Cyclotron resonance, the memory function, and the random potential at the Si:SiO$_2$ interface

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Swept-frequency cyclotron-resonance experiments in the two-dimensional electron gas for intermediate densities ($n_s \gtrsim eH/hc$, where eH/hc is the degeneracy of the lowest Landau level) reveal severely distorted and shifted absorption lines. We show how these data can be inverted to display directly the holomorphic memory function $M(\omega)$ which describes complex frequency-dependent electron scattering in the presence of a strong magnetic field. The relaxation time given by the imaginary part of M exhibits a broad resonance at ω_c while the real part of M which produces effective-mass corrections shows strong dispersion. By comparing these results with the theory of Götze and Hajdu, the strength (U) and range (r_0) of the scattering potential can be extracted. For the particular Si devices studied, typical values for the strength U are 15–30 K and those for the range are $r_0 \sim 50$ Å.

INTRODUCTION

Cyclotron resonance (CR) in the two-dimensional (2D) electron gas in Si inversion layers has been the focus of numerous experimental[1–15] and theoretical[16–33] investigations. In principle, CR should directly reveal the electron mass and scattering rate, but seldom are the observations free of interesting complications. Quantum oscillations,[5] subharmonic structure[15] and dramatic shifts away from the cyclotron-resonance frequency,[3,4,8,11,13,14] and distorted line shapes are some of the interesting observations that have inspired much theoretical discussion and some controversy.[4,8] The focus of this paper is on the apparent mass shifts and line-shape changes that are most dramatic at low densities and yet not so low as to be in the extreme quantum limit. That is to say, $n_s \geq eB/hc$ where e is the electron charge, B the magnetic field, h Planck's constant, and c the speed of light. eB/hc is the degeneracy of the lowest Landau level. Previously, we focused on the behavior of CR of the 2D electron gas in the extreme quantum limit[10,11,13,14] while noting that at intermediate densities the resonance position shifts to low frequency and the line shape is skewed to high frequencies. In the following sections we show how the line shape can be analyzed to display directly the complex memory function $M(\omega)$, and then we compare our analysis with recent theory for M. The latter yields a measure of the strength and range of the random potential.

EXPERIMENTAL RESULTS

The samples reported here were fabricated on (100) surfaces of p-type silicon but have substantially different peak mobility at 4.2 K. Table I outlines their characteristics.

The device geometry and measurement techniques have been described elsewhere.[34] In essence, the frequency-dependent conductivity $\sigma(\omega)$ is determined from 10 to 50 cm^{-1} by measuring the fractional change in transmission $\Delta T/T$ when the electrons are introduced into the inversion layer. The conductivity is related to the fractional change in transmission by

$$\frac{\Delta T}{T} = -\frac{2\mathrm{Re}\sigma(\omega)}{Y_0+Y_{\mathrm{Si}}+Y_G}, \tag{1}$$

where Y_0, Y_{Si}, and Y_G are the wave admittances of free space, silicon, and the gate metallization, respectively.

Typical experimental results are shown in Fig. 1 for device GKB9-8 and in Fig. 2 for GK5-17-78-1. 1. At the lowest densities a resonance is observed with the peak shifted substantially below the cyclotron frequency and a severely distorted line shape. In sample GK5-17-78-1 the line is shifted so low that if one interpreted the resonance shift as a mass change it would require a mass nearly twice as heavy as the bulk mass projected onto the (100) surface. We note that the sample that shows the larger

TABLE I. Characteristics of samples fabricated on (100) surfaces of p-type silicon.

Sample	Peak mobility at 4.2 K (cm^2/V sec) [at n_s (cm^{-2})]	Oxide thickness (Å)	Substrate doping (cm^{-3})
GKB9-8	12 000 (9.3×10^{11})	3680	1.2×10^{15}
GK5-17-78-1	3600 (1.2×10^{12})	1410	2.5×10^{15}

