Background contributions to sound waves at the liquid-helium λ transition

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The noncritical contributions to the sound near $T_\lambda(P)$ in ^{4}He liquid are analyzed quantitatively in terms of the dynamic scaling theory of Ferrell and Bhattacherjee. By using as background the thermodynamic velocity in a zero-height sample, an excellent agreement is found between the theoretical predictions and the experimental data for the dispersion of the velocity. In contrast, the observed background attenuation does not seem to be explained by a simply additive hydrodynamic contribution.

One of the basic assumptions of modern critical phenomena, first formulated explicity by Fixman,[1] is the hypothesis that the critical behavior of a medium arises from divergences in the long-wavelength thermal fluctuations in the microscopic variables. It would follow, then, that near a second-order phase transition the transport coefficients and the quantities derived from them (such as the attenuation and dispersion of sound) would have both a critical contribution associated with the long-wavelength fluctuations, and a background, or noncritical, contribution arising from the shorter-wavelength fluctuations. It is now known, however, that such a division is somewhat arbitrary. In fact, the renormalization-group treatments have shown that the important fluctuations near a second-order phase transition (those that are the cause of critical singularities) do not have a characteristic length.[2] Instead, the critical behavior of the system will be a consequence of the presence of fluctuations having all wavelengths ranging from interatomic distances up to the correlation length. However, in the renormalization-group formalism, calculations are performed in the asymptotic critical region, whereas measurements are by necessity not carried out in this region.[3] So, even in these models, the measured transport coefficients still seem to result from two contributions and a meaningful comparison with the theories would require an independent estimation of the background. In fact, the importance of the background terms was recognized years ago,[4] but until recently the different efforts have been mainly focused on the critical part of each transport coefficient. With the increase of theoretical and experimental precision, the background contribution has become one of the central problems in critical dynamics.[5–7]

In this Communication, I present precise experimental results of both the *total* (measured) attenuation and the velocity of the high-frequency (~ 1 GHz) first sound extrapolated at T_λ in ^{4}He liquid under pressure. These results (which complete our previously published data[8]), as well as other available high-frequency data,[9–13] are analyzed in terms of the recent and very successful dynamic scaling theory of Ferrell and Bhattacharjee (FB).[14–16] This allows one to obtain, to my knowledge for the first time, quantitative information about the important background contributions to the first sound. The FB theory has already explained the scaling behavior of the normalized first sound attenuation.[8,9,14] However, this theory also allows a calculation with no adjustable parameters of the absolute values of both the critical attenuation and the critical velocity at T_λ. In previous comparisons,[8,9,14] the dispersion of the velocity has almost been neglected, although, as we will see here, this parameter plays a crucial role as a test of both the critical and the background terms. The first sound is only weakly coupled to the order-parameter fluctuations and therefore its critical attenuation and velocity does not present singularities at T_λ. This is an essential feature for the present analysis because it permits a reliable extrapolation at T_λ of the experimental data.

The current treatments of the background contributions to the sound may be summarized as follows.[4,17] It is first assumed that a linear superposition of the critical (subscript c) and background (subscript B) parts holds so that the total measured attenuation α_M and velocity u_M are given by $\alpha_M = \alpha_c + \alpha_B$ and $u_M = u_c + u_B$. Then, in second-order phase tranitions in fluids, it is assumed that the background part that arises from conventional noncritical losses can be obtained throughout the transition, using the functional form suggested by classical hydrodynamics (Navier-Stokes equations). The background velocity will then be the classical hydrodynamic velocity, which includes the noncritical dispersion. The background damping constant $D_B^H = 2u_B^3\alpha_B/\omega^2$ will be given by

$$D_B^H = \frac{1}{\rho}\left[\tfrac{4}{3}\eta + \zeta_0 + (\gamma - 1)\frac{K}{C_P}\right] , \qquad (1)$$

where ρ is the density, η and ζ_0 are the steady-state (nonrelaxing) shear and bulk viscosites, K is the thermal conductivity, C_P is the specific heat at constant pressure, and $\gamma = C_P/C_V$. For $\alpha_{c\lambda}$ and $u_{c\lambda}$ I will use the predictions of the FB theory. Let me first emphasize that this theory is based on the Fixman approach: The critical attenuation and velocity are supposed to be due to the critical slowing down of the frequency-dependent specific heat, and the classical losses are not taken into account. For liquid helium, the FB theory predicts at T_λ,[14]

$$\tilde{D}_{c\lambda} = C_1 L_{1\lambda}/[L_{1\lambda}^2 + (\pi/2)^2] \quad , \qquad (2)$$

$$\alpha_{c\lambda} = \pi\omega\tilde{D}_{c\lambda}/2L_{1\lambda}u_{B\lambda} \quad , \qquad (3)$$

where I have introduced the normalized critical dispersion $\tilde{D}_{c\lambda}$ at T_λ defined by

$$\tilde{D}_{c\lambda} \equiv u_{c\lambda}/u_{B\lambda} = (u_{M\lambda} - u_{B\lambda})/u_{B\lambda} \quad .$$

