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Triplet pairing in metals: $TiBe_2$

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The possibility of p-state superconductivity in $TiBe_2$ is investigated. A rough estimate, via the spin-fluctuation model, yields a transition temperature for the clean material of the order of 0.1 K. Assuming this value, we estimate that the transition could be observed at presently attainable temperatures in a sample with a residual resistivity ratio as low as 1000. We also comment briefly on the validity of the theory.

I. INTRODUCTION

Since the discovery of the p-wave superfluid state in ^3He, there has been considerable interest in the possibility of triplet superconductivity in other materials.[1–11] The fact that p-state pairing has not yet been seen in metals is probably due to the low (and difficult to estimate) transition temperatures involved and, perhaps more importantly, to the extreme sensitivity to impurity scattering.[1]

The first candidate to be seriously considered was Pd. Since Pd is an exchange-enhanced paramagnet it was hoped that the spin-fluctuation (SF), or paramagnon, mechanism which is presumably responsible for the transition in ^3He, would also be effective here. The spin fluctuations in Pd are, however, weaker than in ^3He and an estimate of T_c^p due to this mechanism gave 10^{-5} K.[2] Subsequently, the contribution of the phonons to the p-wave pairing interaction in Pd was shown to be very small.[3] Experimental observations down to 1.7 mK showed no sign of superconductivity.[4]

At this time it was pointed out that T_c^p, as a function of the Stoner enhancement factor S, should go through a maximum for S on the order of 100 and then go to zero as the ferromagnetic transition is approached ($S \to \infty$).[2,5] This led to the suggestion that the weak itinerant ferromagnet $ZrZn_2$ would be interesting in this respect because it goes paramagnetic under pressure, and thus the Stoner factor can be varied by changing the pressure.[6,7] We have recently discussed the possibility of p-wave pairing in $ZrZn_2$ in both the paramagnetic and ferromagnetic phases.[8] Recent experiments[9] have shown no evidence of superconductivity down to temperatures of 25 mK and pressures $0.6 \leq P \leq 20$ kbar. This is probably due to the large amount of scattering and the resulting low-residual-resistivity ratios of the samples investigated. On the other hand, the uncertainty in the theoretical estimates of T_c^p (Ref. 8) is particularly large in $ZrZn_2$ because several input parameters of the theory, for example, the many-body effective mass as a function of pressure, are not known.

It has also been suggested that p-wave pairing may occur in thin films[10,11] and in layered compounds.[11] Although the situation in bulk materials is somewhat discouraging at present, we are not quite ready to give up and we consider here another candidate, $TiBe_2$, in which the input parameters to our theory are now fairly well known and are near the optimum values necessary for a high T_c^p. Although no superconductivity has been observed in $TiBe_2$ down to 20 mK,[12] we believe that it is worthwhile to reconsider this material since lower temperatures and better samples are now available. We avoid the continuing controversy[13] over the nature of the low-temperature phase of this material by simply assuming that it remains a strongly enhanced paramagnet at very low temperatures with a Stoner factor of about 62 (Refs. 14–16) and a many-body effective-mass enhancement $m^*/m \simeq 4.1$.[14,15]

II. ESTIMATE OF T_c^p

We estimate T_c^p for $TiBe_2$ within the model described in detail in Refs. 2 and 8. In this model we have

$$k_B T_c^p = (E_F/S)\exp[-(1+\lambda_0)/\lambda_1^{SF}] \quad , \tag{1}$$

where

$$\lambda_0 = \lambda_0^{SF} + \lambda_0^{ph} \simeq m^*/m - 1 \tag{2}$$

and we assume that the p-wave phonon parameter λ_1^{ph} is approximately zero. The SF mass enhancement and pairing parameters are given by

$$\lambda_l^{SF} = g_l N(0) \int_0^{2k_F} \frac{q\,dq}{2k_F^2} p_l \left(1 - \frac{q^2}{2k_F^2}\right) V(q) \quad , \tag{3}$$

where $g_0 = \frac{3}{2}$, $g_1 = \frac{1}{2}$, and, retaining only enhanced terms, $V(q)$ has the random-phase approximation (RPA) form[2,8]

$$V(q) = \frac{I^2(q)\chi_0(q)}{1 - I(q)\chi_0(q)} \quad . \tag{4}$$

Here $\chi_0(q) \equiv \chi_0(|\vec{q}|, q_0 = 0)$ is the static susceptibility of the noninteracting system and $I(q)$ is the (q-dependent) exchange interaction parameter which is related to the Stoner factor by $S = [1 - N(0)I(0)]^{-1}$. For simplicity we choose

$$I(q) = I(0)(1 + b^2 q^2)^{-1} \quad . \tag{5}$$

The model now contains two parameters: S, which is fitted to the measured susceptibility, and the range parameter b, which is adjusted so that the measured m^*/m is obtained from Eqs. (2)–(5). The one-parameter model with $b = 0$ overestimates λ_0^{SF} and yields too large an effective mass. The momentum dependence of I reduces λ_1^{SF} as well as λ_0^{SF}, but T_c^p is not strongly affected because the changes in λ_0^{SF} and λ_1^{SF} in the exponent of Eq. (1) tend to cancel. Of course one must know λ_0^{ph} to carry out the procedure. For $TiBe_2$ a value $\lambda_0^{ph} = 0.54$ has been given by Jarlborg and Freeman,[16] but a better value is probably 0.8.[14] It is seen from Eq. (1) that a large λ_0^{ph} is detrimental to p-wave pairing. Employing the values $S = 62$, $\lambda_0^{ph} = 0.8$, and $m^*/m = 4.1$, we find $b = 0.71/k_F$ and $\lambda_1^{SF} = 0.59$. With $E_F = 6.8$ mRy $= 1072$ K as appropriate for our spherical Fermi-surface model,[14] we obtain $T_c^p = 0.02$ K.