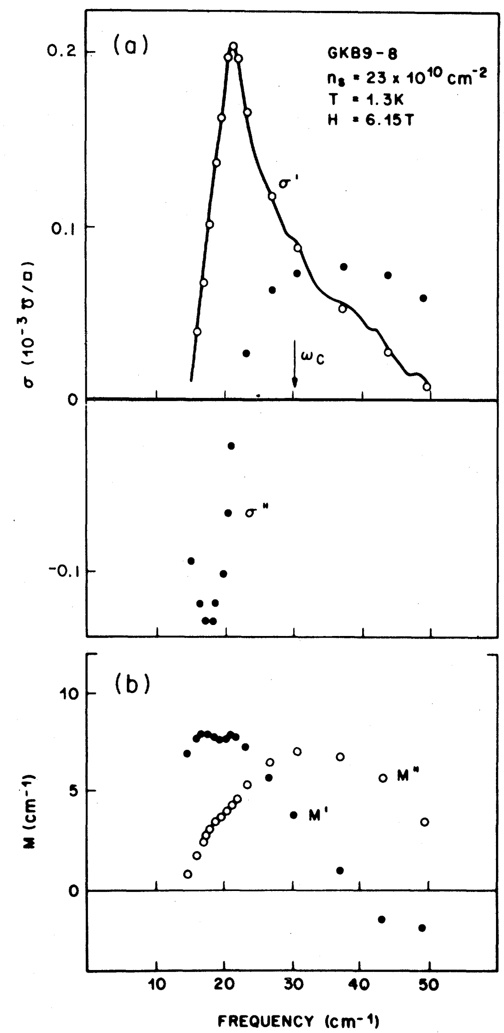

FIG. 1. (a) Cyclotron resonance at 6.15 T. Open circles are the data used for Kramers-Kronig analysis. Closed circles are the imaginary part of the conductivity σ'', deduced by Kramers-Kronig analysis of the real part of the conductivity σ'. (b) Real and imaginary parts of the memory function (M' and M'') deduced from σ' and σ''.

changes has lower peak mobility and hence more scattering at the interface.

EXTRACTION OF THE MEMORY FUNCTION

A key concept in the theory of high-frequency transport is the complex memory function, $M(\omega)=M'(\omega)+iM''(\omega)$, which describes the complex frequency-dependent scattering. In the absence of a magnetic field we may write for the conductivity

$$\sigma(\omega)=\frac{n_s e^2}{m}\frac{i}{\omega+M(\omega)} , \tag{2}$$

where m is the electron mass. In general, $M(\omega)$ will exhibit some frequency dependence and the real and imaginary parts must be related by a Kramers-Kronig relation in order to satisfy causality.

In a magnetic field we may write[23,29,31]

$$\sigma_{\pm}(\omega)=\frac{n_s e^2}{m}\frac{i}{(\omega\mp\omega_c)+M(\omega)} , \tag{3}$$

which implies that $M(\omega)$ is independent of the sense of circular polarization. It is not clear whether this is strictly correct or how it compromises the analysis that follows.

The experiment measures the real part of $\sigma_{xx}(\omega)$:

$$\sigma_{xx}(\omega)=\frac{1}{2}\frac{n_s e^2}{m}\left[\frac{i}{(\omega+\omega_c)+M(\omega)}+\frac{i}{(\omega-\omega_c)+M(\omega)}\right] . \tag{4}$$

This equation can be algebraically inverted at each frequency if we know both the real and imaginary parts of $\sigma_{xx}(\omega)$.[35] The imaginary part is determined by making a piecewise linear Kramers-Kronig analysis. The input data points are indicated in each experimental figure by open circles. We extrapolate to zero frequency and infinite frequency as follows:

$$\begin{aligned}\sigma(\omega)&=(\omega^2/\omega_1^2)\sigma(\omega_1),\quad 0<\omega<\omega_1\\ \sigma(\omega)&=(\omega_N^2/\omega^2)\sigma(\omega_N),\quad \omega_N<\omega ,\end{aligned} \tag{5}$$

where ω_1 and ω_N are the first and last data points, respectively. The resulting imaginary part is shown as solid points in Figs. 1 and 2.

The memory function can now be extracted by a simple inversion of Eq. (4). The results are shown

