Free-carrier absorption in quantizing magnetic fields: Degenerate carriers

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The theory developed to treat free-carrier absorption in nondegenerate semiconductors in the presence of quantizing magnetic fields is modified so as to be applicable to highly doped semiconductors where the carriers are degenerate. When either acoustic-phonon or nonpolar optical-phonon scattering is dominant, the free-carrier absorption coefficient undergoes de Haas–Shubnikov-type oscillations, where the period of the oscillations depends upon the Fermi energy E_F of the carriers and the photon and phonon frequencies. When $E_F >> \hbar\omega_c$, the amplitude of the oscillations is small but when $E_F > \hbar\omega_c$, the amplitude of the oscillations will be comparable to the oscillations which occur in nondegenerate semiconductors.

I. INTRODUCTION

In recent years there has been renewed interest in magneto-optical absorption in semiconductors.[1–4] Experimental observations of the photoconductivity of semiconductors in magnetic fields have shown an oscillatory dependence on the magnetic field.[1] We have recently extended the quantum theory of the free-carrier absorption in semiconductors[5,6] to take into account the presence of quantizing magnetic fields.[7–9] The results we obtained were applicable to nondegenerate semiconductors where either acoustic or nonpolar optical-phonon scattering is dominant. The absorption coefficient was found to be an oscillatory function of the magnetic field with peaks in the absorption whenever $\Omega = n\hbar\omega_c$ for acoustic-phonon scattering and $\Omega \pm \omega_0 = n\hbar\omega_c$ for nonpolar optical-phonon scattering. The symbols used here are the same as defined in our previous work.[7–9]

Since the results we obtained previously were for nondegenerate semiconductors, they are applicable only to lightly doped semiconductors such as those in which the magneto-photoconductivity experiments were done. In highly doped semiconductors, the density of the carriers will be high enough so that the carriers will be degenerate. Therefore, in this paper we wish to modify our previous theoretical results so they will be applicable in such highly doped materials.

II. THEORY

The basic equations which enable us to calculate the free-carrier absorption in semiconductors have been presented in our previous papers.[7–9] The free-carrier absorption coefficient is given by

$$K = \frac{\epsilon^{1/2}}{nc} \sum_i W_i f_i \ , \tag{1}$$

where the symbols have been defined in our previous work.[7–9] Here f_i is the carrier distribution function which for a degenerate gas of electrons is

$$f(E_i) = \begin{cases} 1, & E_i < E_F \\ 0, & E_i > E_F \end{cases} \tag{2}$$

where E_F is the Fermi energy of the carriers. The transition probability W_i has been obtained for acoustic-phonon[7] and nonpolar optical-phonon[8] scattering with the radiation polarized either longitudinal or transverse[9] to the dc magnetic field. For acoustic-phonon scattering, the transition probability is

$$W_i = \left[\frac{2ne^2 E_a{}^2 k_B T}{\epsilon \rho u^2 \hbar^3 \Omega_0} \right] \frac{\omega_c}{\Omega^3} \sum_{n'=0} \frac{\left[K_z^2 - \frac{\Delta_{n'n}}{2} \right]}{(K_z^2 - \Delta_{n'n})^{1/2}} \tag{3}$$

for the radiation polarized along the magnetic field direction and

$$W_i = \frac{ne^2 k_B T m E_a^2}{2\epsilon\Omega\rho u^2 \hbar^4 \Omega_0} \left[\left[\frac{\omega_c}{\omega_c + \Omega} \right]^2 + \left[\frac{\omega_c}{\omega_c - \Omega} \right]^2 \right] \times \sum_{n'=0} \frac{(n'+n+1)}{(k_z^2 - \Delta_{n'n})^{1/2}} \tag{4}$$

for the radiation polarized transverse to the mag-

netic field. Here E_a is the deformation potential, Ω_0 is the volume of the crystal, and

