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Lifetime of a quasiparticle in a two-dimensional electron gas

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We have investigated the inelastic Coulomb lifetime τ_{ee} of a quasiparticle near to the Fermi surface of a two-dimensional electron gas. Within a perturbative approach based upon the random-phase approximation, we find that at low temperature $1/\tau_{ee}$ behaves like $T^2 \ln T$. Furthermore at small quasiparticle excitation energy, the leading contribution to $1/\tau_{ee}$ is inversely proportional to the electronic density and does not depend upon the electric charge. Although the plasmon frequency goes to zero at long wavelength, plasmon emission contributes to the quasiparticle decay only when the quasiparticle excitation energy exceeds a certain threshold. The threshold becomes a small fraction of the Fermi energy in the high-density limit.

I. INTRODUCTION

The effect of Coulomb interaction on the lifetime of the electronic states close to the Fermi surface is a classic problem in many-body theory. For the ordinary three-dimensional (3D) electron gas, the inverse inelastic lifetime $1/\tau_{ee}$ associated with the electron-electron interaction has been evaluated within a perturbative approach by several authors since the pioneering work of Landau and Pomerantschusk.[1-3] At zero temperature for a quasiparticle state with wave vector p close to the Fermi wave vector p_F, it is found[3] that $1/\tau_{ee} \propto (p-p_F)^2$. Luttinger[4] has established the validity of this result at all the orders in perturbation theory. In a one-dimensional electron gas neutralized by a rigid positive background it has been found that $1/\tau_{ee} \propto |p-p_F|$.[4,5] The corresponding calculation for a two-dimensional (2D) electron gas has been performed by Chaplik.[6] The result is $1/\tau_{ee} \propto (p-p_F)^2 \ln|p-p_F|$.

There has been considerable interest in recent years in the physical properties of 2D metals. Electrons confined in silicon inversion layers and to the GaAs layer of GaAs-$Al_xGa_{1-x}As$ heterojunctions provide a vivid realization of such peculiar systems.[7] The inelastic broadening of the electronic states in these conductors plays a major role in the interpretation of magnetoconductance experiments[8-12] and its bearing upon the localization problem.[13-15] This is discussed in detail by Wheeler.[8] Several authors have invesitgated the Coulomb inelastic lifetime of the electronic states of a 2D metal in the presence of a finite concentration of impurities.[12,16] Their analysis, however, is restricted to the diffusive regime and the results cannot be extrapolated to the pure-metal limit. The aim of this work is to present a detailed and comprehensive investigation of the temperature-dependent Coulomb inelastic broadening in the simple case of a pure 2D electron gas. An interesting feature of the 2D situation is the possibility of plasma modes affecting the Coulomb broadening of the electronic states. In the usual 3D case this phenomenon is inhibited by the large energy associated with plasma oscillations. For a 2D metal however, the plasma frequency goes to zero in the long-wavelength limit[17] and plasmon emission can in principle become an available decay channel also for thermal or low-energy electronic excited states.

The paper is organized as follows. In Sec. II the microscopic theory of the Coulomb inelastic lifetime of a quasiparticle is revisited and specialized to the case of a 2D electron gas. In Sec. III we evaluate $1/\tau_{ee}$ and explicitly establish its asymptotic behavior. Section IV provides some discussion with emphasis on the peculiar temperature and charge dependence of the results. Finally three appendixes complete the paper by providing a discussion of a few technical aspects of the theory.

II. INELASTIC LIFETIME OF A QUASIPARTICLE

Consider a degenerate gas of N electrons in its normal ground state. This can be well described in terms of filled Fermi sea. A quasiparticle is obtained by adding to the system an extra electron which occupies an otherwise empty state characterized by a wave vector $\vec{p}$ and a spin projection σ. In complete analogy a quasihole can be obtained by removing an electron from an otherwise occupied state. At $T=0$ K, if p_F is the Fermi wave vector, necessarily $p \geq p_F$ for a quasiparticle and $p \leq p_F$ for a quasihole.

The ground state for these $N \pm 1$ electrons configurations is of course again a filled Fermi sea with the same Fermi wave vector, apart from corrections of order $1/N$. In the absence of any relaxation mechanism, quasiparticle, and quasihole states are stationary. The mutual Coulomb interaction however, provides a way to redistribute energy and momentum among the electrons and causes a quasiparticle (quasihole) state to decay. This leads to a finite inelastic lifetime $1/\tau_{ee}$ for these electronic states.