In Eqs. (2) and (3), $L_{1\lambda} = \ln(\Gamma_c/\omega) + B$, $\Gamma_c = e^{1-B}\Gamma_0$, and $\Gamma_0 = 2B_\psi \kappa_0^2 t_0^{4/3}$. The notation here is that of Ref. 14. In particular, C_1 is a dimensionless coupling constant relating the zero-frequency velocity near T_λ to the specific heat, $L_{1\lambda}$ is the real part of the normalized freqeuncy-dependent specific heat at T_λ, B is the normalized background specific heat, and B_ψ is the order-parameter background.

Equations (1) to (3) provide a complete description of the total (measured) attenuation and velocity of the first sound at T_λ. All the information required to obtain α_M and u_M at T_λ from these equations comes from other sources. So, this model can be compared with the experimental data with no adjustable parameters, although a variety of physical properties has to be known. Fortunately, and in contrast with what occurs in other systems, all this information is actually available near T_λ in liquid helium. In what concerns the parameters arising in Eqs. (2) and (3), I obtain the values of C_1 and t_0 presented in Table I by using the logarithmic representation of the specific heat C_P under pressure.[18] I also use the values[5,14] $\kappa_0 = 7\times10^7$ cm^{-1}, $B = 1.45$, and $B_\psi = 1.5\times10^{-4}$ cm^2/sec, independently of the pressure. As a first check of these parameters, I have compared Eqs. (2) and (3) with the very precise data of Carey, Buchal, and Pobell at $\omega/2\pi = 1$ MHz and under different pressures.[19] At this intermediate frequency the background attenuation is very small ($\alpha_{B\lambda} \lesssim 10^{-3}$ cm^{-1}), and a sizable critical dispersion (see below) is present. I find a very good agreement between these data for both $\alpha_{c\lambda}$ and $\tilde{D}_{c\lambda}$ and the theory within the estimated combined error or 15%.

TABLE I. Parameters at T_λ in ^{4}He liquid under different pressures arising in the theory of Ferrell and Bhattacharjee for the critical contributions.

P (bar)	SVP	17	20	23.1	25	28.5
$t_0\times10^2$	23	11	10	9.0	7.4	6.0
$C_1\times10^2$	4.9	4.7	4.1	4.4	4.5	5.0

The available high-frequency results ($\omega/2\pi \gtrsim 100$ MHz) extrapolated at T_λ at saturated vapor pressure (SVP) or under pressure, including the present data, are summarized in Table II. Two different methods have been used to obtain these data: a Brillouin light scattering technique used in Refs. 11 and 12, and also used to obtain the present data, and ultrasonic techniques used in Refs. 9, 10, and 13. Due to differences in boundary conditions, the values of α and u determined by the two kinds of experiments may differ.[4,20] However, a detailed check of the form of the dynamic structure factor associated with

TABLE II. Background and critical contributions to the sound at T_λ in ^{4}He liquid in the high-frequency range.

P (bar)	$\omega\times10^9$ (sec^{-1})	$u_{B\lambda}(\omega)$ Adjusted (cm/sec)	$\tilde{D}_{c\lambda}\times10^2$ Expt.	$\tilde{D}_{c\lambda}\times10^2$ Theor.	$\alpha_{M\lambda}$ Expt. (cm^{-1})	$\alpha_{c\lambda}$ Theor. (cm^{-1})	$D_{B\lambda}^A\times10^4$ Adjusted (cm^2/sec)
SVP[a]	1.70				~246	163	5.9
SVP[b]	3.17	21690	1.00	0.90	750	374	8.2
SVP[c]	6.28				2430	955	7.6
SVP[d]	6.91		~1.0	1.00	2260	1087	5.0
1.9[e]	4.21	23320	1.00	1.00	852	590	3.6
17[f]	6.05	31850	1.13	1.06	1100	900	3.7
20.4[d]	6.91				1450	995	6.7
23.1[e]	6.06	33470	1.13	1.10	1316	936	7.7
25.4[b]	5.00	34000	1.12	1.12	1100	781	10.0
28.5[e]	6.31	34750	1.44	1.37	1680	1400	5.9

[a]Reference 13.
[b]Reference 11.
[c]Reference 10.
[d]Reference 9.
[e]Present work.
[f]Reference 12.

the first sound in our experiments allows us to conclude that, within the accuracy of the measurements, the light scattering data in Table II have the conventional meaning. Also, note that in this high-frequency range it is not possible to accurately measure the sound velocity by means of ultrasonic techniques. In fact, the present data in Table II provide, for the first time, simultaneous precise information on $\alpha_{M\lambda}$ and $u_{M\lambda}$ in the GHz range, the relative uncertainty being less than 5% for α_M and 0.05% for u_M.

