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Magnetoresistance of weakly disordered electrons

P. A. Lee* and T. V. Ramakrishnan†
Bell Laboratories, Murray Hill, New Jersey 07974
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The magnetoresistance of a weakly disordered electron gas arising from spin splitting of conduction-electron energies is calculated and found to be positive. For large fields $h = g\mu_B H/kT \gg 1$, it goes as $\ln h$ in two and as $\sqrt{h}$ in three dimensions. The sign is opposite that due to incipient localization.

I. INTRODUCTION

Two aspects of the problem of electronic transport in a disorderd medium have received extensive attention recently. The first is the effect of localization,[1–3] and we use the term to describe the properties of a single electron in a random potential. The second is the effect of electron interaction[4–7] which has been worked out in the weak disordered limit, when the electrons are assumed to be diffusive. It turns out that theories based on localization or interaction predict rather similar behavior for the conductivity. On the other hand, the Hall effect based on both theories has been calculated and found to give very different predictions.[5,6,8] It has also been found that the localization theory has the interesting feature that the conductivity is very sensitive to magnetic field, and that the magnetoresistance is always negative.[6] In this paper we investigate the magnetoresistance based on the interaction theory. The orbital effect of the magnetic field has been discussed by Larkin,[9] Altshuler *et al.*,[10] and Fukuyama.[11] In this paper we discuss the magnetoresistance based on the coupling to electron spins.

II. MAGNETORESISTANCE DUE TO SPIN SPLITTING

We recall that in the interaction theory, the dynamically screened Coulomb interaction is treated to first order. The essential feature is the vertex correction shown in Fig. 1 involving ladder diagrams in the particle-hole channel (the diffusion pole). As an example the exchange and Hartree contributions to the self-energy are shown in Figs. 2(a) and (b). Based on similar sets of diagrams, the correction to the conductivity $\delta\sigma(\omega)$, where ω is frequency or temperature, is given by[5,6]

$$\frac{\delta\sigma}{\sigma} = (2-2F)\frac{1}{2\pi s m D}\ln\omega\tau \ , \tag{2.1}$$

where σ is the spin degeneracy, D is the diffusion constant, and

$$F = \int d\hat{\Omega}\, v\left[2k_F \sin\frac{\theta}{2}\right] \Big/ \int d\hat{\Omega}\, v(0) \tag{2.2}$$

is the average over the solid angle $\hat{\Omega}$ of the statically screened Coulomb interaction $v(q)$. The Hartree term shown in Fig. 2(b) is proportional to F because the momentum transfer in the interaction in Fig. 1(b) must be integrated over. Since the electrons that participate in the interaction are near the Fermi surface, the average over momentum transfer can be replaced by the angular average of Eq. (2.2).

We now consider the magnetoresistance due to

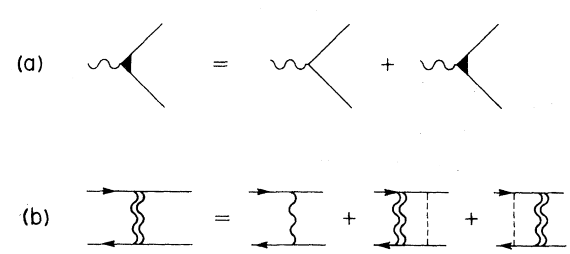

FIG. 1. (a) Vertex correction due to impurity scattering (dashed line); (b) dressing of the interaction line by diagrams in the particle-hole channel.

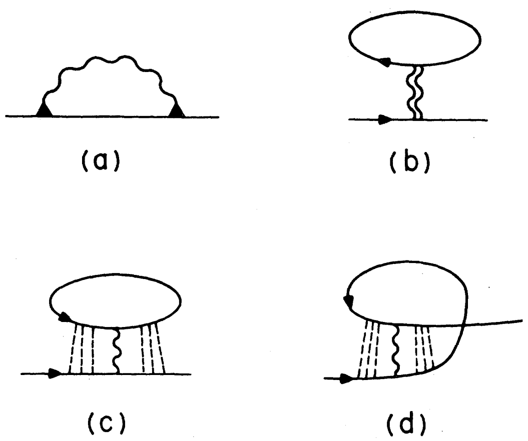

FIG. 2. (a) Exchange and (b) Hartree correction to the self-energy diagram. (c) and (d) are the particle-particle ladder version of (b) and (a), respectively

the splitting of the spin-up and spin-down bands in a magnetic field. This is most simply illustrated for the self-energy correction. The logarithmic correction is due to the correlation between the wave function of the added electron with the wave functions of the occupied electrons that is nearby in energy. The exchange and the equal-spin Hartree terms involve correlation between electrons with the same spin, and is unaffected by the spin splitting. This leaves the Hartree term between opposite spins and the spin splitting produces a gap $g\mu_B H$ between the lowest unoccupied spin-up electron and the highest occupied spin-down electron. The logarithmic divergence of that term is therefore cutoff for $g\mu_B H > \omega$.

