Symmetry breaking in Potts ϕ^3 field theory. Crossover in random ferromagnets

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We investigate the effects of quadratic and trilinear symmetry breaking in a ϕ^3 field theory for the $(n+1)$-state Potts model by means of renormalized perturbation theory to one-loop order in dimension $d=6-\epsilon$. For $n+1=2^m$ the break in quadratic symmetry splits the field into m critical and $n-m$ noncritical components, and we allow for three trilinear couplings v,w,z between different field components. In the limit $n=m=0$ the Hamiltonian represents a bond-diluted Ising ferromagnet near the percolation threshold with anisotropy parameter $\widetilde{m}^2 \approx e^{-2J/T}$ and critical mass $t=p_c(T)-p$, where T is the absolute temperature, p the concentration of occupied bonds, and $p_c(T)$ a point on the critical line. We find that for any nonzero quadratic anisotropy there are only nonsymmetric trilinear fixed-point (FP) couplings $v^*(\widetilde{\mu})$, $w^*(\widetilde{\mu})$, $z^*(\widetilde{\mu})$; $\widetilde{\mu}=\widetilde{m}/\kappa$ and κ is a scale parameter. The multicritical percolation point $v^*_{\rm II}(0)=w^*_{\rm II}(0)=z^*_{\rm II}(0)=(2\epsilon/7)^{1/2}$ is the only symmetric FP in the parameter space (v,w,z), and it is not completely stable under trilinear symmetry breaking even in the absence of quadratic anisotropy, since the stability matrix has a marginal eigenvector. Starting from the percolation point we find that, despite a break in trilinear symmetry induced by quadratic anisotropy, there is a crossover to the critical line with asymptotic mean-field exponents. The flow of the couplings and the effective exponents for the crossover are calculated. A further result is that trilinear symmetry breaking yields a new completely stable, asymmetric FP $v^*_{\rm I}=z^*_{\rm I}=0$, $w^{*2}_{\rm I}(\widetilde{\mu})/(1+\widetilde{\mu}^2)=2\epsilon/7$, with nonclassical critical exponents $\eta_{\rm I}=+\epsilon/21$, $\nu_{\rm I}^{-1}-2=5\epsilon/21$ for all finite $\widetilde{\mu}$.

I. INTRODUCTION

In a quenched bond-diluted Ising model with a fraction $p<1$ of occupied bonds the critical temperature $T_c(p)$ decreases as p is decreased, becoming zero at the percolation concentration p_c. The critical behavior as function of bond concentration, for any fixed and finite temperature T, is assumed to be classical with mean-field exponents above four dimensions. It has been argued that the percolation threshold $p=p_c$ at $T=0$ should be regarded as a higher-order critical point.[1-3]

The thermally driven transition from the percolation threshold to the critical line in random site-diluted magnets with nonmagnetic impurities has become of considerable experimental interest in recent years[2] and it has been suggested that this transition is a realization of crossover in an anisotropic one-state Potts model on a lattice.[4] This is the limit $n=0$ of the $(n+1)$-state Potts model[5] in which the pair interaction $K_i e_i(\vec{r})e_i(\vec{r}\,')$ between the $n+1$ vectors $e_i^\alpha(\vec{r})$ $(\alpha=1,\ldots,n+1;\ i=1,\ldots,n)$ on the lattice site $\vec{r}$ that point to the vertices of a hypertetrahedron in n-dimensional space, is not the same for all components i.

In a previous publication,[6] in the following referred to as I, we report on a renormalization-group (RG) study of crossover induced by quadratic symmetry breaking in a Potts ϕ^3 field theory for the $(n+1)$-state Potts model with emphasis on the $n=0$ limit. This a ϕ^3 field theory in which the trilinear coupling is given exclusively in terms of the "Potts vectors" e_i^α. It is shown there that a trilinear coupling $ud_{ijk}\,\phi_i\phi_j\phi_k$ (summation over repeated indices) that is symmetric in the field components, with a tensorial coefficient $d_{ijk}=e_i^\alpha e_j^\alpha e_k^\alpha$, is a rather nonobvious assumption and we pointed out that the quadratic symmetry breaking that is necessary for the crossover from the percolation point to the critical line may induce a relevant break in trilinear symmetry even very close to the percolation threshold, which could yield to new critical behavior described by a nonsymmetric fixed point of the RG equations.

Multicritical points can be investigated by means of symmetry-breaking perturbations and crossover phenomena from high- to low-symmetry critical behavior in anisotropic spin systems have already been extensively investigated by means of RG procedures in an n-component ϕ^4 field theory.[7–9] It has been shown that anisotropy in the quartic term in the Hamiltonian is irrelevant in the RG sense[10] whenever $n < 4 - O(\epsilon)$, where $\epsilon = 4 - d$, and that the crossover in the disordered phase from $O(n)$ to $O(m)$—symmetric critical behavior ($m < n$) driven by quadratic symmetry breaking can be described by a single, isotropic, quartic coupling, in the case of small n.

The effects of symmetry-breaking perturbations to study multicritical points in ϕ^3 field theory became of more recent concern. The three-state Potts model with linear and quadratic symmetry breaking has been investigated recently assuming trilinear symmetry.[11] An extended Potts ϕ^3 field theory has been suggested to describe the crossover in branched polymers and it was shown that quadratic symmetry breaking induces a crossover from the percolation point to lattice-animal behavior, whereas a break in trilinear symmetry, of a different kind than the one discussed in I and in the present paper, yields the behavior of θ solvent branched polymers.[12] The results of the works done so far in ϕ^3 field theory indicate that the percolation point is a multicritical point in the space of couplings that go into the Hamiltonian.