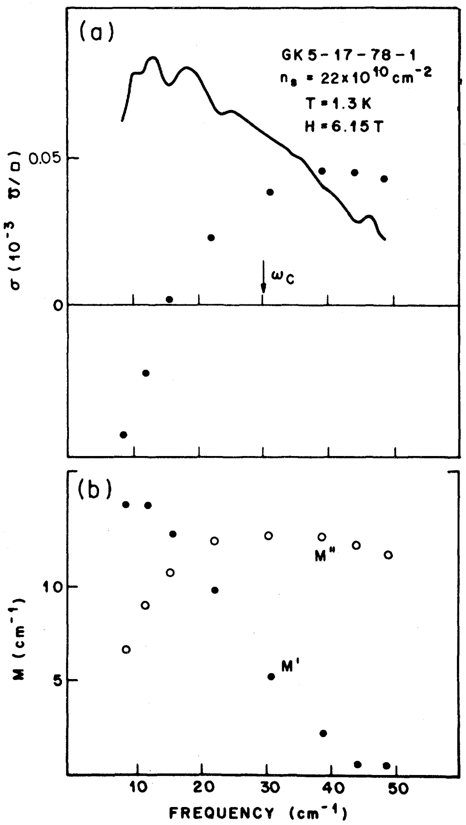

FIG. 2. (a) Cyclotron resonance at 6.15 T. Open circles are the data used for Kramers-Kronig analysis. Closed circles are the imaginary part of the conductivity σ'', deduced by Kramers-Kronig analysis of the real part of the conductivity σ'. (b) Real and imaginary parts of the memory function (M' and M'') deduced from σ' and σ''.

in Figs. 1 and 2. $M''(\omega)$, the scattering part of the memory function exhibits a broad resonance at $\sim\omega_c$, while M', the mass-shift part, exhibits a strong dispersion. *The experiment and analysis demonstrate in a model-independent manner that a correct microscopic theory of CR must develop a resonance in the complex scattering function at or near* ω_c.

We reexamine the assumptions used in the above analysis. There are basically two assumptions: (i) $M(\omega)$ in (3) and (4) is independent of the sense of polarization; (ii) the extrapolation formulas (5) are correct. In the limit that $\omega_c/M''(\omega_c) \gg 1$, neither assumption will seriously compromise the analysis. However, it should be noted that the conductivity will be dominated by the right-most term in (4), and the analysis will essentially return $M_+(\omega)$ in the event that $M(\omega)$ is sensitive to polarization. It is clear that to perform this experiment correctly one should use circular polarization and obtain data for both directions of magnetic field. Failing this difficult goal, high magnetic fields are desirable. The experiments outlined here only satisfy $\omega_c/M''(\omega_c) > 1$ and may suffer some systematic errors. Nonetheless, this study serves as an indicator of how this analysis can proceed and of what kind of information results.

MEMORY FUNCTION AND THE SCATTERING POTENTIAL

Although there have been numerous theoretical discussions of the frequency-dependent memory function to explain results on cyclotron resonance, few have recognized that there may be resonances in $M(\omega)$ at or near ω_c.[22,23,29,31,32] The most transparent discussion from our point of view is by Götze and Hajdu[31] and we compare with their theory here.

The random potential is characterized by a strength U and wave-vector cutoff q_0 or range $r_0 = 1/q_0$. If the distribution function is Gaussian,

$$|U(q)|^2 = 4\pi r_0^2 U^2 \exp(-q^2 r_0^2) . \tag{6}$$

We had taken a different form for $|U(q)|^2$,

$$|U(q)|^2 = \begin{cases} 4\pi U^2/q_0^2, & q < q_0 \\ 0, & q > q_0 , \end{cases} \tag{7}$$

in a previous publication.[36]

The theory of Götze and Hajdu[31] generates spectral widths described by the semielliptical function first obtained by Ando and Uemura,[16]

$$A_n(\omega) = \Gamma_n^{-1}\{1-[(\omega-\omega_n)/2\Gamma_n]^2\}^{1/2} , \tag{8}$$

where

$$\Gamma_0 = U\left[\frac{1}{1+(q_0R)^2/2}\right]^{1/2} , \tag{9}$$

$$\Gamma_1 = U\left[\frac{1+(q_0R)^4/4}{[1+(q_0R)^2/2]^3}\right]^{1/2} , \tag{10}$$

where R is the cyclotron radius given by

$$R = (\hbar/eB)^{1/2} . \tag{11}$$

Then $M''(\omega)$ is given by

$$M''(\omega) = \omega_c \frac{\Gamma_0^2}{(\Gamma_0+\Gamma_1)} b_{01} I_{01} , \tag{12}$$