As will be seen shortly, there is good reason to believe that the two-parameter model *underestimates* T_c^p. First, however, we would like to comment generally on the validity of the theory. We emphasize that we do not claim to *accurately* calculate T_c. This is clearly not possible at present. Our aim is to make a rough, order-of-magnitude, estimate in order to get an idea of whether it is worthwhile to make an experimental effort. Is even this more model goal attainable? We believe that the modified SF modest employed below can provide more than just qualitative results. First, we point out that there is at present no *rigorous* alternative to the SF model. The Landau theory, for example, is not exact at the large momentum transfers included in the integral of Eq. (3), not to mention the difficulties involved in extending the original theory to metals.[17]

From a microscopic point of view, the situation is grim. The RPA form for the effective interaction finds some justification in the work of Hertz[18] who showed that, for a nearly ferromagnetic system at $T = 0$, the mean-field critical exponents are correct. Unfortunately, the single-particle self-energy appears in the linearized gap equation for T_c, and it has not yet been possible to consistently calculate this quantity, including the vertex corrections which are important in the nearly ferromagnetic region.[19,20] It was shown in Ref. 19 that a consistent calculation of the self-energy should lead to a renormalization which would effectively reduce the λ_0 appearing in Eq. (1) and a phenomenological scaling of λ_0^{SF} was suggested. A similar scaling was employed by Levin and Valls.[5]

A strong point in favor of the scaled SF model is that it does work fairly well for the only known p-state condensed system, namely, ^{3}He.[19] Thus if we view the theory as *phenomenological*, it seems quite reasonable to apply it to other nearly ferromagnetic systems. With the parameters appropriate for ^{3}He at the melting pressure, i.e., $S = 21$, $m^*/m = 5.5$, and $\lambda_0^{ph} = 0$, the two-parameter theory yields $b = 0.28/k_F$, $\lambda_1^{SF} = 0.79$, and $T_c^p = 0.22$ mK. Introducing a scaling parameter α by making the replacement $\lambda_0^{SF} \rightarrow \alpha\lambda_0^{SF}$ in Eq. (1), we find that a value $\alpha = 0.57$ is required to bring T_c^p up to the observed value of 2.6 mK. Employing now this same value of α in the $TiBe_2$ calculation, we find $T_c^p(TiBe_2) \simeq 0.1$ K. Since $TiBe_2$ is closer to the ferromagnetic transition than ^{3}He, the arguments of Ref. 19 suggest that the appropriate α may actually be smaller, and hence T_c^p even higher. We thus consider 0.1 K to be a reasonable order-of-magnitude estimate for T_c^p in *pure* $TiBe_2$.

III. EFFECT OF SCATTERING

For small concentrations of scatterers the reduction in T_c^p can be written as[1,21]

$$\Delta T_c^p / T_{c0}^p = -\hbar\pi/(8 T_{c0}^p \tau_{tr}) \quad , \tag{6}$$

where τ_{tr} is the transport lifetime and T_{c0}^p is the previously calculated T_c for the clean system. The physical meaning of Eq. (6) is made clearer by writing the energy uncertainty due to the scattering as $\delta E \simeq \hbar/\tau_{tr} = \hbar v_F / l_{imp}$ and using the BCS relation $\Delta \simeq 1.75 k_B T_c^p$ for the energy gap Δ. Equation (6) becomes

$$\Delta T_c^p / T_{c0}^p \simeq -0.69(\delta E/\Delta) \simeq -2.16(\xi_0 / l_{imp}) \quad , \tag{7}$$

where $\xi_0 = \hbar v_F / \pi k_B T_c^p$ is the coherence length and l_{imp} is the mean free path due to impurities. A quantity of direct interest to experimentalists is the residual resistivity ratio $r \equiv \rho_{300}/\rho_{imp} \simeq l_{imp}/l_{300}$, where ρ_{300} is the resistivity at 300 K. An estimate of $l_{300} = \tau_{300} v_F$ is obtained by setting the energy uncertainty $\hbar/\tau_{300}$ equal to $k_B T$. Thus $l_{imp}^{-1} \simeq 300 k_B/(\hbar v_F r)$ and Eq. (7) yield

$$\Delta T_c^p / T_{c0}^p \simeq -100(r T_{c0}^p)^{-1} \quad . \tag{8}$$

For $T_{c0}^p = 0.1$ K, an r of at least 1000 would be required before there would be any hope of observing a transition even in the microkelvin range. Since the best $TiBe_2$ at present has $r = 140$,[14] more metallurgical effort will be necessary unless nature is kind and our T_c estimate turns out to be a serious underestimate.

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