The aim of the work that we report in this paper is to gain further insight into the multicritical percolation point by studying the thermally driven crossover in random ferromagnets. This enables us to determine the stability of the percolation point to a trilinear symmetry breaking which has, apparently, not been considered by other authors before. Moreover, this is the first time, to our knowledge, that the combined effects of quadratic and trilinear symmetry breaking are discussed.

Our first purpose is to provide the details of the calculations that illustrate the stringent constraint that is implied by the choice of a symmetric trilinear coupling done in I, in order to compare with previous work by Stephen and Grest.[4] Here, however, we go considerably further and exhibit the full effects of trilinear symmetry breaking induced by the break in quadratic symmetry that describes the thermally driven crossover away from the percolation point. We find that there is no crossover to a symmetric trilinear fixed-point coupling and that the only symmetric fixed point is the one associated with percolation. In view that the $(n+1)$-state Potts model is isomorphic to an n-vector model, this is in sharp contrast to the crossover driven by quadratic symmetry breaking with an isotropic quartic coupling in an n-component ϕ^4 field theory for small n. Nevertheless, we show that the nonsymmetric trilinear fixed point that is reached in the crossover from the percolation point yields the classical asymptotic critical behavior with mean-field exponents that one expects to have in dimension $d > 4$.

Our second purpose is to show that there is a break in trilinear symmetry inherent to a Potts ϕ^3 field theory of the one-state Potts model. Indeed, we find that even in the absence of quadratic symmetry breaking the percolation point is not completely stable to trilinear symmetry-breaking perturbations, and that in a Hamiltonian parameter space with three trilinear couplings that are relevant to random ferromagnets there is an eigenvector that corresponds to a marginal eigenvalue. If quadratic symmetry breaking is then turned on we find that, in the limit of extreme anisotropy, the nonsymmetric fixed point that describes the random ferromagnet becomes a completely stable fixed point. We also find a further, completely stable asymmetric fixed point with new critical exponents that is not reached from the percolation point and we call this "asymmetric percolation."

The main difference between the break in trilinear symmetry in the Potts ϕ^3 field theory considered in the present work and that of the works by Harris, Lubensky, and Isaacson,[12] is that the trilinear couplings of these authors are not given exclusively by the $n+1$ n-dimensional Potts vectors e_i^α but rather by set of n n-dimensional vectors and their trilinear symmetry breaking involves always the additional component. The implications of this difference are pointed out in the present paper.

Our starting Hamiltonian with pair interactions between the vectors $e_i^\alpha(\vec{r})$ singles out m one-spin components in a representation of Wallace and Young appropriate for an anisotropic $n+1=2^m$-state Potts model that can be expected to represent a bond-diluted Ising ferromagnet in the replica limit $m=0$. The resulting continuous-field Landau-Ginzburg-Wilson Hamiltonian describes what we call a Potts ϕ^3 field theory, with quadratic symmetry breaking in which the trilinear coupling has the tensorial structure of the d_{ijk} in terms of the Potts-state vectors $e_i^\alpha(\vec{r})$. Without the quadratic symmetry-breaking term this is the Hamiltonian of Amit's work[13] and the essentially equivalent one of Priest and Lubensky[14] for an isotropic trilinear interaction.

The Landau-Ginzburg-Wilson Hamiltonian is given in Sec. II. Quadratic symmetry breaking splits the fields into "longitudinal" (or critical) and "transverse" (or noncritical) components, with critical and noncritical mass, respectively. In accordance, three trilinear couplings are introduced to allow for possible trilinear symmetry breaking between different field components. A fourth trilinear coupling, between critical-field components, that was also renormalized in I for symmetry reasons, need not be included here because it is associated to a tensorial coefficient that vanishes identically in the irreducible three-point vertex functions.

Our RG procedure developed in Sec. III is renormalized-perturbation theory (RPT) with the dimensional regularization (DR) of 't Hooft and Veltman and generalized minimal subtraction (GMS) of dimensional poles, as introduced by Amit and Goldschmidt for ϕ^4 field theory with quadratic symmetry breaking. Our calculations are restricted to one-loop order in dimension $d=6-\epsilon$, and they apply to the disordered phase. The bare two- and three-point vertex functions have dimensional poles in ϵ that are subtracted out by means of wave-function and coupling-constant renormalization, as in the usual procedure of minimal subtraction. There are additional logarithmic terms in the scaled noncritical mass (or anisotropy parameter) $\tilde{\mu}=\tilde{m}/\kappa$, where κ is a momentum-scale parameter, that become singular in the limit of large $\tilde{\mu}$ and that are also subtracted in GMS. The calculation of the tensorial coefficients is deferred to Appendix A and the RG procedure, particularly the subtraction of the large $\tilde{\mu}$ terms, is checked against a cutoff-dependent version of RPT in Appendix B. The RG equations for the irreducible vertex functions can be found there. The Wilson β functions that determine the fixed points are discussed in Sec. IV, where it is shown that the symmetric solution for the renormalized trilinear couplings, $v^*(\tilde{\mu})=w^*(\tilde{\mu})=z^*(\tilde{\mu})$, does not solve the fixed-point equations for nonzero $\tilde{\mu}$, except with the trivial $v^*=w^*=z^*=0$. The new asymptotic nonsymmetric fixed point and the asymmetric-percolation fixed point are also discussed in Sec. IV. The critical crossover exponents are obtained there by means of the other Wilson functions $\gamma_\phi^{(i)}$ and $\gamma_{\phi^2}^{(i)}$, for $i=1$ and 2. It is also shown that asymmetric percolation has critical exponents that are both nonclassical and different from the usual percolation exponents. Our results and further implications are summarized in Sec. V.