It is simple to modify the interaction calculation to include the spin-splitting effect. Let us denote by $\sigma_{\uparrow\downarrow}$ the Hartree correction to the up spin-electron conductivity due to the down-spin electrons. It is given by

$$\sigma_{\uparrow\downarrow}=F\frac{4De^2}{\pi\hbar}\int_0^\infty d\Omega\left[\frac{\partial}{\partial\Omega}[\Omega N(\Omega/kT)]+\tfrac{1}{2}\right]f_{\uparrow\downarrow}(\Omega), \tag{2.3}$$

where

$$f_{\uparrow\downarrow}(\Omega)=\frac{1}{d}\int\frac{d\vec{q}}{(2\pi)^d}\frac{Dq^2}{[-i(\Omega+g\mu_B H)+Dq^2]^3}, \tag{2.4}$$

where d is the dimensonality,

$$N(\omega)=[\exp(\omega)-1]^{-1},$$

and F is the angular average of the statically screened Coulomb interaction as defined in Eq. (2.2). We note that the Hartree interaction always involves the static interaction, which explains the difference in the integrand in Eq. (2.4) from that in the exchange contribution, as given, for instance, in Ref. 4, Eq. (33). (We note that there is apparently an error of a factor of 2 between Eqs. (33) and (32) in Ref. 4 and the factor $\frac{1}{2}$ in [ ] in Eq. (2.3) is also missing.) Similarly we have the down-spin contribution to the conductivity $\sigma_{\downarrow\uparrow}$ and $f_{\downarrow\uparrow}$ which is obtained from Eq. (2.4) by $g\mu_B H \to -g\mu_B H$. The $\vec{q}$ integration can be done and it is convenient to perform an integration by parts in Ω. The total correction $\delta\sigma_I$ to the conductivity from the interaction theory can be summarized as

$$\delta\sigma_I(H,T)=\delta\sigma_I'(T)+\delta\sigma_I''(H,T), \tag{2.5}$$

where $\delta\sigma_I'$ is the field-independent exchange and equal-spin Hartree contribution. We have

$$\delta\sigma_I'=\frac{e^2}{\hbar}\frac{1}{4\pi^2}(2-F)\ln(T\tau) \tag{2.6a}$$

for two dimensions (2D), and

$$\delta\sigma_I'=\frac{e^2}{\hbar}\frac{1}{4\pi^2}\frac{1.3}{\sqrt{2}}(\tfrac{4}{3}-F)\left[\frac{T}{D}\right]^{1/2}. \tag{2.6b}$$

for three dimensions (3D). The field-dependent part is given by $\delta\sigma_I''=\sigma_{\uparrow\downarrow}+\sigma_{\downarrow\uparrow}$,

$$\delta\sigma_I''(H,T)-\delta\sigma_I''(0,T)=-\frac{e^2}{\hbar}\frac{F}{4\pi^2}g_2(h) \tag{2.7a}$$

for 2D, and

$$\delta\sigma_I''(H,T)-\delta\sigma_I''(0,t)=-\frac{e^2}{\hbar}\frac{F}{4\pi^2}\left[\frac{T}{2D}\right]^{1/2}g_3(h), \tag{2.7b}$$

for 3D, where

$$g_2(h)=\int_0^\infty d\Omega\frac{d^2}{d\Omega^2}[\Omega N(\Omega)]\ln\left|1-\frac{h^2}{\Omega^2}\right| \tag{2.8a}$$

in 2D and

$$g_3(h)=\int_0^\infty d\Omega\frac{d^2}{d\Omega^2}[\Omega N(\Omega)]\times(\sqrt{\Omega+h}+\sqrt{|\Omega-h|}-2\sqrt{\Omega}) \tag{2.8b}$$

in 3D and $h=g\mu_B H/kT$. The zero-field contribution $\delta\sigma_I''(0,T)$ is the usual one, and is equal to the term proportional to F in Eq. (2.6). The functions g_2 and g_3 can be computed numerically. They have the limiting behavior

$$g_2=\begin{cases}\ln(h/1.3), & h\gg 1\\ 0.084h^2, & h\ll 1\end{cases} \tag{2.9}$$

and

$$g_3=\begin{cases}\sqrt{h}-1.3, & h\gg 1\\ 0.053h^2, & h\ll 1\ .\end{cases} \tag{2.10}$$

In Eq. (2.10), the constant 1.3 in the $h\gg 1$ limit is simply the $H=0$ term subracted in Eq. (2.7). It is worth noting that in the large-field limit, $h\gg 1$, $\delta\sigma_I''(HT)$ becomes temperature independent. The only temperature dependence due to interactions is given by Eq. (2.6b) and the coefficient of the $\sqrt{T}$ term is always positive. This is in contrast to the $H=0$ case, where the $\sqrt{T}$ coefficient is proportional to $\frac{4}{3}-2F$ and can change sign depending on the value of F which varies between zero and unity.[12]

