# Band structure, intercalation, and interlayer interactions of transition-metal dichalcogenides: TiS<sub>2</sub> and LiTiS<sub>2</sub>

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The layered structure dichalcogenide  $TiS_2$  was studied in thin-film and bulk models, with the use of the self-consistent linearized augmented-plane-wave (LAPW) scheme. Bandstructure calculations were performed on the single sandwich (S-Ti-S), the double sandwich  $(S-Ti-S)_2$ , and the bulk to reveal the strength and effect of interlayer interactions on the electronic structure. Comparisons are made with existing bulk band models. The fully intercalated LiTiS<sub>2</sub> compound was studied by the same LAPW methods. Changes in the electronic density of states and charge density induced by the alkali intercalate are described.

## I. INTRODUCTION

There has been considerable interest in the structural, electrical, optical, and superconducting properties of the transition-metal dichalcogenides.<sup>1</sup> One of the distinctive properties of these materials is that most of them have a layered crystal structure consisting of chalcogen-metal-chalcogen sandwiches that are internally strongly bonded but are only weakly coupled to each other. Some of their physical properties, such as electrical conductivity and shear resistance, are highly anisotropic.<sup>2</sup> They can be cleaved easily into thin platelets. Several of these materials exhibit charge-density-wave phase transitions.<sup>3</sup> An interesting feature of these materials, which is related to the weak intersandwich interaction, is that they can be intercalated, i.e., foreign atoms can be introduced into the intersandwich region. This is a phenomenon of technological importance because of the possibility of the use of the alkali intercalated materials in lightweight batteries.<sup>4</sup> It has been found that in some of these materials a magnetic moment is developed upon intercalation with a transition metal.<sup>5</sup> Also, intercalation can either enhance or suppress the superconducting transition temperature.<sup>6</sup>

The fundamental nature of electronic states in TiS<sub>2</sub> and related materials has long been the subject of controversy from both experimental and theoretical points of view. The bulk band structure of  $TiS_2$ has been calculated by several authors, $^{7-11}$  and the experimental literature has become very extensive. Nevertheless, it is still questionable whether "ideal"  $TiS_2$  is a semimetal or semiconductor. With the advent of accurate band-structure methods for treating thin-film geometries it has become literally possible to take apart crystals in order to analyze interactions between atomic planes. In this way it becomes possible to make quantitative conclusions about critical features of the band structure, which have been previously discussed qualitatively on the basis of symmetry arguments. Since intercalation modifies the interlayer interactions it is important to develop consistent theoretical models for treating ideal host and intercalated systems. Such models will hopefully provide a basis for a better understanding of transport and excitation phenomena.

In this paper we study the intersandwich interaction of TiS<sub>2</sub> by calculating the band structure and density of states (DOS) of a single sandwich of TiS<sub>2</sub>, a double sandwich (S-Ti-S)<sub>2</sub>, and the bulk material, and comparing them with each other. The semirelativistic linearized augmented-plane-wave (LAPW) method is used for both thin-film and bulk calculations in order to make meaningful compar-



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TABLE I. Crystal struct in the z direction, $u$ is the LAPW's, and $\vec{k}$ points are	ure constant intrasandwic wave vectors	s and LAPW h TiS <sub>2</sub> layer t used in self	V band parameters $V$ band parameters $D$ is spacing, $D$ is -consistency it	eters. All le the boundai erations in	engths in Boh ry of film, $D'$ units of $2\pi/a$	r units. $a_0$ is the distant $t_0$ , $2\sqrt{3}\pi/a$ .	is the lattice of the lattice of $10^{\circ}$ , $2\pi/c_0$ .	constant in efining the	$x$ - $y$ plane, $c_0$ is the lattice constant z-component Fourier expansion of
				M	uffin-tin radi	ns			k points used in self-
	$a_0$	$c_0$	n	Π	S	Li	D	D'	consistent iterations
TiS <sub>2</sub> single sandwich	6.448		0.25 <i>c</i> 0	2.78	1.813		9.007	12.0	$(\frac{2}{9},0),(\frac{4}{9},0),(\frac{5}{9},\frac{1}{3})$
(TiS <sub>2</sub> ) <sub>2</sub> double sandwich	6.448		$0.25c_{0}$	2.78	1.813		19.769	23.0	$(\frac{2}{9},0),(\frac{4}{9},0),(\frac{5}{9},\frac{1}{3})$
TiS <sub>2</sub> bulk	6.448	10.762	0.25 <i>c</i> <sub>0</sub>	2.78	1.813				$(0, \frac{2}{9}, \pm \frac{1}{4}), (0, \frac{4}{9}, \pm \frac{1}{4}), (0, \frac{8}{9}, \pm \frac{1}{4})$
									$(\frac{1}{9}, \frac{5}{9}, \pm \frac{1}{4}), (\frac{1}{9}, \frac{7}{9}, \pm \frac{1}{4}), (\frac{2}{9}, \frac{8}{9}, \pm \frac{1}{4})$
Li-(TiS <sub>2</sub> ) <sub>2</sub> sandwich	6.529		$0.232c_{0}$	2.78	1.813	2.5	20.765	23.0	same as TiS <sub>2</sub> sandwich
Li-TiS <sub>2</sub> bulk	6.529	11.707	$0.232c_{0}$	2.78	1.813	2.5			same as TiS <sub>2</sub> bulk

isons between different systems. The self-consistent band structures of both film and bulk predict that  $TiS_2$  is a *semimetal*, in contrast to most previous work. We analyze the self-consistent valence charge density and decompositions of the density of states to obtain a bonding model consistent with the quasi-two-dimensional nature of the electronic structure. We have also studied the change in the electronic structure upon intercalation of a full layer of Li into the interlayer region of the double sandwich and the bulk material. Donation of the Li conduction electron to the lattice causes an interesting rearrangement of the Ti-based occupancy of the valence bands.