For $T=0$ K the situation is readily analyzed via standard time-dependent perturbation theory and $1/\tau_{ee}$ is given by the decay rate of the corresponding plane-wave state. At finite temperature the situation is more complicated and $1/\tau_{ee}$ is defined by the relaxation rate of the occupation number $n_{\vec{p},\sigma}$, as obtained by an approach based on a transport equation of the type[18]

$$\frac{\partial n_{\vec{p},\sigma}}{\partial t} = -\frac{n_{\vec{p},\sigma} - n^0_{\vec{p},\sigma}}{\tau_{ee}} , \tag{1}$$

where $n^0_{\vec{p},\sigma}$ is the distribution function at equilibrium. In both cases $1/\tau_{ee}$ can be evaluated within perturbation theory, with the use of the usual Fermi golden rule,[19]

$$\frac{1}{\tau_{ee}} = \frac{2\pi}{\hbar} \sum_{\vec{k},\vec{q},\sigma'} n^0_{\vec{k},\sigma'}(1-n^0_{\vec{k}-\vec{q},\sigma'})(1-n^0_{\vec{p}+\vec{q},\sigma}) \, | V_c(\vec{p},\vec{q}) |^2 \delta(E_{\vec{p}+\vec{q}} + E_{\vec{k}-\vec{q}} - E_{\vec{p}} - E_{\vec{k}}) , \tag{2}$$

where $V_c(\vec{p},\vec{q})$ is the matrix element of suitable electron-electron interaction potential.

Some discussion is in order as far as the proper choice of V_c is concerned. As pointed out by Quinn and Ferrell,[3] the use in Eq. (2) of the bare Coulomb potential matrix element $v(q)$ for V_c leads to the unphysical result $1/\tau_{ee} = \infty$. Such a difficulty can however be surmounted by allowing for screening effects. This is readily done within the random-phase approximation (RPA).[3,19] Accordingly we choose a dynamically screened interaction of the form

$$V_c(\vec{p},\vec{q}) = \frac{v(q)}{\epsilon(q,(E_{\vec{p}} - E_{\vec{p}+\vec{q}})/\hbar)} , \tag{3}$$

where $\epsilon(q,\omega)$ is the wave vector and frequency-dependent dielectric function of a two-dimensional electron gas.[6,20] ϵ is here evaluated at the frequency $(E_{\vec{p}} - E_{\vec{p}+\vec{q}})/\hbar$ corresponding to the energy transferred to the electron gas by the extra electron (hole) during the scattering. Notice that the use of a dynamical screening (as compared to a static one) makes V_c a complex quantity.

The sum over $\vec{k}$ and σ' appearing in Eq. (2) can be performed and, with the use of the fluctuation and dissipation theorem, expressed in terms of the imaginary part of the susceptibility $\chi^0(q,\omega)$ of a noninteracting electron gas,[19]

$$\sum_{\vec{k},\sigma'} n^0_{\vec{k},\sigma'}(1-n^0_{\vec{k}-\vec{q},\sigma'})\delta(E_{\vec{k}-\vec{q}} - E_{\vec{k}} - \omega) = -\frac{\mathrm{Im}\chi^0(q,\omega)}{S\pi(1-e^{-\hbar\omega/k_BT})} , \tag{4}$$

where S is the total surface and k_B is Boltzmann's constant.[21] Using this result and Eq. (3) in (2), $1/\tau_{ee}$ can be expressed as

$$\frac{1}{\tau_{ee}(\Delta)} = \frac{1}{\hbar S(1+e^{-\Delta/k_BT})} \sum_{\vec{q}} v(q) \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left[\coth\left[\frac{\hbar\omega}{2k_BT}\right] - \tanh\left[\frac{\hbar\omega - \Delta}{2k_BT}\right] \right] \times \mathrm{Im}\left[\frac{1}{\epsilon(q,\omega)}\right] \delta\left[\omega - \frac{\Delta + \mu - E_{\vec{p}+\vec{q}}}{\hbar}\right] , \tag{5}$$

where μ is the chemical potential and we have introduced the quantity $\Delta = E_{\vec{p}} - \mu$. Δ is just the excitation energy of the quasiparticle (hole) state. This expression for $1/\tau_{ee}$ applies equally well to the usual three-dimensional case (see Appendix C).[22]