$$\Delta_{n'n}=(2m/\hbar)[\Omega-(n'-n)\omega_c]\ .$$

Using Eqs. (2)–(4) in Eq. (1), we obtain the free-carrier absorption coefficient for a degenerate semiconductor for radiation polarized along the magnetic field

$$K_{\parallel}(B)=\frac{E_a^2}{2\pi^2}\left[\frac{e^2m^2k_BT\omega_c^2}{\hbar^6\rho u^2c\epsilon^{1/2}\Omega^3}\right]\sum_{n=0}^{n_1}\sum_{n'=0}^{n_2}[E_F-(n+\tfrac{1}{2})\hbar\omega_c]^{1/2}[E_F+\hbar\Omega-(n'+\tfrac{1}{2})\hbar\omega_c]^{1/2}\ , \tag{5}$$

and for radiation polarized transverse to the magnetic field

$$K_{\perp}(B)=\frac{E_a^2}{8\pi^2}\left[\frac{e^2k_BTm^2\omega_c}{\epsilon^{1/2}c\Omega\rho u^2\hbar^5}\right]\left[\left[\frac{\omega_c}{\omega_c+\Omega}\right]^2+\left[\frac{\omega_c}{\omega_c-\Omega}\right]^2\right]$$
$$\times\sum_{n=0}^{n_1}\sum_{n'=0}^{n_2}(n'+n+1)\ln\left|\frac{[E_f-(n+\frac{1}{2})\hbar\omega_c]^{1/2}+[E_F+\hbar\Omega-(n'+\frac{1}{2})\hbar\omega_c]^{1/2}}{[E_f-(n+\frac{1}{2})\hbar\omega_c]^{1/2}-[E_F+\hbar\Omega-(n'+\frac{1}{2})\hbar\omega_c]^{1/2}}\right|\ . \tag{6}$$

Here n_1 and n_2 are the largest integers satisfying the conditions

$$n_1\le\frac{E_f}{\hbar\omega_c}-\frac{1}{2},\quad n_2\le\frac{E_F}{\hbar\omega_c}+\frac{\Omega}{\omega_c}-\frac{1}{2}\ . \tag{7}$$

The free-carrier absorption coefficient in the absence of the magnetic field for degenerate carriers is

$$K(0)=\frac{2E_a^2}{9\pi^2}\left[\frac{e^2m^2k_BT}{\hbar^8\rho u^2c\epsilon^{1/2}\Omega^3}\right]E_F^3\left[1+\frac{\hbar\Omega}{E_F}\right]^{3/2}\ . \tag{8}$$

Using Eq. (8) in Eqs. (6) and (7), we can express our results in terms of the ratio of the absorption coefficient in the presence of the magnetic field to its zero-field value for both the longitudinal and transverse polarizations.

$$\frac{K_{\parallel}(B)}{K(0)}=\frac{9}{4}\frac{\left[\frac{\hbar\omega_c}{E_F}\right]}{\left[1+\frac{\hbar\Omega}{E_F}\right]^{3/2}}\sum_{n'=0}^{n_1}\sum_{n=0}^{n_2}\left[1-(n+\tfrac{1}{2})\frac{\hbar\omega_c}{E_F}\right]^{1/2}\left[1+\frac{\hbar\Omega}{E_F}-(n'+\tfrac{1}{2})\frac{\hbar\omega_c}{E_F}\right]^{1/2}\ , \tag{9}$$

and

$$\frac{K_{\perp}(B)}{K(0)}=\frac{9}{16}\frac{\left[\frac{\hbar\omega_c}{E_F}\right]\left[\frac{\hbar\Omega}{E_F}\right]^2}{\left[1+\frac{\hbar\Omega}{E_F}\right]^{3/2}}\left[\left[\frac{\omega_c}{\Omega+\omega_c}\right]^2+\left[\frac{\omega_c}{\Omega-\omega_c}\right]^2\right]\sum_{n=0}^{n_1}\sum_{n'=0}^{n_2}\ , \tag{10}$$
$$\ln\left|\frac{[E_F-(n+\frac{1}{2})\hbar\omega_c]^{1/2}+[E_F+\hbar\Omega-(n'+\frac{1}{2})\hbar\omega_c]^{1/2}}{[E_F-(n+\frac{1}{2})\hbar\omega_c]^{1/2}-[E_F+\hbar\Omega-(n'+\frac{1}{2})\hbar\omega_c]^{1/2}}\right|\ .$$