II. GENERAL FORMULATION AND EFFECTIVE HAMILTONIAN

It has been showed by Stephen and Grest[4] that within the replica formalism the effective Hamiltonian for a random Ising ferromagnet with a fraction $p<1$ of occupied bonds corresponds to an anisotropic $(n+1)$-state Potts model, $n+1=2^m$, in the replica limit $m=0$. The anisotropy parameter that breaks the symmetry among the Potts operators is proportional to e^{-2K}, for an Ising pair interaction $K=J/T$ and T being the absolute temperature.

We indicate by σ_μ, $\mu=1,\ldots,m$, the replicated spin operators and we introduce $n=2^m-1$ operators e_i, $i=1,\ldots,n$, formed by all distinct products of the σ_μ's:

$$e_\mu=\sigma_\mu,\quad \mu=1,\ldots,m$$
$$e_q=\ldots(\sigma_\mu\sigma_\nu),\ldots,(\sigma_\mu\sigma_\nu\sigma_\eta),\ldots,(\sigma_1\sigma_2,\ldots,\sigma_m)\,,\quad q=m+1,\ldots,n\,. \tag{2.1}$$

In terms of the operators $e_i(\vec{r})$ defined on the lattice site $\vec{r}$ the effective Hamiltonian of Ref. 4 is

$$H=-\sum_{NN}\left\{K_0\sum_i^n e_i(\vec{r})e_i(\vec{r}\,')-K_1\sum_{s\geq 2}2(s-1)\sum_{\{s\}}e_q(\vec{r})e_q(\vec{r}\,')\right\}, \tag{2.2}$$

where the last sum runs over those e_q that are a product of s σ_μ's. Here,

$$2^m K_0=-\ln(1-p)+(m-2)K_1$$

and $K_1=[p/(1-p)]e^{-2K}$ is the symmetry-breaking field that favors the ordering of the spin variables e_μ in Eq. (2.1). In the following greek subindices run over longitudinal (critical) components and latin over transverse (noncritical) ones as indicated in Eq. (2.1) *except* i,j,k, that run over all. Also summation over repeated indices is used all through this paper.

The e_i's defined in Eq. (2.1) form a representation of the $n+1$ n-component vectors e_i^α, $\alpha=1,\ldots,n+1$, that point to the vertices of a hypertetrahedron in n-dimensional space and that describe the $(n+1)$-state Potts model. In this representation α indicates the 2^m states of the m spin variables $\sigma_\mu=\pm1$ and a sum over α is understood as a trace over the spin states.[5] The e_i^α satisfy the Potts constraint

$$\sum_\alpha^{n+1} e_i^\alpha=0\,, \tag{2.3a}$$

$$e_i^\alpha e_i^\beta=(n+1)\delta_{\alpha\beta}-1\,, \tag{2.3b}$$

$$e_i^\alpha e_j^\alpha=(n+1)\delta_{ij}\,. \tag{2.3c}$$

For a renormalization-group study of crossover from the percolation threshold to the critical line in a random ferromagnet it is sufficient to consider a slightly simplified Hamiltonian with a single pair interaction[6] $K_1 e_q(\vec{r}) e_q(\vec{r}\,')$ among transverse components in Eq. (2.2). The Landau-Ginzburg-Wilson (LGW) effective Hamiltonian, in dimension $d=6-\epsilon$, is then derived by using standard Gaussian integration techniques and it has a quadratic symmetry-breaking term for finite temperature T. As we showed in I that quadratic anisotropy could generate trilinear anisotropic couplings through the RG procedure,[6] we allow here for three different trilinear couplings to start with and obtain

$$H=\int d^d x \left[\tfrac{1}{2}(\nabla\phi)^2+\tfrac{1}{2}t\phi^2+\tfrac{1}{2}\tilde{m}^2\phi_2^2+\tfrac{1}{2}\kappa^{\epsilon/2}v_0 d_{\mu\nu p}\phi_\mu\phi_\nu\phi_p+\tfrac{1}{2}\kappa^{\epsilon/2}w_0 d_{\mu pq}\phi_\mu\phi_p\phi_q+\frac{1}{3!}\kappa^{\epsilon/2}z_0 d_{pqr}\phi_p\phi_q\phi_r\right], \tag{2.4}$$

where ϕ is an n-component real field with m longitudinal and $(n-m)$ transverse components, ϕ_1 and ϕ_2, such that $\phi^2=\phi_1^2+\phi_2^2$ with

$$\phi_1^2=\phi_\mu(\vec{x})\phi_\mu(\vec{x}),\quad \phi_1^2=\phi_p(\vec{x})\phi_p(\vec{x})\ . \tag{2.5}$$