In order to analyze these data, I will first consider the critical velocity dispersion $\tilde{D}_{c\lambda}$. It is essential to notice that near T_λ the t dependence of $u_M(\omega)$ becomes very weak for $\omega \gtrsim 1$ MHz, as a consequence of the critical contributions.[8-13,19] Here $t \equiv |T-T_\lambda|/T_\lambda$ is the reduced temperature. Therefore at these frequencies $u_{M\lambda}(\omega)$ is not affected by the gravitational inhomogeneities arising in a sample of finite height,[21,22] and its corresponding background velocity $u_{B\lambda}(\omega)$ must be calculated in a *zero-high* sample. To estimate $u_{B\lambda}(\omega)$, I have used the t dependence of the thermodynamic sound velocity in a zero-high sample $u^*(0)$ proposed by Ahlers,[18,22] who assumes the same logarithmic t dependence of C_P that I have used to obtain C_1 and t_0 in Table I. Although far from T_λ [where both the critical and gravitational effects on $u_M(\omega)$ are negligible] such a calculation gives absolute values of $u^*(0)$ lower than the measurements, its temperature dependence fits the high-frequency data very well for $t \gtrsim 10^{-2}$ on both sides of T_λ. This result is consistent with the dynamic scaling ideas[14-16] according to which $u_M(\omega)$ must join the noncritical velocity in the vicinity of $\omega/\Gamma = 1$, where $\Gamma \equiv 2B_\psi \kappa_0^2 t^{4/3}$ is the characteristic relaxation rate in liquid helium. Therefore the absolute values of $u^*(0)$ are matched to agree with the high-frequency data at $t = 10^{-2}$. This background velocity will include the noncritical dispersion at $t = 10^{-2}$, which is expected to be very close to the one existing at T_λ.

The critical dispersion at T_λ, $\tilde{D}_{c\lambda}$, obtained by subtracting the adjusted values of $u_\lambda^*(0)$ from the measured velocity extrapolated at T_λ, is presented in Table II, together with the predictions of the FB theory. Again a striking good agreement between theory and experiments is found. This strongly supports the correctness of both the FB theory and the values used here for the parameters arising in this theory. This last point is important, not only because we are now able to consistently analyze the corresponding attenuation, but also because some of these parameters (whose extraction at this time involves important discrepancies[6]) arise also on other dynamic properties near T_λ in liquid helium.[5,6] These results seem also to confirm the mechanism proposed above for the reduction of the gravitational effects on $u_M(\omega)$ near T_λ. This conclusion can be extended to the sound near the critical point in pure fluids and also to binary liquids near the consolute temperatures.[15,17] Although the gravitational effects in these transitions have been studied theoretically in detail,[21] their important implications on the dispersion of the velocity have not been, to my knowledge, observed at yet.

The total (measured) attenuation $\alpha_{M\lambda}$, the critical attenuation $\alpha_{c\lambda}$ [obtained from Eq. (3)], and the *adjusted* background damping constant, defined as $D_{B\lambda}^A \equiv 2u_{B\lambda}^3(\alpha_{M\lambda} - \alpha_{c\lambda})/\omega^2$, are also presented in Table II. These values of D_B^A are to be compared with those of the hydrodynamic damping constant D_B^H given by Eq. (1). For the sake of brevity, I report here only the main features of this comparison. Note first that the scatter between the different values of $D_{B\lambda}^A$ at similar pressures is much bigger than the total combined errors in $\alpha_{M\lambda}$ and $\alpha_{c\lambda}$, which at these frequencies is estimated to be less than 20%. Also, the values of $D_{B\lambda}^A$ disagree with those of $D_{B\lambda}^H$ obtained by using the available data[3] for the parameters appearing in Eq. (1) and assuming, as it is usual for dense fluids,[17,20] $\zeta_0 = \eta$. For instance, using at SVP,[3,23] $\rho = 0.146$ g/cm^3, $\eta_\lambda = 2.47 \times 10^{-5}$ P, γ (at $t = 10^{-5}$) $= 1.037$, and[5] $(K_B/\rho C_P)_\lambda = 1.2 \times 10^{-4}$ cm^2/sec, we obtain $D_{B\lambda}^H = 4.0 \times 10^{-4}$ cm^2/sec. In contrast, D_B^H calculated far from T_λ ($t \gtrsim 5 \times 10^{-2}$) is in reasonable agreement at all pressures with the one found from the measured attenuation in this noncritical region.[24] These results for the attenuation clearly confirm the suspicions existing at this time on the phenomenological treatments of the background.[6,7] It is now urgent to reconsider some of the accepted notions about these important contributions to the ultrasonic attenuation near second-order phase transitions in fluids.

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