If we use for the single-particle energy $E_{\vec{k}}$ the free-electron value $\hbar^2 k^2/2m$, the angular part of the integration involved in Eq. (5) can be carried out analytically. For a 2D system the result is

$$\int_0^{2\pi} d\phi\, \delta\left[\omega - \frac{\Delta+\mu-E_{\vec{p}+\vec{q}}}{\hbar}\right] = \begin{cases} 0, & \omega > \Omega(q)\,, \\ \dfrac{2}{\{[\Omega(q)-\omega][\Omega(q)+\hbar q^2/m+\omega]\}^{1/2}}\,, & -\Omega(q)-\dfrac{\hbar q^2}{m} \le \omega \le \Omega(q) \\ 0, & \omega < -\Omega(q)-\hbar q^2/m \end{cases} \tag{6}$$

where $\hbar\Omega(q)$ (see Fig. 1) is the maximum value of the energy transfer for a scattering process in which the extra electron (hole) changes its wave vector by q,

$$\Omega(q) = \frac{\hbar p q}{m} - \frac{\hbar q^2}{2m}\,. \tag{7}$$

With (6) in (5) and $v(q)=2\pi e^2/q$ we finally obtain for a 2D electron gas,

$$\frac{1}{\tau_{ee}(\Delta)} = -\frac{e^2}{\pi\hbar(1+e^{-\Delta/k_BT})}\left[\int_{-\infty}^{0} d\omega \int_{-q_-(\omega)}^{q_+(\omega)} dq + \int_0^{(\mu+\Delta)/\hbar} d\omega \int_{q_-(\omega)}^{q_+(\omega)} dq\right] \times \left[\coth\left[\frac{\hbar\omega}{2k_BT}\right] - \tanh\left[\frac{\hbar\omega-\Delta}{2k_BT}\right]\right] \frac{\mathrm{Im}(1/\epsilon(q,\omega))}{\{[\Omega(q)-\omega][\Omega(q)+(\hbar q^2/m)+\omega]\}^{1/2}}\,, \tag{8}$$

where $q_\pm(\omega)$ are the solutions of the equation $\Omega(q)=\omega$, i.e.,

$$q_\pm(\omega) = p\left[1 \pm \left[1-\frac{\hbar\omega}{\mu+\Delta}\right]^{1/2}\right]. \tag{9}$$

Figure 1 illustrates the geometrical constraints imposed by energy and momentum conservation to the decay processes in a 2D electron gas.

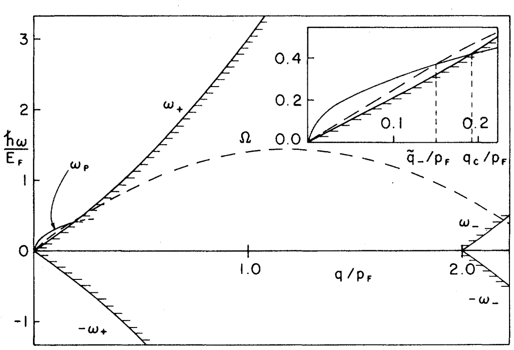

FIG. 1. Geometry of the q,ω space for a 2D electron gas. Single-particle excitations are possible only within the electron-hole continuum defined by $\omega_- \le |\omega| \le \omega_+$, with $\omega_\pm(q)=\hbar p_F q/m \pm \hbar q^2/2m$. Quasiparticle decay into electron-hole pairs is allowed only for q,ω in the electron-hole continuum and such that $-\Omega(q)-hq^2/m \le \omega \le \Omega(q)$ [Eq. (7) in the text]. ω_p is the plasmon dispersion relation. The inset is an expansion of the small q,ω region and depicts a situation in which plasmon emission is possible. For illustration we have chosen here $p=1.2p_F$ and $r_s=0.318$.

A completely equivalent approach to this problem is to evaluate to the lowest order in the screened interaction the self-energy $\Sigma(\vec{p},E_{\vec{p}})$ of the quasiparticle (quasihole). The corresponding diagram is shown in Fig. 2. $1/\tau_{ee}$ is then obtained via[3,19]

$$\frac{1}{\tau_{ee}} = -\frac{2}{\hbar}\,\mathrm{Im}\Sigma(\vec{p},E_{\vec{p}})\,. \tag{10}$$

III. DECAY PROCESSES

We turn now to the analysis of the elementary processes by which a quasiparticle (quasihole) state can decay, as described by the imaginary part of the inverse dielectric function in (8). Within RPA we can divide $\mathrm{Im}(1/\epsilon)$ as follows:

$$\mathrm{Im}\frac{1}{\epsilon(q,\omega)} = \mathrm{Im}\frac{1}{\epsilon(q,\omega)}\bigg|_{e\text{-}h} + \mathrm{Im}\frac{1}{\epsilon(q,\omega)}\bigg|_{\mathrm{pl}}\,. \tag{11}$$

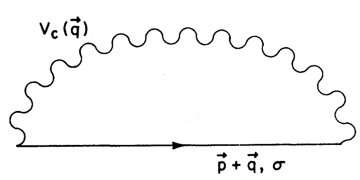

FIG. 2. The simplest self-energy diagram for an electron of wave vector $\vec{p}$ and spin projection σ. The dashed line represents the screened Coulomb potential, Eq. (3).