The coupling constants v_0, w_0, z_0 are dimensionless and κ is an arbitrary momentum scale parameter. The tensorial coefficients

$$d_{ijk}=\sum_{\alpha=1}^{n+1} e_i^\alpha e_j^\alpha e_k^\alpha,\quad i,j,k=1,\ldots,n \tag{2.6a}$$

$$d_{\mu\nu\eta}\equiv 0\ , \tag{2.6b}$$

are evaluated with the operators e_i^α of Eq. (2.1), from where the last equality in Eq. (2.6b) follows. The square of the critical mass t, and the noncritical mass $\tilde{m}$ are related to the scaling fields of the random bond-diluted Ising model,[1] $\mu_1 \simeq p_c-p$ and $\mu_2 \simeq e^{-2K}$ in the low-temperature limit through

$$t\simeq\mu_1+A\tilde{m}^{2/\phi},\quad \tilde{m}^2\approx\mu_2 \tag{2.7}$$

with p_c the mean-field value of the percolation concentration. We stress the absence of a trilinear coupling among all longitudinal components due to the vanishing of $d_{\mu\nu\eta}$ in Eq. (2.6b).

In dimension $d>4$ the quartic terms in the LGW Hamiltonian, which have not been written out, are irrelevant in the RG sense and for a study of the disordered phase, to which the present work is restricted, they are not needed for the stability of the thermodynamic quantities and, consequently, they will be ignored in what follows.

III. DIMENSIONAL REGULARIZATION AND GENERALIZED MINIMAL SUBTRACTION

We renormalize H by dimensional regularization (DR) with[13,15] generalized minimal subtraction[8] (GMS). As we are studying a fully anisotropic ϕ^3 field theory close to the critical dimensionality $d_c=6$, we need to introduce two wave-function renormalization constants, $Z_\phi^{(1)}$ and $Z_\phi^{(2)}$, for longitudinal and transverse fields together with three dimensionless renormalized couplings v,w,z, and also $Z_{\phi^2}^{(1)}$, $Z_{\phi^2}^{(2)}$ that renormalize the longitudinal and transverse two-point function with a ϕ^2 insertion.[10] The renormalization is done at the critical theory for fixed $\tilde{m}$. For a random ferromagnet this means renormalization at a point on the critical line. There are no critical-mass subtraction terms in dimensional regularization and we can proceed as usual by taking the critical line as given by mean-field theory. Here $\tilde{m}^2$ is the true transverse-inverse susceptibility at criticality for the longitudinal components and the fact that the theory can be correctly renormalized by keeping $\tilde{m}$ fixed follows along the same lines as in ϕ^4 field theory.[8,9] Moreover, this implies that the dimensionless $\tilde{\mu}=\tilde{m}/\kappa$ grows to infinity in the asymptotic region $\kappa\to 0$ and the renormalization procedure should be carried out so as to keep the theory finite in this limit. This is done by the requirement that the renormalization constants should cancel the leading dimensional pole and the next-to-leading logarithmic singularity in $\tilde{\mu}$ order by order in the renormalized couplings v, w, z, and ϵ, in the dimensionally regularized theory.

The bare two- and three-point vertex functions $\Gamma_{ij}^{(2)}$ and $\Gamma_{ijk}^{(3)}$ transform like the tensors δ_{ij} and d_{ijk} in Eq. (2.6), and we obtain for the bare functions at criticality

$$\Gamma_{\mu\nu}^{(2)}=\delta_{\mu\nu}(k^2-\Sigma_1)=\delta_{\mu\nu}\Gamma_1^{(2)}\ , \tag{3.1a}$$

$$\Gamma_{pq}^{(2)}=\delta_{pq}(\tilde{m}^2+k^2-\Sigma_2)=\delta_{pq}\Gamma_2^{(2)}\ , \tag{3.1b}$$

$$\Gamma_{\mu\nu}^{(2,1)}=\delta_{\mu\nu}\Gamma_1^{(2,1)}\ , \tag{3.2a}$$

$$\Gamma_{pq}^{(2,1)}=\delta_{pq}\Gamma_2^{(2,1)}\ , \tag{3.2b}$$

$$\Gamma^{(3)}_{\mu\nu p} = d_{\mu\nu p}\Gamma^{(3)}_{v} , \tag{3.3a}$$

$$\Gamma^{(3)}_{\mu pq} = d_{\mu pq}\Gamma^{(3)}_{w} , \tag{3.3b}$$

$$\Gamma^{(3)}_{pqr} = d_{pqr}\Gamma^{(3)}_{z} , \tag{3.3c}$$

where the diagrams that contribute to the self-energy parts and to the two-point vertex with ϕ^2 insertion $\Gamma^{(2,1)}_{ij}$ are shown in Figs. 1 and 2 while those for the three-point vertex functions are shown in Figs. 3 and 4, to one-loop order. The evaluation of these diagrams includes a tensorial factor due to the summation over the internal indices of the coefficients d_{ijk} associated to each trilinear coupling in the Hamiltonian of Eq. (2.4). The calculation of these tensorial factors is lengthy although straightforward and it is performed through the systematic use of Eqs. (2.3) and Eqs. (2.6), as it is shown in Appendix A. It is, however, of importance to point out here that the result of the tensorial sums for any particular diagram depends also on the external indices that are not summed over. This not only brings out an overall tensorial factor δ_{ij} or d_{ijk} for $\Gamma^{(2)}_{ij}$ or $\Gamma^{(3)}_{ijk}$ as indicated in Eq. (3.1)–(3.3), but also changes the dependence on n and m of the weight of each diagram. For instance the weight of diagram d in Fig. 3 is $(n+3-3m)$ while the weight of diagram h in the same figure is $(n+2-3m)$.