## **II. COMPUTATIONAL DETAILS**

## A. Crystal structure and symmetry

TiS<sub>2</sub> crystallizes in the CdI<sub>2</sub> crystal structure. A projected view of the crystal structure is shown in Fig. 1. The arrangement of the Ti and S atoms within the layers is hexagonal. The stacking of the layers is *ABC-ABC-*..., where the S atoms occupy the *A* and *C* sites and the Ti atoms occupy the *B* sites. The S layer is separated by  $0.25c_0$  from the Ti layer in the pure material, where  $c_0$  is the lattice constant in the *z* direction. Thus the intrasandwich and the intersandwich separation of the S layers are equal.<sup>12</sup>

Our calculation of the intercalated material is for a full layer of Li on the octahedral B sites in the intersandwich region. In this case the space group of the crystal remains unchanged. The volume of the crystal expands by 11.5% (Ref. 12) upon intercalation, most of the increase being in the z direction and most of that again being in the interlayer S-S distance, whereas the intralayer distance remains almost unchanged, as we would expect. In Table I we show the values of the crystal-structure constants and also some of the other inputs used in our calculations.<sup>13</sup>



FIG. 2. Brillouin zone for the  $TiS_2$  structure.

TABLE II. Small group of  $\vec{k}$  for symmetry points in the Brillouin zone, and point-group operation used for partial symmetrization of the LAPW basis.

k points	Small group of $\vec{k}$	Operation used in symmetrization
Γ,Α	$D_{3d}$	i
M,L	$C_{2h}$	i
K,H	$D_3$	$C_2 = i\sigma$
$\Sigma, R, U$	$C_{1h}$	σ
T,T',S,S'	$C_2$	$C_2 = i\sigma$
Δ	$C_{3v}$	σ
Р	$C_3$	

The space group of the crystal is  $D_{3d}^3(P\overline{3}m1)$ . In Fig. 1 we have marked in the rotation axes and mirror planes of the corresponding point group  $D_{3d}(\overline{3}m)$ . The Brillouin zone is shown in Fig. 2 with the irreducible wedge outlined. At symmetry points that have a subgroup of order 2 linear combinations of the basis functions are formed that transform as the symmetric and antisymmetric representations, and thereby block diagonalizing the overlap and Hamiltonian matrices. In Table II we give the small group of k for the symmetry points shown in Fig. 2 and the operation used in symmetrizing the basis functions, if any. Compatibility relations deduced from the character table are useful in drawing the band structure.<sup>13</sup> Not only does the symmetrization of LAPW's reduce the computation time, but the resulting symmetry labels denote the interplane nodal structure essential for intepretation of level ordering and splittings.

## B. LAPW band method

The linearized augmented-plane-wave (LAPW) method used in this work has been described in the literature<sup>13</sup>; here we only present an outline of the basic approach. A variational basis of the form

$$\phi_{i\vec{k}}(\vec{r}) = \sum_{lm} [A_{lm}^{i}(\vec{k})u_{l}(E_{l},r) + B_{lm}^{i}(\vec{k})\dot{u}_{l}(E_{l},r)]Y_{lm}(\hat{r})$$
(1)

is used to expand the Bloch functions inside spherical regions around each atom. The  $E_l$  energy parameters control the shape of the radial functions and can be given different values for each l and each atom type. The energy derivative functions  $\dot{u}_l = \partial u_l / \partial E |_{E=E_l}$  are chosen to allow an efficient "Taylor-series" expansion of the eigenstates for energies close to  $E_l$ . The  $E_l$  are normally chosen to lie near the center of the bands of interest.

Outside the atomic muffin-tin spheres in intersti-

tial regions, the eigenstates are expanded in plane waves. In the case of film geometry only twodimensional periodicity holds and the interstitial LAPW's are represented as

$$\phi \frac{mn}{V}(\vec{\mathbf{r}}) = \Omega^{-1/2} e^{i \vec{\mathbf{K}} mn \cdot \vec{\mathbf{r}}}, \qquad (2)$$

where  $\Omega$  is a normalizing factor, and  $\vec{K}_{mn} = \vec{k} + \vec{g}_m + g_n \hat{z}$ . Here  $\vec{k}$  is the two-dimensional (2D) Bloch vector,  $\vec{g}_m$  is a 2D reciprocal-lattice vector, and  $g_n$  labels Fourier components for the expansion in the aperiodic z direction.

The expansion coefficients  $A^i, B^i$  of Eq. (1) are obtained by matching the LAPW's and their radial derivatives across the atomic sphere boundaries. Again for film geometry, it is convenient to consider *vacuum* regions to develop an efficient expansion for |z| > D, where D is the film thickness. In this region we write

$$\phi_{\text{vac,}\vec{k}}^{mn}(\vec{r}) = [A_{mn}^{\text{vac}}(\vec{k})u_m(E_v,z) + B_{mn}^{\text{vac}}(\vec{k})\dot{u}_m(E_v,z)]e^{i\vec{k}\cdot\vec{m}\cdot\vec{r}}$$
(3)

in analogy with Eq. (1). The expansion bases  $u_m, \dot{u}_m$  are solutions of Schrödinger's equation for the average of the full potential over the x-y plane. Depending upon the value of the vacuum energy parameter  $E_v$ , the basis function may have either damped or oscillatory behavior for large |z|.