The first term is associated with single electron-hole pair excitations with wave vector q and energy $\hbar\omega$. The second describes inelastic processes involving excitation of plasma modes. Since in a three-dimensional electron gas the plasma frequency is always finite, for small excitation energies Δ, single electron-hole pair excitations represent the only relevant dissipative processes. In a 2D system however, the plasma frequency (see Appendix A) goes to zero at long wavelength[17] and, as already mentioned above, plasmons became available at small energies. Multipairs excitations are also possible but they lead to a small effect at low excitation energies, and are disregarded in RPA.

A. Decay into single-particle excitations

Within RPA single electron-hole pair excitations are possible only for q and ω inside the electron-hole continuum (see Fig. 1). At low temperature and small excitation energy Δ only the region of small ω is relevant. In this case Eq. (8) can be considerably simplified.

We first notice that because of the singular behavior of the integrand in Eq. (8) for $\omega=\Omega(q)$ [i.e., $q=q_{\pm}(\omega)$], the main contribution to the decay rate at low energies comes from scattering processes involving a small wave-vector change q of the order of $p-p_F$. Accordingly, we write

$$\operatorname{Im}\frac{1}{\epsilon(q,\omega)}\bigg|_{e\text{-}h}=-\frac{\hbar\omega}{2e^2p_F}\left[1-\left[\frac{m\omega}{\hbar q p_F}\right]^2\right]^{1/2}, \tag{12}$$

where we have made use in (A3) of the small q and ω expansion of the electronic susceptibility [Eqs. (5), (A1), and (A2)].[23] For q and ω outside of the electron-hole continuum, Im $(1/\epsilon)_{eh}$ is zero.

At $T=0$ K, the frequency integral of Eq. (8) is restricted to the interval 0, $\Delta/\hbar$. In this case, making use of (12) in (8), $1/\tau_{ee}\,|_{e\text{-}h}$ can be reduced, after some straightforward manipulations, to a single quadrature. A direct inspection allows us to extract the leading contribution. We find

$$\frac{1}{\tau_{ee}(\Delta)}\bigg|_{e\text{-}h}\simeq-\frac{E_f}{4\pi\hbar}\left[\frac{\Delta}{E_f}\right]^2\left[\ln\left[\frac{\Delta}{E_F}\right]-\frac{1}{2}-\ln\left[\frac{2q_{\mathrm{TF}}^{(2)}}{p_F}\right]\right],\quad T=0\text{ K},\quad \Delta\ll\frac{\hbar^2p_Fq_{\mathrm{TF}}^{(2)}}{m} \tag{13}$$

a result previously obtained by Chaplik.[6] In Eq. (13) $E_F=\hbar^2p_F^2/2m$ is the Fermi energy of the electronic system and $q_{\mathrm{TF}}^{(2)}$ is the Thomas-Fermi screening wave vector in 2D, given by $2me^2/\hbar^2$. The result of the numerical integration is shown in Fig. 3.

At finite temperatures the integrals involved in Eq. (8) are not feasible. However, in the region of temperatures much larger than Δ/k_B and much smaller than E_F/k_B, we have been able to evaluate the relevant contribution. The result is

$$\frac{1}{\tau_{ee}(\Delta)}\bigg|_{e\text{-}h}\simeq-\frac{E_F}{2\pi\hbar}\left[\frac{k_BT}{E_F}\right]^2\left[\ln\left[\frac{k_BT}{E_F}\right]-\ln\left[\frac{q_{\mathrm{TF}}^{(2)}}{p_F}\right]-\ln 2-1\right],\quad \Delta\ll k_BT\ll E_F\,. \tag{14}$$

B. Decay into plasma modes

At zero temperature, the contribution to $\operatorname{Im}(1/\epsilon)$ associated with the collective modes is given in RPA by[10]

$$\operatorname{Im}\frac{1}{\epsilon(q,\omega)}\bigg|_{\mathrm{pl}}=-\pi\left[\frac{\partial\,\mathrm{Re}\epsilon(q,\omega)}{\partial\omega}\right]^{-1}\delta(\omega-\omega_P(q))\,, \tag{15}$$

where $\omega_P(q)$ is the plasma dispersion relation, as discussed in Appendix B. This expression is defined only for values of q and ω lying outside of the electron-hole continuum (see Fig. 1). For a 2D electron gas the quantity $\partial\,\mathrm{Re}\epsilon/\partial\omega$ can be readily evaluated using Eqs. (A3) and (A1).