III. DISCUSSION

In Sec. II we have discussed only the particle-hole diffusion pole modification of the conductivity. In Ref. 6 it was noted that Figs. 2(c) and 2(d) are the particle-particle version of Figs. 2(b) and 2(a) and yield equal contribution. These diagrams should be sensitive to the orbital effects of the magnetic field. The resulting magnetoresistance have been evaluated by Fukuyama[11] and by Altshuler *et al.*[10] Fukuyama considered first-order perturbation theory in some coupling constant (his g_2 and g_4) whereas Altshuler *et al.*[10] pointed out that it is necessary to sum a ladder involving repeated interactions between the electrons. A typical Hartree diagram is shown in Fig. 3. This replaces the coupling constant λ by the effective coupling

$$\tilde{\lambda}=\frac{\lambda}{1+\lambda\ln(E_F/T_0)}\ , \tag{3.1}$$

where $T_0=\max(T,DeH)$. Equation (3.1) is a very similar to the theory of superconductivity, except that for repulsive interaction the coupling constant scales to weak coupling. Indeed if a phonon induced attractive coupling λ_p is also present, the λ in Eq. (3.1) should be replaced by

$$\lambda\rightarrow\lambda_p+\frac{\mu}{1+\mu\ln(E_F/\omega_D)}\ , \tag{3.2}$$

where μ is the electron-electron interaction and E_F in Eq. (3.1) should be replaced by ω_D. Equation (3.2) is very familiar in the theory of superconductivity and it is more proper to think of the effect as due to superconducting fluctuations. The surprising element is that even for replusive interaction, relatively strong temperature-dependent effects are predicted for the density of states and conductivity, even though the overall size of the effect is small compared with that given by Figs. 2(a) and 2(b) because of the renormalization of λ given by Eq. (3.1). Since the renormalization depends only logarithmically on T_0, the ladder sum can be approximated by a phenomenological coupling constant. From this point of view Fukuyama's theory is in basic agreement with Altshuler *et al.*[10] if his g_2 and g_4 are understood to be phenomenological constants smaller than g_3 except when superconducting fluctuations are important.

We should also emphasize that the particle-particle channel is important only for short range interaction. There are two reasons for this. First, the evaluation of Figs. 2(c) and 2(d) requires averaging over the Fermi surface, so that the bare coupling λ is proportional to F. Second, it can be shown that for small momentum transfer q such that $ql\lesssim 1$ there are additional diagrams which cancel the leading singularity in Figs. 2(c) and 2(d). Thus the interaction potential must have significant component for $ql\gtrsim 1$. However, in the $k_Fl\gg 1$ limit, this requirement is less stringent than the requirement that F is nonnegligible.

According to Refs. 10 and 11, the particle-particle channel leads to positive magnetoresistance when the Landau-orbit size becomes comparable to the thermal length $(D/T)^{1/2}$, i.e.,

$$\frac{2eH}{\hbar c}>\frac{kT}{D}\ . \tag{3.3}$$

For $k_Fl\gg 1$ this occurs at a smaller field than the requirement for spin splitting discussed in Sec. II,

$$g\mu_BH>kT \tag{3.4}$$

for normal values of g. Larkin[9] has pointed out that the Maki-Thompson diagram for superconductivity can be applied to the case of repulsive interac-

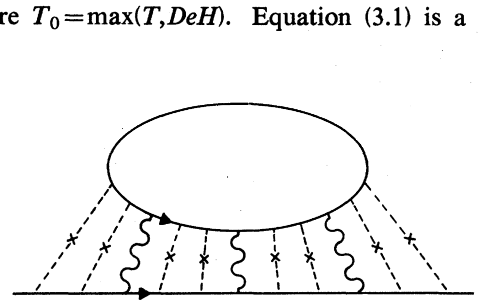

FIG. 3. Typical Hartree diagram included in Ref. 10.

tion as well. This produces a positive magnetoresistance when the field satisfies

$$\frac{2eH}{\hbar c} > \frac{1}{D\tau_{\rm in}} \ . \tag{3.5}$$

Usually the inelastic scattering rate $\tau_{\rm in}^{-1}$ is smaller than kT so that this occurs at an even weaker field than Eq. (3.3). In fact this effect takes the same form as the magnetoresistance of noninteracting electrons due to the suppression of localization, except the overall magnitude is very small for normal metals, being proportional to $\pi^2\tilde{\lambda}/6$ for $|\tilde{\lambda}| \ll 1$.

The combination of the spin-splitting effect on the interacting model and the localizing effect already leads to very rich behavior and magnetoresistance is clearly a powerful tool for disentangling the two contributions. This is especially true in 2D, since the orbital contribution is sensitive only to the magnetic field component normal to the plane whereas the spin-splitting term should be isotropic.[13] Many positive magnetoresistance data can be analyzed using the spin splitting and the localization terms only[13,12] which presumably mean that $\tilde{\lambda}$ is small for these systems. There are apparently also other systems where $\tilde{\lambda}$ is not negligible and it will be very interesting to experimentally separate the three magnetic field regimes discussed in Eqs. (3.3)–(3.5).

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*Present address: Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139.

†Present address: Indian Institute of Science, Bangalore 560012, India.

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