where

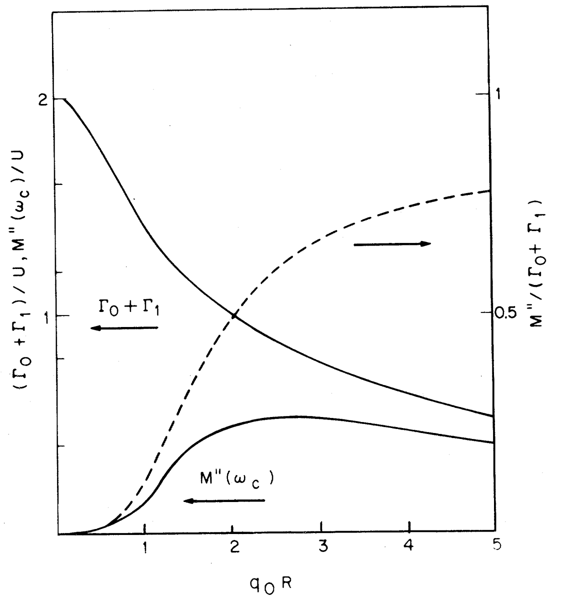

FIG. 3. Halfwidth and peak height of resonance in $M''(\omega)$ vs q_0R. The dashed line is the ratio of peak height to half-width.

$$b_{01}=\frac{(q_0R)^4/2}{[1+(q_0R)^2/2]^2}\,, \tag{13}$$

$$I_{01}=\frac{eB}{hcn_s}\int\frac{d\omega'}{\pi}(\Gamma_0+\Gamma_1)A_1(\omega')A_0(\omega'-\omega)\times\left[\frac{f(\omega'-\omega)-f(\omega')}{\omega}\right], \tag{14}$$

and $f(\omega)$ is the Fermi function.

We note that the spin and valley degeneracy require that only the lowest Landau level is occupied. If there is no spin-flip or valley-valley scattering, then the integral in (14) may be simply performed for the different spin and valley levels occupied to generate a total I_{01} and $M''(\omega)$.

Without doing an explicit calculation, Götze and Hajdu[31] indicate that the peak of $M''(\omega)$ should occur near ω_c and should be given by

$$M''(\omega_c)\approx\frac{32}{3\pi}\frac{\Gamma_0^2}{(\Gamma_0+\Gamma_1)}\frac{(q_0R)^4/4}{[1+(q_0R)^2/2]^2}\,, \tag{15}$$

and the halfwidth of $M''(\omega)$ should be $\sim(\Gamma_0+\Gamma_1)$.

In Fig. 3 we plot the peak $M''(\omega_c)$ and the halfwidth $\Gamma_0+\Gamma_1$ in units of U against q_0R. Also shown is the ratio of peak to width. As pointed out by Götze and Hajdu, in the limit of $q_0R \ll 1$ the width of the CR given by $M''(\omega_c)$ is much smaller than the frequency scale on which $M''(\omega)$ changes, and we expect a Lorentzian line centered at ω_c. This case corresponds to long-wavelength potential scattering and if the cyclotron radius is much less than this characteristic wavelength, the CR linewidth will not exhibit features of the density of states. On the other hand, if $q_0R>1$, then the CR linewidth will be sufficiently large so that the line shape will sample the dispersion in $M(\omega)$, and departures from a simple Lorentzian line shape will occur.

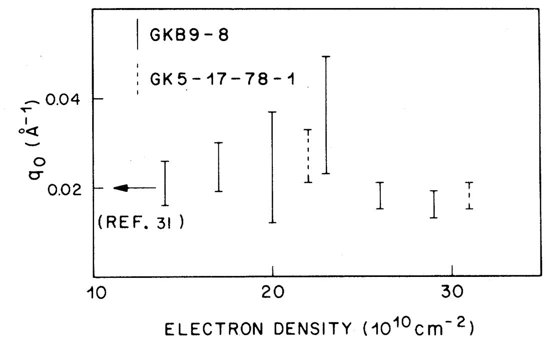

FIG. 4. Cutoff wave vector q_0 for the random interface potential determined by comparing experimental resonance in $M''(\omega)$ with theory of Ref. 31.

In Fig. 4 we show the range parameter $q_0=1/r_0$ deduced from the ratio of peak $M''(\omega_c)$ to the halfwidth $\Gamma_0+\Gamma_1$. The error bars indicate uncertainty in determining the ratio from the experimental $M''(\omega)$ and not any systematic errors due to limitations of the theory. The range of the potential fluctuations are of the order of 50 Å.