Inserting Eq. (15) in (8) we find after some straightforward manipulations,

$$\frac{1}{\tau_{ee}(\Delta)}\bigg|_{\mathrm{pl}}=\frac{2e^2}{\hbar}\int_0^{q_c}dq\,\frac{\Theta(\Omega(q)-\omega_+(q))}{\{[\Omega(q)-\omega_P(q)][\Omega(q)+(hq^2/m)+\omega_P(q)]\}^{1/2}}\,\frac{1}{[\partial\mathrm{Re}\epsilon(q,\omega_P(q))/\partial\omega]}$$
$$\times\int_{\min[\Delta/\hbar,\Omega(q)]}^{\min[\Delta/\hbar,\omega(q)]}d\omega\,\delta(\omega-\omega p(q)\,, \tag{16}$$

where q_c is the critical wave vector for plasma modes (see Appendix B) and min$[a,b]$ is the minimum between a and b.

Finite contributions to $1/\tau_{ee}$ in Eq. (16) come only from wave vectors q for which the condition $\omega_+(q) \le \omega_P(q) \le \Omega(q)$ is satisfied (see Fig. 1). Furthermore, at zero temperature, the excitation energy Δ must be larger than $\hbar\omega_P(\tilde{q}_-)$, with $\tilde{q}_-$ defined in Eq. (B6). This leads to the existence of a finite excitation-energy threshold Δ_c for the decay into plasmons. By using Eq. (B6) in (B2) we obtain for Δ_c the following equation:

$$\Delta_c = \left[\frac{32me^2E_F(E_F+\Delta_c)^{1/2}}{3\hbar} \right]^{1/2} \cos\left[\frac{\pi}{3} + \tfrac{1}{3} \arccos \left[\frac{E_F+\tilde{\Delta}(r_s)}{E_F+\Delta_c} \right]^{3/4} \right] , \tag{17}$$

where $\tilde{\Delta}$ is given by Eq. (B5). In the high-density limit, Eq. (17) reduces to the simpler form,

$$\Delta_c \simeq \sqrt{2} r_s E_F , \quad r_s \ll 0 . \tag{18}$$

Here r_s is the average interelectronic distance measured in Bohr radii.

In the general case Eq. (17) must be used. The values of Δ_c as given by Eq. (17) and (18) are compared for small r_s in Fig. 4. At metallic densities Eq. (17) gives quite large values for the excitation energy threshold Δ_c, and the quasiparticle decay into plasma modes is inhibited. In the high-density regime however, Δ_c can be still considered as a small fraction of the Fermi energy. In this case, for small Δ and Δ_c, we can make use in (16) of the approximate form

$$\left. \frac{\partial \mathrm{Re}\epsilon(q,\omega)}{\partial\omega} \right|_{\omega=\omega_P(q)} \simeq \frac{2}{\omega_P(q)} , \tag{19}$$

valid at small q. The result for $1/\tau_{ee}\,|_{\mathrm{pl}}$ is

$$\left. \frac{1}{\tau_{ee}(\Delta)} \right|_{\mathrm{pl}} \simeq \frac{2e^3 m^{1/2}}{\hbar^2} \Theta(\Delta-\Delta_c)(q_m - \tilde{q}_-)^{1/2} , \tag{20}$$

with $q_m = \min[\Delta^2/2e^2E_F , q_c]$ and $\tilde{q}_-$ is given in Eq. (B6). For Δ slightly larger than Δ_c, (20) reduces to

$$\left. \frac{1}{\tau_{ee}(\Delta)} \right|_{\mathrm{pl}} \simeq \frac{2\sqrt{2}me^2}{\hbar^3 p_F} \Delta_c \left[\frac{\Delta-\Delta_c}{\Delta_c} \right]^{1/2} , \quad \Delta \simeq \Delta_c . \tag{21}$$

Finally, as Δ exceeds $\hbar\omega_P(q_c) = (2e^2E_Fq_c)^{1/2}$, Eq. (20) can be written as

$$\left. \frac{1}{\tau_{ee}(\Delta)} \right|_{\mathrm{pl}} \simeq \frac{2e^4 m}{\hbar^3} \left[\frac{\Delta}{E_F} \right]^{1/2} , \quad \Delta \ge \hbar\omega_P(q_c) . \tag{22}$$