The LAPW's are again matched at the vacuumfilm boundary, and a secular equation is set up to determine the eigenenergies and corresponding eigenvectors for a discrete grid of  $\vec{k}$  values. The wave functions of the form

$$\psi_n(\vec{k},\vec{r}) = \sum_i C_{ni}(\vec{k})\phi_{i\,\vec{k}}(\vec{r}) \tag{4}$$

are then occupied according to Fermi statistics to generate the charge density required for selfconsistent iterations. The energy spectrum  $E_n(\vec{k})$  is decomposed or projected in various ways to aid in interpretation of the atomic character of the states.

# C. Self-consistent crystal potential and charge density

The special  $\vec{k}$  points of Chadi and Cohen<sup>14</sup> were used for calculating the self-consistent charge density. They are listed in Table I. For the film calculations the 3  $\vec{k}$ -point set was found to yield a potential that was in surprisingly good agreement with the 6  $\vec{k}$ -point set, the largest difference being less than 4 mRy. Hence, the 3  $\vec{k}$ -point set was used in the self-consistency iterations. In the bulk calculations we used the 12  $\vec{k}$ -point set obtained from



FIG. 3. Self-consistent LAPW band structures. Calculated data points for even and odd parity states with respect to the operations listed in Table II are indicated by  $\times$  and  $\circ$ , respectively: (a) single sandwich, S-Ti-S, (b) double sandwich, (S-Ti-S)<sub>2</sub>, (c) bulk TiS<sub>2</sub>.

Chadi and Cohen's 6 k-point set for  $D_{6h}$  symmetry, since the irreducible wedge of a  $D_{3d}$  crystal is twice as large as that of a  $D_{6h}$  crystal with the same lattice constants.

The calculations are performed in the warped muffin-tin approximation, i.e., no shape approximations are made in the interstitial and vacuum regions but the potential in the atomic spheres is spherically averaged. The warping terms in the interstitial region are found to produce level shifts of as much as 1 eV, compared to levels of the uncorrected muffin-tin potential. We believe the atomic nonspherical terms that we have neglected are much less important for the following reason: A trial calculation was carried part way to selfconsistency with the radius of the Ti sphere reduced from 2.78 to 2.34 a.u., and with the radius of the S sphere increased by the same amount. The largest difference found in the eigenvalues was 0.2 eV; part of this is probably due to the fact that the calculation with the changed radii was not run all the way to self-consistency. Since the nonspherical terms would be largest at the sphere boundary, this shows that intrasphere angular terms are not very important (see, however, discussion of the S p – Ti d band gap below).

In order to produce accurate charge-density plots of the individual states useful in discussing bonding interactions, it is necessary to keep the nonspherical wave-function terms. Spherical harmonics up to l = 6 were included in the atomic sphere regions for the wave functions, and up to l=8 in the charge density for this purpose. At the end of the selfconsistent iterations a finer k grid was used to calculate properties so as to minimize interpolation errors. The density of states (DOS) for bulk and film were calculated by the linear tetrahedron and triangle methods, 15, 16 respectively. Twenty-three  $\vec{k}$ points in the irreducible triangle were used for the film calculations and a uniform mesh of 75 k points in the irreducible wedge was used for the bulk calculations.

# III. BAND STRUCTURE AND DENSITY OF STATES A. Level structure and bonding

In Figs. 3(a)-3(c) we give the band structure of (a) the single sandwich S-Ti-S, (b) the double

sandwich  $(S-Ti-S)_2$ , and (c) bulk  $TiS_2$ . The zero of the energy is chosen to be at the Fermi level  $E_F$ . The two low-lying bands are S 3s-like, the six bands (twelve for the double sandwich) lying almost entirely below the Fermi energy are S p-like, and the five bands lying almost entirely above  $E_F$  are Ti 3d-like, though there is considerable admixture in some of the states as will appear further in the partial densities of states. The nearest S atoms form an approximately regular octahedron around the Ti atoms. Hence the d bands split into a  $t_{2g}$ -like triplet and an  $e_g$ -like doublet of subbands, as one would expect from crystal-field theory.

If we were to start with two sandwiches far apart from each other we would have a pair of degenerate levels for each level in the single sandwich. If the two sandwiches are brought closer to each other the levels will split and the extent to which they split is a measure of the intersandwich interaction. Similarly, to compare with the bulk structure, we may start with an infinite number of sandwiches far apart from each other; we then have an additional quantum number  $k_z$  that is not present in the single sandwich. States with all values of  $k_z$  will be degenerate, i.e., there will be no dispersion of the bands along the z direction. On bringing the sandwiches closer to each other this degeneracy is lifted; thus the band dispersion in the z direction is a measure of the intersandwich interaction.