The renormalization procedure by DR (Refs. 13 and 15) and GMS (Ref. 8) proceeds as follows. A given bare-vertex function, such as $\Gamma^{(2)}_1$ in Eq. (3.1), is given explicity to one-loop order by

$$\Gamma^{(2)}_1 = k^2-(n+1)^2[v_0^2(m-1)A_1 + \tfrac{1}{2}w_0^2(n+1-2m)A_2] , \tag{3.4}$$

with A_1 and A_2 being the loop integrals with one and two transverse propagators of diagrams a and b of Fig. 1:

$$A_1 = \kappa^2 \int^{\Lambda/\kappa} d^dp(\tilde{\mu}^2+p^2)^{-1}\left| p+\frac{\vec{k}}{\kappa}\right|^{-2} , \tag{3.5a}$$

$$A_2 = \kappa^2 \int^{\Lambda/\kappa} d^dp(\tilde{\mu}^2+p^2)^{-1}\left[\tilde{\mu}^2+\left|\vec{p}+\frac{\vec{k}}{\kappa}\right|^{2}\right]^{-1} , \tag{3.5b}$$

where Λ is an arbitrary cutoff and

$$\tilde{\mu} = \tilde{m}/\kappa . \tag{3.6}$$

Both integrals go like Λ^{d-4} and are ultraviolet divergent for $d>4$. DR consists[15] in performing

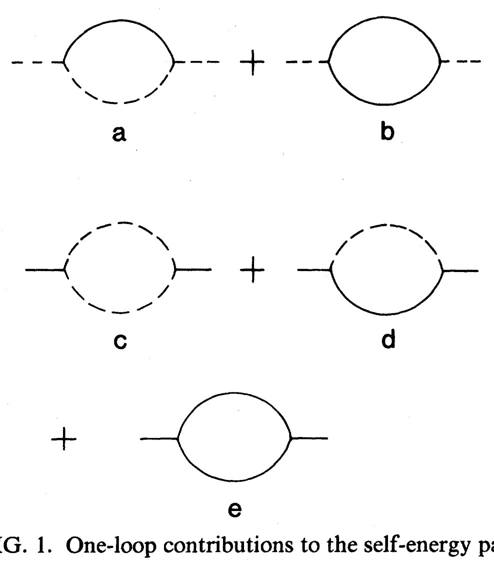

FIG. 1. One-loop contributions to the self-energy parts Σ_1 (a and b) and Σ_2 ($c-e$) of the longitudinal and transverse two-point vertex functions in Eq. (3.1). Dashed and solid internal lines represent longitudinal and transverse free-field propagators, respectively. To each trilinear interaction with one, two, or three solid lines is associated a factor $v_0 d_{\mu\nu p}$, $w_0 d_{\mu pq}$, or $z_0 d_{pqr}$, respectively, and summations over internal indices is implied.

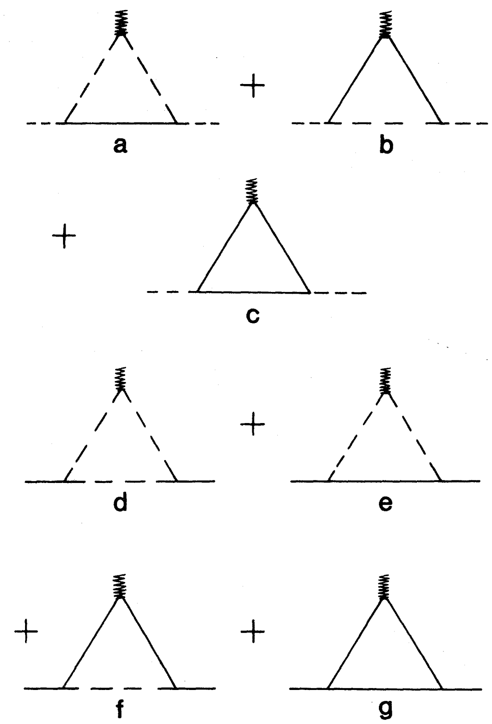

FIG. 2. One-loop contributions to the two-point vertex functions with ϕ^2 insertions $\Gamma^{(2,1)}_{\mu\nu}$ ($a-c$) and $\Gamma^{(2,1)}_{pq}$ ($d-g$). Internal lines and trilinear interactions are as specified in Fig. 1. The wiggly line indicates the insertion point.