IV. DISCUSSION

In this paper we have calculated within a perturbative approach the temperature-dependent

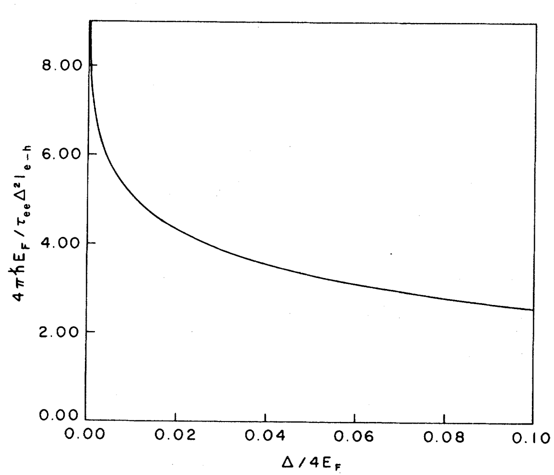

FIG. 3. Plot of $(\tau_{ee}\Delta^2)^{-1}$ measured in units of $4\pi\hbar E_F$ versus $\Delta/4E_F$, as obtained via direct numerical computation. For illustration we have taken here $r_s=2$.

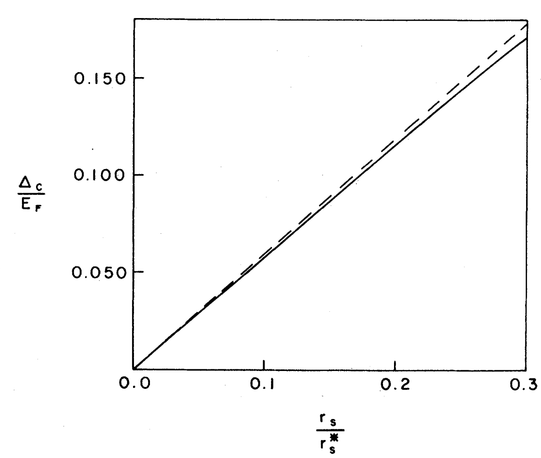

FIG. 4. Plasmon emission threshold Δ_c (measured in units of E_F) versus r_s/r_s^* ($r_s^* = 8\sqrt{2}/27$) in the high-density region. Δ_c is obtained solving numerically Eq. (17). The dashed line is the asymptotic formula $\Delta_c/E_F = \sqrt{2} r_s$ [Eq. (18)].

Coulomb inelastic lifetime τ_{ee} of a quasiparticle in a 2D electron gas. Our findings [Eqs. (13) and (20)] complemented by the results of earlier calculations, are schematically summarized in Table I. It is clear that the customary textbook "phase-space argument"[1] leads to the correct answer only in the 3D case.

In a 2D system, τ_{ee} displays an extra logarithmic dependence.[6] This peculiar result stems from the concurrent effects of the planar geometry and the conservation of energy and momentum in the electronic collision processes, as expressed by Eq. (6).[24] This has been overlooked by several previous investigators.

Another interesting feature is the complete independence of τ_{ee} from the electric charge, as manifest in Eqs. (13) and (14). This is just one of the consequences of the analytic dependence of the screened Coulomb potential regarded as a function of electric charge and wave vector.

We have investigated for comparison the dependence on the electric charge e of τ_{ee} in the usual 3D case. In the high-density limit Quinn and Ferrel[3] find that $1/\tau_{ee}$ is simply proportional to e. In the general case however, this dependence is much more involved as shown by the calculation of $1/\tau_{ee}$ for a 3D electron gas presented in Appendix C.

Quite generally, the dependence of $1/\tau_{ee}$ upon e is dictated by which values of the wave-vector transfer q are the most relevant ones in the decay process. In a 3D metal all the q values between zero and $2p_F$ provide a relevant contribution to $1/\tau_{ee}$ leading to the complicated structure of Eq. (C1). For a 2D system the singular behavior displayed in Eq. (6) makes the q values of the order of $p-p_F$ to contribute the leading term at low excitation energy. Since at long wavelength the screened Coulomb potential is independent of e so does $1/\tau_{ee}$.

The situation resembles the dirty-metal case.[14] In the presence of a finite concentration of impurities momentum conservation is relaxed and diffusion dominates the dynamics at low energy. In this case, in all dimensions, the most relevant contributions to the inelastic Coulomb lifetime come from q values of the order $(k_BT/D)^{1/2}$, where D is the diffusion constant. As a consequence $1/\tau_{ee}$ does not depend upon e both in two[16] and three[25] dimensions.