We can make the following correspondence between the single sandwich levels at the zone center  $\overline{\Gamma}$  and the double sandwich and bulk levels:

Single sandwich	Double sandwich	Bulk		
$\overline{\Gamma}_{1+}$	$\overline{\Gamma}_{1+}$ and $\overline{\Gamma}_{2-}$	$\Gamma_{1+}$ and $A_{1+}$		
$\overline{\Gamma}_{2-}$	$\overline{\Gamma}_{2-}$ $\overline{\Gamma}_{1+}$	$\Gamma_{2-}$ $A_{2-}$		
$\overline{\Gamma}_{3+}$	$\overline{\Gamma}_{3+}$ $\overline{\Gamma}_{3-}$	$\Gamma_{3+}$ $A_{3+}$		
$\overline{\Gamma}_{3-}$	$\overline{\Gamma}_{3-}$ $\overline{\Gamma}_{3+}$	$\Gamma_{3-}$ $A_{3-}$		

We illustrate this with a simple example, the S 3s states. In a single sandwich these give rise to a  $\overline{\Gamma}_{1+}$  and a  $\overline{\Gamma}_{2-}$  [see Fig. 3(a)]; the phase of the wave function with respect to adjacent layers can be schematically represented as  $\frac{+}{+}$  and  $\frac{+}{-}$ , respectively. In the double sandwich [Fig. 3(b)] each of these gives rise to a  $\overline{\Gamma}_{1+}$  and a  $\overline{\Gamma}_{2-}$  and in the bulk [Fig. 3(c)] they give rise to a continuum of levels along the  $\Gamma$ -A direction. We visualize the interplane nodal structure from the following diagrams, keeping in mind that the unit cell of the double sandwich has two formula units and in the bulk there is one formula unit per unit cell, while  $k_z$  represents the change in phase from one unit cell to the next in the z direction:

Predominant atomic	TiS <sub>2</sub> si	ingle sandwich	(Ti	$(\mathbf{S}_2)_2$ double sar	ndwich		TiS <sub>2</sub> bulk	
character	State	Energy	State	Energy	Diff. <sup>a</sup>	State	Energy	Diff. <sup>t</sup>
S 3 <i>s</i>	$\overline{\Gamma}_{1+}$	-13.55	$\overline{\Gamma}_{1+}$ $\overline{\Gamma}_{2-}$	-13.71 -13.50	0.21	$\Gamma_{1+}$ $A_{1+}$	-13.57 -13.17	0.40
S 3 <i>s</i>	$\overline{\Gamma}_{2-}$	-12.36	$\overline{\Gamma}_{1+}$ $\overline{\Gamma}_{2-}$	-12.57 -12.26	0.31	$A_{2-}$ $\Gamma_{2-}$	-12.51 -11.89	0.62
$S 3p_z$	$\overline{\Gamma}_{1+}$	-4.62	$rac{\overline{\Gamma}_{1+}}{\overline{\Gamma}_{2-}}$	4.87 4.14	0.73	$\Gamma_{1+} A_{1+}$	-5.23 - 3.83	1.45
$S 3p_{xy}$	$\overline{\Gamma}_{3+}$	-2.58	$\overline{\Gamma}_{3+}$ $\overline{\Gamma}_{3-}$	-2.42 -2.38	0.04	$\Gamma_{3+}$ $A_{3+}$	-2.43 -2.37	0.06
<b>S</b> 3 <i>p</i> <sub>z</sub>	$\overline{\Gamma}_{2-}$	-1.13	$\overline{\Gamma}_{1+} \ \overline{\Gamma}_{2-}$	-1.64 -0.50	1.14	$A_{2-}$ $\Gamma_{2-}$	-2.13 0.25	2.43
\$ 3 <i>p</i> <sub>xy</sub>	$\overline{\Gamma}_{3-}$	0.09	$\overline{\Gamma}_{3+}$ $\overline{\Gamma}_{3-}$	0.11 0.17	0.06	$A_{3-}$ $\Gamma_{3-}$	0.18 0.34	0.16
Ti 3 <i>d</i>	$\overline{\Gamma}_{3+}$	0.28	$\overline{\Gamma}_{3-}$ $\overline{\Gamma}_{3+}$	0.37 0.42	0.05	$A_{3+}$ $\Gamma_{3+}$	0.42	0.01
Ti 3 <i>d</i>	$\overline{\Gamma}_{1+}$	0.97	$\overline{\Gamma}_{1+}$ $\overline{\Gamma}_{2-}$	0.91 1.27	0.36	$\Gamma_{1+}$ $A_{1+}$	0.79 1.36	0.57
Ti 3 <i>d</i>	$\overline{\Gamma}_{3+}$	3.53	$\frac{\overline{\Gamma}_{3+}}{\overline{\Gamma}_{3-}}$	3.57 3.69	0.12	$\Gamma_{3+}$ $A_{3+}$	3.53 3.85	0.32

TABLE III. Energy levels of single sandwich  $TiS_2$  at  $\overline{\Gamma}$  and corresponding levels of double sandwich and bulk. Energies are given relative to  $E_F$  in eV.

<sup>a</sup>Splitting between interlayer bonding and antibonding levels.  ${}^{b}k_{z}$  dispersion in bulk bands.

In Table III we give the energy levels of the single sandwich at  $\Gamma$  and the corresponding levels of the double sandwich and the bulk. The splittings within pairs originating from the same state in the single sandwich are seen to be small in most cases. Two states do have a fairly large splitting of 0.7 and 1.1 eV in the double sandwich and 1.4 and 2.4 eV in the bulk. The predominant atomic character of the states was determined by integrating the *l*component charge densities within the atomic spheres, and by examining wave-function contour maps. These states are of S  $p_z$  character, the very states we would expect to extend the furthest into the intersandwich region, and which had been identified by Mattheiss<sup>17</sup> as the critical states for interlayer interactions. Very recently Benesh and Woolley have called attention to the effects of shifts in the bulk  $\Gamma_{2-}$  state due to applied pressure.<sup>18</sup>

The dispersion of the bulk bands in the  $\Gamma$ -A direction is approximately double the splitting of the bands at  $\overline{\Gamma}$  in the double sandwich. In fact, it is interesting to note that the single-sandwich bands are approximately the average of the bulk bands along the  $\Gamma MK$  and the ALH directions and the double-sandwich bands are similar to those obtained by superposing the bulk bands from planes one-

quarter and three-quarters of the way up from the  $\Gamma MK$  and the *ALH* planes.