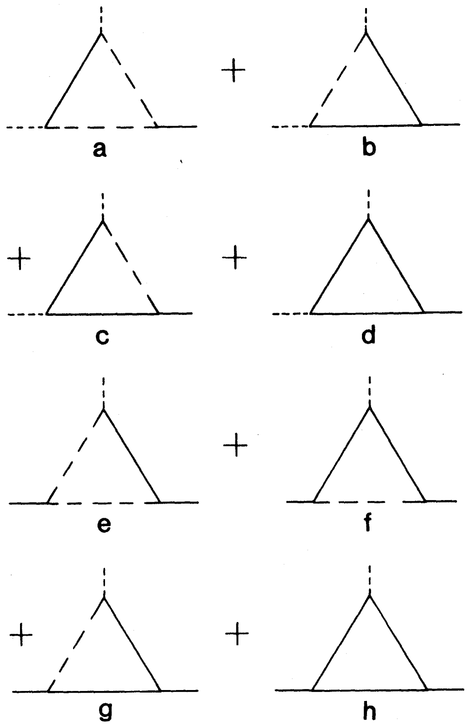

FIG. 3. One-loop contributions to the vertex functions $\Gamma^{(3)}_{\mu\nu p}$ $(a-d)$ and $\Gamma^{(3)}_{\mu pq}$ $(e-h)$. Internal lines and trilinear interactions are as specified in Fig. 1.

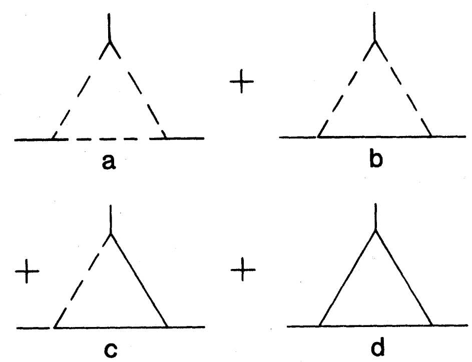

FIG. 4. One-loop contributions to the vertex functions $\Gamma^{(3)}_{pqr}$. Internal lines and trilinear interactions are as specified in Fig. 1. The tensorial sum associated to diagram b vanishes identically.

the integrals for $d<4$, when they are convergent, by letting $\Lambda\to\infty$ and then to continue analytically for other values of d. The singularities at $d=6-\epsilon$ will appear now as a contribution to the mass term that is absorbed in the critical $t=0$, and as a dimensional pole in ϵ in the coefficient of k^2. The singular parts[15] of A_1 and A_2, to the next-to-leading order, are given by

$$[A_1]_{\text{sing}}=A_{\text{sing}}+k^2\int_0^1 dx\, x(1-x)\left\{\ln\left[\tilde{\mu}^2x+x(1-x)\left[\frac{k}{\kappa}\right]^2\right]-\ln(1+\tilde{\mu}^2)\right\},$$
$$[A_2]_{\text{sing}}=A_{\text{sing}}+k^2\int_0^1 dx\, x(1-x)\left\{\ln\left[\tilde{\mu}^2+x(1-x)\left[\frac{k}{\kappa}\right]^2\right]-\ln(1+\tilde{\mu}^2)\right\}, \tag{3.7}$$

$$A_{\text{sing}}=-k^2\frac{1}{3\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right]. \tag{3.8}$$

Since the last terms in both Eqs. (3.7) are regular in ϵ and finite for $\tilde{\mu}\to\infty$, they do not contribute to this order. The singular contribution to both integrals is the same and given by Eq. (3.8). The leading singularity is a dimensional pole independent of the anisotropy $\tilde{\mu}$ that is canceled by the renormalization constants as in the theory with minimal subtraction (MS). However, the next-to-leading term becomes singular when $\tilde{\mu}\to\infty$, and GMS consists in requiring that the renormalization constants also cancel these singularities so as to render the theory finite when $\tilde{\mu}\to\infty$. We notice that the singular contribution to all diagrams is the same as far as they contain at least one internal transverse propagator, while the integral for diagram c in Fig. 1 gives

$$[A_0]_{\text{sing}}=-k^2\frac{1}{3\epsilon}. \tag{3.9}$$

Similarly, the singular part of the integrals that contribute to $\Gamma^{(3)}_{ijk}$ and $\Gamma^{(2,1)}_{ij}$ are

$$L_{\text{sing}}=\frac{1}{\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right], \tag{3.10}$$

for diagrams $a-c$ and $e-g$ in Fig. 2, all diagrams in Fig. 3, and diagrams $b-d$ in Fig. 4, while

$$[L_0]_{\rm sing}=\frac{1}{\epsilon}\ , \tag{3.11}$$

for diagrams d in Fig. 2 and a in Fig. 4.

Here, as usual, a factor $K_d=2^{d-1}\pi^d\Gamma(d/2)$ was absorbed in the definition of v_0,w_0,z_0. Combining the tensorial factors for each diagram with Eqs. (3.8)–(3.11) we obtain for the singular parts of the relevant bare-vertex functions in Eqs. (3.1)–(3.3)

$$\Gamma_1^{(2)}\approx k^2\left[1+\frac{(n+1)^2}{3\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right][(m-1)v_0^2+\tfrac{1}{2}(n+1-2m)w_0^2]\right]\ , \tag{3.12a}$$

$$\Gamma_2^{(2)}\approx\tilde{m}^2+k^2\left[1+\frac{(n+1)^2}{6\epsilon}v_0^2+\frac{(n+1)^2}{3\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right][(m-1)w_0^2+\tfrac{1}{2}(n-2m)z_0^2]\right]\ , \tag{3.12b}$$

$$\Gamma_1^{(2,1)}\approx 1+\frac{(n+1)^2}{\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right][2(m-1)v_0^2+(n+1-2m)w_0^2]\ , \tag{3.13a}$$