TABLE I. Asymptotic behavior of the inelastic Coulomb lifetime τ_{ee} for $p \to p_F$ in a 3D, 2D, and 1D degenerate electron gas.

	$1/\tau_{ee}$
3D[a]	$(p-p_F)^2$
2D[b]	$(p-p_F)^2 \ln \lvert p-p_F \rvert$
1D[c]	$\lvert p-p_F \rvert$

[a]Landau and Pomerantschusk (Ref. 1), Baber (Ref. 2), and Quinn and Ferrell (Ref. 3).
[b]Chaplik (Ref. 6) and this work.
[c]Luttinger (Ref. 4).

In the comparison with experiment the dependence of the leading term in a temperature expansion of $1/\tau_{ee}$ upon the electronic density is usually of interest. For a 3D metal $1/\tau_{ee} \propto A_3 T^2$ with A_3 proportional to $n^{-3/2}$. Our analysis of the 2D case gives [see Eq. (14)] $1/\tau_{ee} \propto A_2 T^2 \ln T$ with A_2 inversely proportional to n.

As discussed in Sec. III, at $T=0$ K, there exists a finite energy threshold for quasiparticle decay into plasma modes. This threshold is a substantial fraction of the Fermi energy at metallic densities. Such a fraction however decreases as the electronic concentration increases [see Eq. (18)]. At finite temperatures the calculation becomes involved and no clear-cut statement can be made. We expect, however, the existence of a typical temperature threshold T_c of the order $r_s E_F/k_B$ for which the plasma decay mechanism becomes as important as the single-particle processes in the broadening of quasiparticle states. Accordingly, above T_c, $1/\tau_{ee}$ will display an additional contribution proportional to $(T-T_c)^{1/2}$ and then to $T^{1/2}$ as T is further increased.

ACKNOWLEDGMENT

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APPENDIX A: DIELECTRIC FUNCTION OF A 2D ELECTRON GAS

The susceptibility $\chi^0(q\omega)$ of a noninteracting electron gas in 2D is readily evaluated at $T=0$ K, via linear response theory. The result is[20]

$$\mathrm{Re}\chi^0(q,\omega) = -N_0 \left[1 - \Theta(\lvert x+y \rvert -1)\frac{\operatorname{sgn}(x+y)}{2x}[(x+y)^2-1]^{1/2} - \Theta(\lvert x-y \rvert -1)\frac{\operatorname{sgn}(x-y)}{2x}[(x-y)^2-1]^{1/2} \right], \qquad \text{(A1)}$$

$$\mathrm{Im}\chi^0(q,\omega)=-\frac{N_0}{2x}\{\Theta(1-|x+y|)[1-(x+y)^2]^{1/2}-\Theta(1-|x-y|)[1-(x-y)^2]^{1/2}\}\ , \tag{A2}$$

where $x=q/2p_F$, $y=m\omega/\hbar qp_F$, and N_0 is the density of states at the Fermi energy. $\Theta(x)$ is the usual step function. Within RPA the dielectric constant $\epsilon(q,\omega)$ is then given by

$$\epsilon(q,\omega)=1-v(q)\chi^0(q,\omega)\ , \tag{A3}$$

with $v(q)=2\pi e^2 q$.

APPENDIX B: PLASMA WAVES IN A 2D ELECTRON GAS

The dispersion relation $\omega_P(q)$ for plasma waves can be readily established within RPA. Making use of the results of Appendix A, the collective mode condition $\epsilon(q,\omega_P(q))=0$ gives at long wavelength,[17,6,20]

$$\omega_P^2(q)\simeq\alpha q+\beta q^2\ , \tag{B1}$$

with $\alpha=2e^2E_F/\hbar^2$ and $\beta=3E_F/2m$. The second term in (B1) is relevant only for $q\geq(4/3\sqrt{2})r_s p_F$, r_s being the average interelectronic distance in Bohr radii. For $r_s\geq1$, and in any case at low frequencies, the expression

$$\omega_P(q)\simeq(\alpha q)^{1/2} \tag{B2}$$

provides a satisfactory approximation.