It is not always possible to make a clear distinction between bonding and antibonding states in anything more complicated than a linear molecule, but it is often possible to make qualitative arguments in terms of these concepts. We have found that it is possible to characterize all the occupied single-particle states at  $\Gamma$  and A as being either bonding or antibonding states in the intrasandwich and intersandwich S-S bond directions, with the Ti atoms merely mediating the interaction. With the use of these concepts it is possible to explain many of the features of the band structure of TiS<sub>2</sub> and some of the changes that occur upon intercalation.

From the schematic diagrams just given we see that the bulk  $\Gamma_{1+}$  and the  $A_{1+}$  states have bonding character in the *intra*sandwich region, whereas the  $\Gamma_{2-}$  and the  $A_{2-}$  are antibonding in this region. The  $\Gamma_{1+}$  and the  $A_{2-}$  are bonding in the *inter*sandwich region, whereas the  $\Gamma_{2-}$  and the  $A_{1+}$  are antibonding in this region. When we examine the charge density of individual states we see that although the intrasandwich and the intersandwich distances are equal, there is appreciably more



FIG. 4. Self-consistent LAPW band structures. Calculated data points for even and odd parity states with respect to the operations listed in Table II, are indicated by  $\times$  and  $\circ$ , respectively. (a) Intercalated sandwich (S-Ti-S)-Li-(S-Ti-S), (b) bulk LiTiS<sub>2</sub>, intercalated structure.

charge in the intrasandwich S-S bond than in the intersandwich S-S bond. On the basis of the usual overlap arguments, this confirms our expectation that the intrasandwich bonds are stronger than the intersandwich bonds. This being so we would predict that the states derived from some particular atomic orbitals would be in order of increasing energy  $\Gamma_{1+} < A_{1+} < A_{2-} < \Gamma_{2-}$ . From Fig. 3 and Table III we see that this is true in all cases. Further, we would predict that an increase in the intrasandwich S-S distance would reduce both the  $\Gamma_{1+}-\Gamma_{2-}$  and the  $A_{1+}-A_{2-}$  splittings, whereas an increase in the intersandwich S-S distance would reduce the  $\Gamma_{1+}$ - $\Gamma_{2-}$  splitting but increase the  $A_{1+}$ - $A_{2-}$  splitting. Upon intercalation, the intrasandwich and the intersandwich distances between the S planes increase by 1% and 16.5%, respectively. Both these changes act to decrease the  $\Gamma_{1+}$ - $\Gamma_{2-}$  splitting but they have opposite effects on the  $A_{1+}$ - $A_{2-}$  splitting. Comparing the pure TiS<sub>2</sub> results of Fig. 3 with the Li-intercalated results given in Fig. 4, we see that indeed the calculated  $\Gamma_{1+}$ - $\Gamma_{2-}$  splittings are considerably reduced, whereas there is a slight increase in the  $A_{1+}$ - $A_{2-}$  splitting.

#### B. Band features and optical properties

The question whether stoichiometric  $TiS_2$  is semiconducting or semimetallic has been a matter of great controversy. It was originally classified as a semiconductor.<sup>2,19</sup> Some years ago most experimentalists believed that it was semimetallic,<sup>20–23</sup> but the prevalent opinion now is that stoichiometric  $TiS_2$  is semiconducting<sup>24–26</sup> and that the observed semimetallic properties are due to excess Ti.

Table IV compares some calculated and experimentally determined band features of TiS<sub>2</sub> obtained by several authors. Our calculations are in fair agreement with experiment and in good agreement linear combination of with the atomic orbitals-discrete variational method (LCAO-DVM) results of Zunger and Freeman<sup>10</sup> with the exception of the value of the p-d gap. We calculate a semimetal with an indirect  $\overline{\Gamma}$ - $\overline{M}$  overlap of 0.1 and 0.2 eV in the single and double sandwiches, respectively, and an indirect  $\Gamma$ -L overlap of 0.5 eV in the bulk. It is worth mentioning here that Mattheiss<sup>17</sup> has observed that inclusion of the nonspherical terms in the muffin-tin spheres,

TABLE IV. Comparison of calculated and experimentally determined band features of  $TiS_2$ . Energies in eV. (1) Semiempirical LCAO (Ref. 7), (2) non-self-consistent Korringa-Kohn-Rostoker method (Ref. 8), (3) self-consistent OPW (Ref. 9), (4) self-consistent LCAO-DVM (Ref. 10), and (5) LCAO (Ref. 11).

	Experiment	(1)	(2)	(3)	(4)	(5)	Present work
Width of S s band	$2-3^{a}, 1-2^{b}$			2.5	2.0	2.6	2.0
Width of S $p$ band	6-7 <sup>a</sup> ,4-4.5 <sup>b</sup> 6-7 <sup>c</sup>	2.9	5.0	6.6	5.5	6.1	5.6
Width of Ti d band	4 <sup>d</sup>	2.1	2.6	7.1	4.4	3.3	4.5
S s - S p gap	6-7 <sup>b</sup>			5.5	6.8	5.9	6.3
S $p-Ti d$ gap	$0.2 - 0.3^{e}$	0.8	2.0 - 2.7	1.5	0.2	0.7	-0.5
$t_{2g}-e_g$ splitting	2.2 <sup>d</sup> ,2.1 <sup>c</sup>	1.4	1.3	$t_{2g}, e_g$ are not distinct	2.3	2.0	2.6

<sup>a</sup>He photoemission, Ref. 32.