By presenting our results in terms of these dimensionless ratios, our results depend only upon the magnetic field, the photon frequency, and the Fermi energy of the carriers and not the parameters of the particular material, except as they enter into the cyclotron frequency and Fermi energy. In Figs. 1 and 2, these ratios are plotted as a function of magnetic field for the longitudinal and transverse polarizations of the radiation relative to the magnetic fields. The parameters which have been used in the calculations leading to Figs. 1 and 2 are those characteristic of highly doped n-InSb with a carrier concentration of 10^{17} cm^{-3} at liquid-

helium temperatures (4.2 K) with the photon frequency corresponding to that of the 10.6-μm radiation from a CO_2 laser. For both polarizations, the free-carrier absorption coefficient is found to be an oscillatory function of magnetic field with two characteristic periods of oscillation, one dependent on the Fermi energy and the other on the Fermi energy and the photon energy. The periods of oscillation of the de Haas–Shubnikov types of oscillation are

$$\Delta\left[\frac{1}{H}\right]=\frac{mcE_F}{e} \text{ and } \Delta\left[\frac{1}{H}\right]=\frac{mc(E_F+\hbar\Omega)}{e}.$$

When $E_F >> \hbar\omega_c$, we can use Poisson's summation formula[10] to rewrite Eq. (9) in the form

$$\frac{K_{||}(B)}{K(0)}=\left[1-\frac{3\pi\sqrt{2}}{4}\frac{k_BT}{\hbar\omega_c}\left[\frac{\hbar\omega_c}{E_F}\right]^{3/2}\sum_{r=1}\frac{(-1)^r}{r^{1/2}}\frac{\cos\left[\frac{2\pi rE_F}{\hbar\omega_c}+\frac{\pi}{4}\right]}{\sinh\left[\frac{2\pi^2rk_BT}{\hbar\omega_c}\right]}\right]$$
$$\times\left[1-\frac{3\pi\sqrt{2}}{4}\frac{k_BT}{\hbar\omega_c'}\left[\frac{\hbar\omega_c}{E_F+\hbar\Omega}\right]^{3/2}\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{s^{1/2}}\frac{\cos\left[\frac{2\pi s(E_F+\hbar\Omega)}{\hbar\omega_c}+\frac{\pi}{4}\right]}{\sinh\left[\frac{2\pi^2sk_BT}{\hbar\omega_c}\right]}\right], \quad (11)$$

which specifically illustrates the oscillatory dependence of the free-carrier absorption coefficients on $1/B$ and the existence of two sets of periods for the de Haas–Shubnikov type of oscillations.[11] When $E_F >> \hbar\omega_c$, the amplitude of the oscillatory terms are of order $(\hbar\omega_c/E_F)^{3/2}$ and $(\hbar\omega_c/E_F+\hbar\Omega)^{3/2}$, respectively, and therefore the amplitude of the oscillatory term is small. However, when $E_F > \hbar\omega_c$, the amplitude of the oscillatory term is comparable to the nonoscillatory part of the absorption coefficient as can be seen from Fig. 1.

For the transverse configuration, the absorption coefficient diverges logarithmically when $\Omega=(n'-n)\omega_c$. When this condition is satisfied, the denominator in the argument of the logarithm will vanish for at least one term in the summation in Eq. (10). The origin of this logarithmic divergence is similar to that which occurs in nondegenerate materials and is due to the breakdown of the Born approximation for the scattering of the free carriers.