$$\Gamma_2^{(2,1)}\simeq 1+\frac{(n+1)^2}{\epsilon}v_0^2+\frac{(n+1)^2}{\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right][2(m-1)w_0^2+(n-2m)z_0^2]\ , \tag{3.13b}$$

$$\Gamma_v^{(3)}\simeq\kappa^{\epsilon/2}\left[v_0+\frac{(n+1)^2}{\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right][v_0^3+(m-2)v_0^2z_0+2(m-2)v_0w_0^2+(n+3-3m)z_0w_0^2]\right]\ , \tag{3.14}$$

$$\Gamma_w^{(3)}\simeq\kappa^{\epsilon/2}\left[w_0+\frac{(n+1)^2}{\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right][2v_0^2w_0+(m-2)w_0^3+2(m-2)v_0w_0z_0+(n+2-3m)w_0z_0^2]\right]\ , \tag{3.15}$$

$$\Gamma_z^{(3)}=\kappa^{\epsilon/2}\left[z_0+\frac{(n+1)^2}{\epsilon}v_0^3+\frac{(n+1)^2}{\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right][3(m-1)w_0^2z_0+(n-3m)z_0^3]\right]\ . \tag{3.16}$$

It is easy to check that in the isotropic limit $\tilde{\mu}=0$ Eqs. (3.12)–(3.16) reproduce the vertex functions of the symmetric Potts model in Ref. 13.

To remove the dimensional poles and the large-$\tilde{\mu}^2$ behavior of the vertex functions in Eqs. (3.1)–(3.3) we renormalize as follows:

$$Z_\phi^{(1)}\Gamma_{\mu\nu}^{(2)}=\Gamma_{R\mu\nu}^{(2)}\ , \tag{3.17a}$$

$$Z_\phi^{(2)}\Gamma_{pq}^{(2)}=\Gamma_{Rpq}^{(2)}\ , \tag{3.17b}$$

$$Z_{\phi^2}^{(1)}\Gamma_{\mu\nu}^{(2,1)}=\Gamma_{R\mu\nu}^{(2,1)}\ , \tag{3.18a}$$

$$Z_{\phi^2}^{(2)}\Gamma_{pq}^{(2,1)}=\Gamma_{Rpq}^{(2,1)}\ , \tag{3.18b}$$

$$Z_\phi^{(1)}(Z_\phi^{(2)})^{1/2}\Gamma_{\mu\nu p}^{(3)}=\Gamma_{R\mu\nu p}^{(3)}\ , \tag{3.19a}$$

$$(Z_\phi^{(1)})^{1/2}Z_\phi^{(2)}\Gamma_{\mu pq}^{(3)}=\Gamma_{R\mu pq}^{(3)}\ , \tag{3.19b}$$

$$(Z_\phi^{(2)})^{3/2}\Gamma_{pqr}^{(3)}=\Gamma_{Rpqr}^{(3)}\ , \tag{3.19c}$$

by means of dimensionless functions $v_0(v,w,z,\epsilon,\tilde{\mu})$, $w_0(v,w,z,\epsilon,\tilde{\mu})$, $z_0(v,w,z;\epsilon,\tilde{\mu})$, $Z_\phi^{(i)}(v,w,z;\epsilon,\tilde{\mu})$, $Z_{\phi^2}^{(i)}(v,w,z;\epsilon,\tilde{\mu})$ ($i=1$ or 2), and where v,w,z are renormalized couplings. For the thermally driven crossover in random ferromagnets near the percolation point we take as usual, in the following, the limit $m=n=0$. The wave-function renormalization constants are

$$Z_\phi^{(1)}=1-\frac{1}{3\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right](-v^2+\tfrac{1}{2}w^2)\ ,$$

$$Z_\phi^{(2)}=1-\frac{1}{6\epsilon}v^2+\frac{1}{3\epsilon}\left[1-\frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right]w^2\ . \tag{3.20}$$

Note that these are not the same, in distinction to the symmetric theory considered in I.

The expansion of the bare couplings in terms of renormalized ones obtained from Eqs. (3.14)–(3.16) and Eq. (3.20) by requiring that the expansion coefficients cancel the resultant singularities are

$$v_0 = v + \frac{1}{12\epsilon}v^3 + \frac{1}{\epsilon}\left[1 - \frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right] \times(-\tfrac{4}{3}v^3 + 2v^2z + 4vw^2 - 3zw^2)\,, \quad (3.21)$$

$$w_0 = w + \frac{1}{6\epsilon}wv^2 + \frac{1}{12\epsilon}\left[1 - \frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right] \times(-26wv^2 + 21w^3 + 48wvz - 24wz^2)\,, \quad (3.22)$$

$$z_0 = z + \frac{1}{4\epsilon}(zv^2 - 4v^3) + \frac{5}{2\epsilon}\left[1 - \frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right]zw^2\,. \quad (3.23)$$

From Eqs. (3.18) and (3.13) it follows that

$$Z^{(1)}_{\phi^2} = 1 - \frac{1}{\epsilon}\left[1 - \frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right](-2v^2 + w^2)\,, \quad (3.24a)$$

$$Z^{(2)}_{\phi^2} = 1 - \frac{1}{\epsilon}v^2 + \frac{2}{\epsilon}\left[1 - \frac{\epsilon}{2}\ln(1+\tilde{\mu}^2)\right]w^2\,, \quad (3.24b)$$

and these are again different from the single Z_{ϕ^2} that appears in I. In fact, the renormalization of $\Gamma^{(2,1)}_{ij}$ is not an independent one because the diagrams combine the tensorial coefficients of $\Gamma^{(2)}_{ij}$ with the loop integrals for three internal propagators.