<sup>b</sup>X-ray photoelectron spectra, Ref. 33. <sup>c</sup>Appearance potential spectra, Ref. 34.

<sup>d</sup>X-ray emission and absorption, Ref. 20.

<sup>e</sup>See text.

which we have neglected, caused the *p*-*d* band gap in another transition-metal dichalcogenide NbSe<sub>2</sub> to increase by 0.3 eV. This shift is in the right direction but is not large enough to bring our band structure into agreement with experimental evidence. Our LAPW calculations are semirelativistic,<sup>13</sup> i.e., the main level shifts due to relativistic effects are included self-consistently. Spin-orbit splittings of a few tenths eV are expected in a full relativistic treatment; it would be interesting to know whether this is sufficient to "open the gap" again.<sup>27</sup>

The optical reflectivity<sup>19</sup> and the optical transmission coefficient<sup>21,28</sup> of TiS<sub>2</sub> have been measured. The reflectivity has been measured in the energy range from 0-12 eV and shows much structure. Unfortunately, it has not been transformed (extrapolation of the high-energy region would be necessary) to obtain the absorption coefficient, which is the quantity that can be most directly calculated. The optical transmission of nearly stoichiometric thin films was measured from about 0.5 to 3.5 eV by Perry<sup>21</sup> revealing a semimetal with a sharp onset of absorption at 1.03 eV with additional structure at  $\sim 1.5, 2.1, 3.2, and$ 3.4 eV. The results are qualitatively similar to those obtained by Beal et al. on highly nonstoichiometric samples. Free-carrier absorption is observed in the infrared region.

A theoretical calculation of the absorption coefficient would involve evaluating the matrix elements of the operator  $\hat{\epsilon} \cdot \nabla e^{i \vec{k} \cdot \vec{r}}$  where  $\vec{k}$  is the

wave vector of the photon and  $\hat{\epsilon}$  is its polarization. We have not evaluated the matrix elements; however, structure in the absorption spectrum can at times be correlated with direct transitions at highsymmetry points. The bands must have zero slope at some high-symmetry points and therefore are parallel there, giving large DOS contributions. In Table V we give the dipole-allowed transitions from the S s,p bands to the Ti d bands. Since current experimental evidence seems to favor the interpretation that stoichiometric TiS<sub>2</sub> is a semiconductor with a band gap of 0.2 - 0.3 eV, we have shifted the calculated Ti d bands up by 0.75 eV to make quantitative comparisons. In Fig. 5 we present a histogram of the allowed transitions for  $\vec{E}||x$  or y polarization since the experiments are performed with unpolarized light incident perpendicularly to the x-y plane. Our results show that the edge noted at  $\sim 1.0$  eV is probably due to direct transitions at  $\Gamma$  and A regions of the Brillouin zone (BZ). The shoulder at 1.6 eV is represented by the beginning of transitions around the L point. The strong peak at  $\sim 2.2$  eV represents the point where M transitions begin, but it is likely that large regions of the BZ also contribute to the transitions in this energy range. The shoulders at 3.2 - 3.3 eV observed at the extreme end of the measured transmission spectra are seen to be the beginning of a very intense band spanning from  $\sim 3.2-7$  eV. Electron-energy-loss experiments would be useful to resolve this major feature of the electronic structure.

	irred.	Pola	rization
k point	rep.		
		$\vec{E}  x \text{ or } y$	$\vec{E}  z$
Г	$2^- \rightarrow 1^+$		1.30,13.44
	$2^- \rightarrow 3^+$	0.93,4.04,13.07,16.17	
	$3^- \rightarrow 3^+$	0.84,3.94	0.84,3.94
	$3^- \rightarrow 1^+$	1.30	
A	$2^{-} \rightarrow 1^{+}$		4.29,14.62
	$2^- \rightarrow 3^+$	3.34,6.77,13.68,17.14	
	$3^- \rightarrow 3^+$	0.98,4.41	0.98,4.41
	$3^- \rightarrow 1^+$	1.93	
		$\vec{\mathbf{E}}  \mathbf{x} $	$\vec{\mathbf{E}}  \mathbf{y}$ or $\mathbf{z}$
М	$1^- \rightarrow 1^+$	2.23,3.95,6.01	
	$1^- \rightarrow 2^+$		3.57,5.88
	$2^- \rightarrow 1^+$		2.23,3.95,4.49,6.01,6.21
			8.27,12.7,14.42,16.48
	$2^- \rightarrow 2^+$	4.75,5.83,7.06,8.14	
		14.04,16.35	
L	$1^- \rightarrow 1^+$	1.72,3.7,6.23	
	$1^- \rightarrow 2^+$		3.35,5.31
	$2^{-} \rightarrow 1^{+}$		1.6,3.58,4.98,6.11,6.97
			9.49,12.23,14.21,16.74
	$2^- \rightarrow 2^+$	3.23,5.19,6.61,3.57,	
		13.85,15.81	

TABLE V. Dipole-allowed optical transitions in bulk  $\text{TiS}_2$  from S *s,p* bands to Ti *d* bands at four high-symmetry  $\vec{k}$  points. The Ti *d* bands have been shifted up by 0.75 eV relative to the S *s,p* bands. Energies are in eV.