When nonpolar optical-phonon scattering dominates the free-carrier absorption process, the calculation exactly parallels that for acoustic-phonon scattering. As was the case for acoustic-phonon scattering, the absorption coefficient undergoes de Haas–Shubnikov oscillations with several kinds of periods. Here, however, there are three kinds of periods, one depending only on the Fermi energy and the other two on the photon and optical-phonon frequencies as well. The latter two periods arise from processes involving either the absorption or emission of optical phonons and the relative amplitudes of these oscillation terms will be temperature dependent because of the temperature

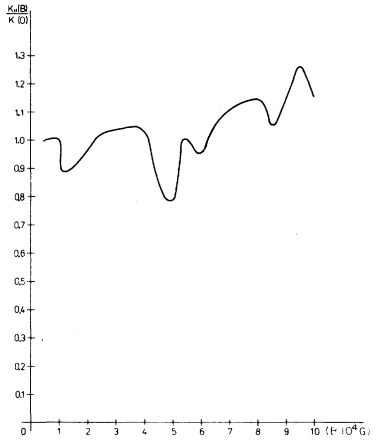

FIG. 1. Free-carrier absorption coefficient for radiation polarized along the magnetic field normalized to its zero-field value is shown as a function of magnetic field for n-InSb with a carrier concentration of 10^{17} cm^{-3} at 4.2 K in a radiation field of 10.6-μm wavelength when acoustic-phonon scattering is dominant.

dependence of the Bose-Einstein distribution. At low temperatures such that $\hbar\omega_0 \gg k_B T$, where ω_0 is the optical phonon frequency, the optical phonons will be frozen out and only the term corresponding to the emission of optical phonons will contribute to the absorption coefficient. Results for the transverse configuration can also be obtained. Here, too, there will be additional structure in the oscillatory behavior of the absorption coefficient due to the finite frequency of the optical phonons.

III. CONCLUSIONS AND SUMMARY

The free-carrier absorption coefficient in highly doped semiconductors, where the carriers are degenerate, will undergo de Haas—Shubnikov oscillations with several sets of periods. The periods will depend on the Fermi energy of the carriers as well as the photon and phonon frequencies. The amplitude as the oscillations will be small when $E_F >> \hbar\omega_c$ but will be comparable in magnitude to those predicted to occur in nondegenerate materials[7–9] when $E_F > \hbar\omega_c$. The behavior of the absorption coefficient depends on the polarization of the radiation relative to the magnetic field.

Katayama and Mills[12] have calculated the high-frequency relaxation time in highly doped polar semiconductors in the presence of quantizing magnetic fields using a Green's-function approach. Direct comparison cannot be made with the results presented in this report since they considered scattering by polar optical phonons and ionized impurities while we have considered scattering by acoustic phonons and nonpolar optical phonons. They did not examine oscillatory contributions to the absorption but such contributions should arise from careful examination of Eqs. (3.3)—(3.9) of their paper. The reason such oscillatory effects did not show up in their numerical calculations is that

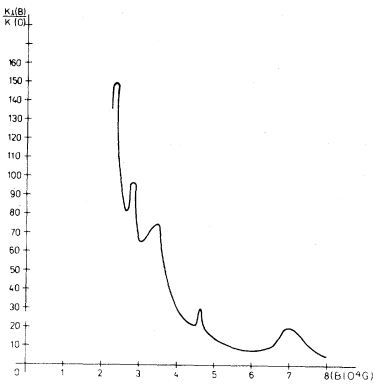

FIG. 2. Normalized free-carrier absorption coefficient for radiation polarized transverse to the magnetic field for the same conditions as in Fig. 1.

at a magnetic field of 50 kG in n-type PbSe, the cyclotron frequency exceeds the photon frequency over the range of photon frequencies they considered and in the regime $\omega_c > \Omega$, we would not expect the relaxation time to oscillate as a function of frequency. Also, since the relaxation time was calculated as a function of Ω for fixed magnetic field, we would not expect any de Haas—Shubnikov oscillations to show up.

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