Plasma waves are well-defined collective modes only for q less than a critical wave vector q_c defined by $\omega_P(q_c)=\omega_+(q)$, $\omega_+(q)$ being the upper edge of the electron-hole continuum (see Fig. 1). From the condition $\epsilon(q_c,\omega_+(q))=0$ we obtain

$$q_c(r_s)=\frac{8p_F r_s}{3\sqrt{2}}\left[\cos\left[\frac{\phi(r_s)}{3}\right]-\frac{1}{2}\right]\ ,\quad r_s\geq\tilde{r}_s \tag{B3}$$

with

$$\phi(r_s)=\arccos[1+(r_s/2\tilde{r}_s)^{-2}-4(r_s/\tilde{r}_s)^{-1}]^{1/2}$$

and

$$q_c(r_s)=2p_F\left[\left[\frac{r_s}{2}\right]^{2/3}\left\{\left[1-\frac{r_s}{2\tilde{r}_s}+\left[1-\frac{r_s}{\tilde{r}_s}\right]^{1/2}\right]^{1/3}+\left[1-\frac{r_s}{2\tilde{r}_s}-\left[1-\frac{r_s}{\tilde{r}_s}\right]^{1/2}\right]^{1/3}\right\}-\frac{\sqrt{2}}{3}r_s\right],\quad r_s\leq\tilde{r}_s\ . \tag{B4}$$

In (B3) and (B4) $\tilde{r}_s=27\sqrt{2}/32\simeq1.19$. For q larger than q_c plasmons suffer Landau damping.

The plasmon frequency $\omega_P(q)$ intersects $\Omega(q)=\hbar pq/m-\hbar q^2/2m$ (see text) only if $\Delta=E-\mu$ is larger than the threshold value $\Delta(r_s)$. With the use of Eq. (B2) we obtain

$$\tilde{\Delta}(r_s)=\left[\left[\frac{r_s}{r_s^*}\right]^{2/3}-1\right]E_F\ , \tag{B5}$$

where $r_s^*=8\sqrt{2}/27\simeq0.42$. In this case the condition $\omega_P(q)=\Omega(q)$ is satisfied by $\tilde{q}_\pm$m

$$\tilde{q}_\pm(r_s)=\tfrac{8}{3}p\cos^2\left[\frac{\pi}{3}\mp\tfrac{1}{3}\arccos\left[\frac{E_F+\tilde{\Delta}(r_s)}{E_F+\Delta}\right]^{3/4}\right]\ , \tag{B6}$$

where $\tilde{q}_-\leq\tilde{q}_+$.

APPENDIX C: CHARGE DEPENDENCE OF $1/\tau_{ee}$

In a 3D electron gas the only contribution to $1/\tau_{ee}$ comes from single electron-hole particle excitations. The calculation can be carried out using Eq. (5) and the standard formulas for $\epsilon(q,\omega)$ and $v(q)$.[19] At $T=0$ K we obtain[26]

$$\left.\frac{1}{\tau_{ee}(\Delta)}\right|_{3\mathrm{D}}\simeq\frac{e^2p_F}{32\hbar}\left[\frac{1}{1+(q_{\mathrm{TF}}^{(3)}/2p_F)^2}+\frac{2p_F}{q_{\mathrm{TF}}^{(3)}}\tan^{-1}\left[\frac{2p_F}{q_{\mathrm{TF}}^{(3)}}\right]\right]\left[\frac{\Delta}{E_F}\right]^2\ , \tag{C1}$$

where $q_{\mathrm{TF}}^{(3)}$ is the usual 3D Thomas-Fermi screening wave vector. The extreme RPA limit (i.e., high densities) for $1/\tau_{ee}$, as calculated by Quinn and Ferrell,[3] is recovered in the limit of $q_{\mathrm{TF}}^{(3)} \ll p_F$,

$$\left.\frac{1}{\tau_{ee}(\Delta)}\right|_{3\mathrm{D}} \simeq \frac{e^2 p_F^2 \pi}{32\hbar q_{\mathrm{TF}}^{(3)}} \left[\frac{\Delta}{E_F}\right]^2, \text{ high-density limit .} \tag{C2}$$

Since $q_{\mathrm{TF}}^{(3)} \propto e$ we observe that $1/\tau_{ee}\,|_{3\mathrm{D}}$ is proportional to e in the high-density limit, whereas in the general case its charge dependence is fairly complicated [see Eq. (C1)].

In obtaining Eq. (C1) we have used for $\epsilon_1(q,\omega)$ the approximate expression $1+(q_{\mathrm{TF}}^{(3)}/q)^2$. Had we disregarded the one with respect to $(q_{\mathrm{TF}}^{(3)}/q)^2$, $1/\tau_{ee}\,|_{3\mathrm{D}}$ would have been charge independent. This is however not justified since in 3D, unlikely in the 2D case, all the wave-vector values between zero and $2p_F$ lead to a contribution of the same order of magnitude to the sum of Eq. (5).

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