#### C. Comparison with intercalated structure

A semiempirical LCAO calculation of the energy bands of Li-TiS<sub>2</sub> has been made by McCanny.<sup>29</sup> McCanny finds that the following changes occur upon intercalation: (i) The S 3s bands move down to lower energies and are split apart further at  $\Gamma$ . (ii) Some bands, particularly the S  $p_z$ -like bands, have a smaller dispersion along  $\Gamma$ -A. (iii) There is an increase of about 0.8 eV in the S p – Ti d gap. (iv) A state that lies above the Ti 3d-like states in pure TiS<sub>2</sub> is pulled down in energy at  $\Gamma$  by about 7 eV into the Ti 3d band region.



FIG. 5. Optical absorption of  $TiS_2$ , estimated from joint density of states due to dipole-allowed transitions.

Our calculations confirm some of these changes but disagree with others. We find that the S 3s and 3p bands move down in energy by  $\sim 0.7$  eV relative to the Ti  $3d-t_{2g}$  bands; the Ti  $3d-e_g$  bands also move down relative to the Ti  $3d-t_{2g}$  bands but by a smaller amount,  $\sim 0.3$  eV. As discussed before we obtain a decrease in the  $\Gamma$ -A dispersion in agreement with McCanny and our bonding argument based upon the lattice expansion. The  $\Gamma_{1+}$ - $\Gamma_{2-}$  splitting is found to decrease in agreement with our prediction, but in disagreement with McCanny's results. Comparing Figs. 3(c) and 4(b) we see that high-lying states are pulled down by about 1.5 eV relative to the Ti  $3d-e_g$  bands but that this shift is not large enough for the band to intersect the Ti 3d bands, as McCanny finds. Aside from these shifts the band structures of the stoichiometric and lithiated films are indeed very similar. There is no evidence of the formation of additional "Li-bonding" bands in the valence region, supporting the simple idea of charge donation to form Li<sup>+</sup> upon intercalation. However, we can see from the relative band-structure shifts, and from analysis of the charge density, that the rigidband model is an oversimplification.



FIG. 6. Layer projected density of states, in units of states per (eV spin). (a) Single sandwich,  $TiS_2$ , (b) double sandwich,  $(TiS_2)_2$ , (c) bulk  $TiS_2$ .

In Figs. 6(a)-6(c) we show the layer projected density of states for  $TiS_2$  films and bulk, broadened by convolution with a Gaussian of width 0.2 eV, in the same sequence as the bands in Fig. 3. In most metallic materials the DOS obtained from a thin-film calculation are considerably narrower than the bulk DOS because there are fewer neighbors with which to interact.<sup>30</sup> The similarity of the single-sandwich, double-sandwich, and bulk DOS seen in Figs. 6(a) - 6(c) is indicative of relatively weak intersandwich interaction, and thus of the quasi-two-dimensional nature of the electronic structure. In particular, we note that although the inner and the outer S atoms in the dou-



ble sandwich are inequivalent they have nearly the same DOS. In all cases we note the sizable participation of Ti 3d 4sp states in the occupied valenceband region, clearly indicating shortcomings of the fully ionic Ti<sup>4+</sup>, S<sup>2-</sup> bonding models. The  $t_{2g}$  and  $e_g$  subbands are nevertheless seen as expected in the region just above  $E_F$ . Splitting of the  $t_{2g}$  band into two subbands is seen even in the single layer, and is thus a feature of the intrasandwich bonding.

The DOS of the intercalated double sandwich, shown in Fig. 7(a), reveals a difference between inequivalent sulfur layers. The states that have most of their weight on the inner S atoms are lower in energy by about 0.7 eV than those with most of their weight on the outer atoms, clearly a result of the attractive Li-site potentials. In the bulk material all S sites are equivalent; however, an experimental technique that is sensitive to both the surface and the bulk should be able to detect the upward shift of the S s and p bands on the surface atoms. Chemical shift measurements on S core states in stoichiometric versus lithiated materials would provide similar information.

In order to display the Ti-S bonding in bulk  $TiS_2$  and the distribution of Li character in the intercalated materials, we present the layer decomposition of charge for states at  $\Gamma$ , near the Fermi energy, in Table VI. This table, along with Fig. 7, makes it clear that the Li contributions to the spectral density are spread across both chalcogen and metal valence and conduction bands.

## **IV. CHARGE DENSITY**

In Fig. 8 we show the charge density of the 16 valence electrons (Ti 1s, 2s, 2p, 3s, 3p and S 1s, 2s, 2p states were included in the core) of bulk  $TiS_2$ : (a) in the y-z plane through the atoms in the unit cell, (b) in the x-y plane through the Ti atoms, and (c) in the x-y plane halfway between the sandwiches (empty octahedral site at the center). Figure 9(a)shows the same plot for the 17 valence electrons of bulk Li-TiS<sub>2</sub>. From Fig. 9 we notice that the charge density is highest around the S atoms. Since the occupied valence levels are of mostly S s or S p character, with nominal valence state  $S^{2-}$ , this is to be expected. The charge around the Ti atom in TiS<sub>2</sub> shows some noticeable angular polarization, but in Li-TiS<sub>2</sub> it is considerably more spherical. In both cases there is a clear qualitative difference compared to the orthogonalized-planewave (OPW) results<sup>9</sup> of von Boehm et al. The present LAPW densities exhibit metal-ligand charge sharing consistent with a mixed covalentionic bonding structure, while the OPW densities appear to represent a purely ionic structure. The midpoint of the intrasandwich S-S bond is at the midpoint of the side edges of the contour plots (point A) and the midpoint of the intersandwich S-S bond is at the top right or the bottom left corners of the plots (point B). As mentioned before we see that even in the unintercalated materials, where the intrasandwich and the intersandwich



FIG. 7. Layer projected density of states, in units of states per (eV spin). (a) Intercalated sandwich, (b) bulk LiTiS<sub>2</sub>.