From the strength of the scattering at ω_c we can deduce the strength of the potential fluctuations. In Fig. 5 we show U for both samples as a function of density. There is a trend downward with increasing

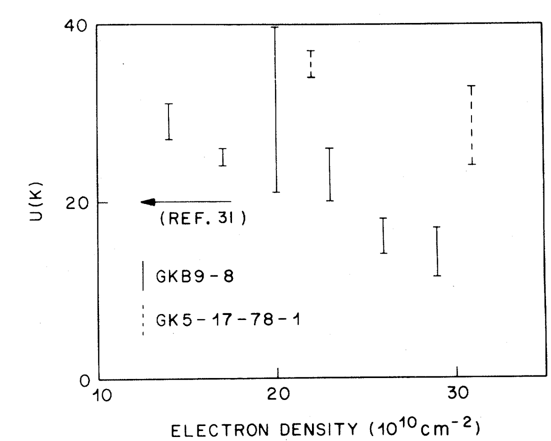

FIG. 5. Amplitude U of the random interface potential determined by comparing experimental resonance in $M''(\omega)$ with theory of Ref. 31.

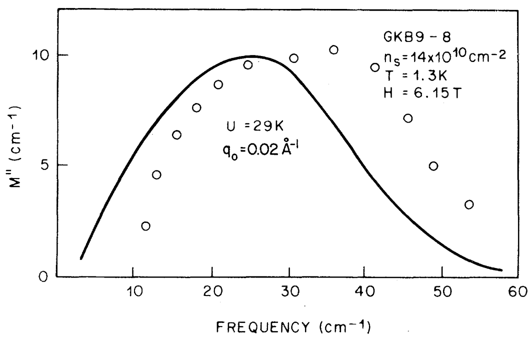

FIG. 6. Solid line is the theoretical resonance in $M''(\omega)$ calculated with theory from Ref. 31. Open circles are experimental $M''(\omega)$ determined by inverting cyclotron resonance.

n_s, with considerable error reflecting uncertainty in linking the experiment and the model. The lower mobility sample has, as expected, a larger U.

In a previous work[36] we had modeled the temperature- and frequency-dependent conductivity in the strongly localized regime with U and q_0 as adjustable parameters. The sample studied was similar to GKB9-8 and required $U \sim 20$ K and $q_0 = 0.02$ Å^{-1}, which are consistent with the values deduced from this inversion of CR.

In Fig. 6 we show $M''(\omega)$ calculated with Eqs. (12) and (14). It is in rough agreement with the experimental $M''(\omega)$. By focusing attention on Fig. 6 it is apparent that we cannot demand more than semiquantitative agreement with experiment. To compare with experiment we have singled out the peak in $M''(\omega)$ at ω_c. There should be peaks at all values of $n\omega_c$ including $n = 0$. Considering the breadth of the resonance shown in Fig. 6, it is clear that there should be overlap of the resonances in $M(\omega)$. Thus the comparison between theory and experiment can be taken as semiquantitative, yielding rough estimates of the scattering potential.

CONCLUSIONS

By performing swept-frequency CR measurements of the 2D electron gas in Si inversion layers, one can obtain sufficient data to perform a Kramers-Kronig analysis for the imaginary or reactive part of the conductivity. The frequency-dependent complex memory function can be extracted by a straightforward inversion of the experimental $\sigma(\omega)$. In this manner broad resonances in $M(\omega)$ can be displayed. By comparison with the theory of Götze and Hajdu[31] one concludes that the length scale for the random potential is smaller than the cyclotron radius and estimates of both its strength and cutoff wave vector can be obtained. These results agree with the observations of Kennedy *et al.*[4] that the apparent mass shifts seen in cyclotron resonance may be caused by electron scattering at the interface. Although this point of view has remained controversial,[8] the present work indicates that the length scale for the fluctuations plays a crucial role. As a result, there will not be a universal relationship between mass shift and scattering rate. For a given scattering rate both the amplitude and length scale of the fluctuations must be known in order to predict the appearance of a mass shift in cyclotron resonance.

It is apparent that further refinement of the theory and experiment will provide a valuable tool for measuring the interface random potential. To be a more quantitative test of the theory and/or a more quantitative measure of the random potential, the experiments should be performed in the regime where $\omega_c / M''(\omega_c) \gg 1$ in order to avoid ambiguities associated with the inversion of $\sigma(\omega)$ for $M(\omega)$. This regime is also easier to deal with theoretically since the resonances in $M(\omega)$ at $n\omega_c$ should be widely separated. On the theoretical side, a more explicit development with formulas for $M'(\omega)$ as well as for $M''(\omega)$ would be helpful.

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