = \tau(p)$, the first form of Eq. (B1) yields

$$\begin{aligned}
B_{\mathcal{F}} = C' \int \frac{d^3p}{(2\pi)^3} \mathcal{F}(\omega\tau(p)) & \frac{\exp \left[-\frac{\left[\vec{p} + \frac{\vec{q}}{2} \right]^2}{2m^*T} \right] - \exp \left[-\frac{\left[\vec{p} - \frac{\vec{q}}{2} \right]^2}{2m^*T} \right]}{\omega + \frac{i}{\tau(p)} - \frac{\vec{p} \cdot \vec{q}}{m^*}} \\
= C' \int_0^{\infty} dp p^2 \exp \left[-\frac{\left[p^2 + \frac{q^2}{4} \right]}{2m^*T} \right] & \mathcal{F}(\omega\tau(p)) \int_{-1}^{+1} dx \frac{(-2)\sinh \left[\frac{pq}{2m^*T} x \right]}{\omega + \frac{i}{\tau(p)} - \frac{pq}{m^*} x}. \tag{B6}
\end{aligned}$$

The x integration can be performed by expansion of \sinh ; the final p integration must be done numerically.

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