S-S distances are equal, there is appreciably more charge in the intrasandwich bond than in the intersandwich bond.

The charge density in the x-y plane through the Ti atoms, except for the region close to the atom site, is surprisingly uniform in both materials. That is, there are regions of conduction-electron-

like density not found in ionic bonded compounds. This emphasizes again the complex mixed-bonding character of the transition-metal chalcogenides similar to that found for the oxides.

Except for a very narrow peak around the Li site, and the reduced Ti polarization, the introduction of Li into the intersandwich region has affect-

TABLE VI. Layer decomposition of selected states for bulk  $TiS_2$  and  $LiTiS_2$ . Energies are given relative to  $E_F$  in eV.

		TiS <sub>2</sub> Fraction of	f charge in		Li-TiS <sub>2</sub> Fraction of charge in layer			
State	Energy	Ti	S	Energy	Ti	S	Li	
Γ <sub>1+</sub>	-13.57	0.17	0.83	-14.05	0.15	0.77	0.06	
$\Gamma_{2-}$	-11.89	0.09	0.91	-12.81	0.07	0.86	0.05	
$\Gamma_{1\pm}$	-5.27	0.41	0.58	5.88	0.38	0.37	0.21	
$\Gamma_{3+}$	-2.43	0.49	0.50	-2.77	0.43	0.55	0.01	
Γ2_	+ 0.24	0.24	0.75	-1.41	0.22	0.59	0.18	
Γ <sub>3-</sub>	+ 0.33	0.06	0.93	-0.53	0.06	0.88	0.04	



FIG. 8. Valence electron charge density for bulk TiS<sub>2</sub> (contour levels differ by a factor of  $\sqrt{2}$ ), in units of electrons per unit cell. (a) y-z plane including atoms of unit cell, (b) x-y plane through Ti atoms, (c) x-y plane halfway between sandwiches.



FIG. 9. Valence electron charge density for the intercalation compound LiTiS<sub>2</sub> (contour levels differ by a factor of  $\sqrt{2}$ ), in units of electrons per unit cell. (a) y-z plane including atoms of unit cell, (b) x-y plane through Ti atoms, (c) x-y plane through Li sites, midway between sandwiches.

ed the shape of the density contours very little. In order to obtain a better idea of the redistribution of charge upon intercalation we have plotted the charge density averaged over the x-y plane,  $\rho_{\perp}$ , as a function of z in Fig. 10,

$$\rho_{\perp}(z) = A^{-1} \int_{\mathcal{A}} \rho(\vec{\mathbf{r}}) dx \, dy \; .$$

The additional charge due to intercalation is seen to lie primarily between the Li and the S planes. This is at first a very surprising result since the additional electron "donated" by Li occupies a band that is predominantly Ti 3*d*-like. We find that this charge is compensated in that the other valence states relax in such a way as to transfer part of their charge away from the Ti layer. We have studied the effect of intercalation on the fraction of the charge in layers associated with each atom type, the layer boundaries being defined to lie halfway between the planes containing atoms of a given type. We have found a shift in the charge



FIG. 10. Charge density averaged over x-y plane and plotted vs z.

away from the Ti layer to occur for all the occupied states studied throughout the Brillouin zone. In Table VI the layer weights for a few states are shown for bulk TiS<sub>2</sub> and bulk Li-TiS<sub>2</sub> in order to display this relaxation of the occupied orbitals.

### **V. CONCLUSIONS**

The band structures and DOS of TiS2 were calculated for a single sandwich of the material, a double sandwich, and the bulk. The similarity of the DOS of the three systems indicates weak intersandwich bonding; at the zone center  $\Gamma$  we can identify the two states, dominantly of S  $p_z$  character, that interact appreciably across the intersandwich region. The band structure and DOS of a double film with a full intercalated layer of Li and of the bulk-lithiated material were also calculated. Upon intercalation the S s and p bands move down in energy by about 0.7 eV and the Ti  $d-e_g$  bands move down by about 0.3 eV relative to the Ti  $d-t_{2g}$  bands. Many features of the band structure of TiS<sub>2</sub> and some of the changes upon intercalation can be explained by making simple arguments concerning the bonding or antibonding character of states in the intrasandwich and intersandwich S-S bond directions. Our semirelativistic LAPW band results predict that both thin-film and bulk  $TiS_2$  are semimetallic with a band overlap of 0.5 eV in the bulk.

Contour plots of the valence charge density were presented and used to characterize general features of the bonding. None of the occupied levels has dominant Li character, according to a state-bystate analysis. The additional charge of the "Li electron" is not localized around the Li atom but is spread out in the region between the Li and the S layers, consistent with ideas of the character of the mobile Li<sup>+</sup> ion in this material derived from NMR and other measurements.<sup>31</sup>

After this work was completed we became aware of the angle-resolved photoemission experiments of Chen et al. on TiSe<sub>2</sub> and TiS<sub>2</sub>.<sup>35</sup> The uppermost sulfur p band of TiS<sub>2</sub> that they observe comes close to  $E_F$  at  $\Gamma$  but does not cross  $E_F$ . From this they conclude that  $TiS_2$  is a semiconductor. However, they also find an appreciable filling of a Ti d band near M and L. It is possible to have completely filled sulfur p bands and a partially filled Ti dband only if the sample contains excess Ti. However, the authors claim that their sample is stoichiometric to better than 0.5%. Hence, it seems possible that the experiment may have failed to see a sulfur p band that crosses  $E_F$ .

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