IV. SYMMETRIC AND ASYMMETRIC PERCOLATION

In terms of the dimensional couplings,

$$\lambda_v = v_0\kappa^{\epsilon/2},\quad \lambda_w = w_0\kappa^{\epsilon/2},\quad \lambda_z = z_0\kappa^{\epsilon/2}\,, \quad (4.1)$$

the three Wilson β functions are obtained through[15]

$$\beta_v = \left[\kappa\frac{\partial}{\partial\kappa}v\right]_{\lambda_j},\quad \beta_w = \left[\kappa\frac{\partial}{\partial\kappa}w\right]_{\lambda_j},\quad \beta_z = \left[\kappa\frac{\partial z}{\partial\kappa}\right]_{\lambda_j}\,, \quad (4.2)$$

where the derivatives are taken at fixed λ_j, $j=v,w,z$. Equations (3.21)–(3.23) can be easily be inverted to this order to give the renormalized couplings in terms of the λ_j in Eq. (4.1), and taking into account the dependence of $\tilde{\mu}$ on κ in Eq. (3.6) we obtain from Eq. (4.2),

$$\beta_v = -\frac{\epsilon}{2}v + \tfrac{1}{12}v^3 + (1+\tilde{\mu}^2)^{-1} \times[-\tfrac{4}{3}v^3 + 2v^2z + 4vw^2 - 3zw^2)\,, \quad (4.3)$$

$$\beta_w = -\frac{\epsilon}{2}w + \tfrac{1}{6}wv^2 + \tfrac{1}{12}(1+\tilde{\mu}^2)^{-1} \times(-26wv^2 + 21w^3 + 48wvz - 24wz^2)\,, \quad (4.4)$$

$$\beta_z = -\frac{\epsilon}{2}z - v^3 + \tfrac{1}{4}zv^2 + \tfrac{5}{2}(1+\tilde{\mu}^2)^{-1}zw^2\,. \quad (4.5)$$

The fixed point (FP) equations satisfied by $v^*(\tilde{\mu})$, $w^*(\tilde{\mu})$, and $z^*(\tilde{\mu})$ are obtained by setting simultaneously to zero β_v, β_w, and β_z in Eqs. (4.3)–(4.5). It is readily verified that for vanishing anisotropy $\tilde{\mu}$ they have the symmetric solution

$$v^*(0) = w^*(0) = z^*(0) = (2\epsilon/7)^{1/2} \quad (4.6)$$

that describes the ordinary percolation transition.[13,14] For finite anisotropy, no matter how small, the solutions to the FP equations are nonsymmetric.

The other Wilson functions are obtained from Eqs. (3.20) and (3.24) as

$$\gamma^{(r)}_{\phi} = \left[\kappa\frac{\partial}{\partial\kappa}\ln Z^{(r)}_{\phi}\right]_{\lambda_j}\,, \quad (4.7)$$

$$\gamma^{(r)}_{\phi^2} = \left[\kappa\frac{\partial}{\partial\kappa}\ln Z^{(r)}_{\phi^2}\right]_{\lambda_j}\,,\quad r=1 \text{ or } 2\,, \quad (4.8)$$

and we observe that w^2 enters all the Wilson functions always divided by $(1+\tilde{\mu}^2)$. It is then convenient to define

$$y(\tilde{\mu}) = w^*(\tilde{\mu})^2/(1+\tilde{\mu}^2)\,,\quad x(\tilde{\mu}) = v^*(\tilde{\mu})^2\,, \quad (4.9)$$

where $[y(\tilde{\mu})]^{1/2}$ appears as an effective trilinear coupling. In terms of these new variables we obtain from Eq. (4.7),

$$\gamma_\phi^{(1)}(\tilde{\mu})=\tfrac{1}{6}y(\tilde{\mu})-\tfrac{1}{3}x(\tilde{\mu})/(1+\tilde{\mu}^2)\ , \qquad (4.10a)$$

$$\gamma_\phi^{(2)}(\tilde{\mu})=\tfrac{1}{6}x(\tilde{\mu})-\tfrac{1}{3}y(\tilde{\mu})\ , \qquad (4.10b)$$

$$\gamma_{\phi^2}^{(r)}=6\gamma_\phi^{(r)},\quad r=1,2\ . \qquad (4.10c)$$

These yield the effective longitudinal crossover exponents,

$$\eta(\tilde{\mu})=\gamma_\phi^{(1)}(\tilde{\mu})\ , \qquad (4.11)$$

$$\nu^{-1}(\tilde{\mu})-2=\gamma_{\phi^2}^{(1)}(\tilde{\mu})-\gamma_\phi^{(1)}(\tilde{\mu})=5\gamma_\phi^{(1)}(\tilde{\mu})\ , \qquad (4.12)$$

that describe the critical

$$\Gamma_{\mu\nu}^{(2)}(k,\tilde{\mu})\propto\delta_{\mu\nu}k^{2-\eta(\tilde{\mu})}$$

and the longitudinal correlation length

$$\xi_{\|}(t,\tilde{\mu})=t^{-\nu(\tilde{\mu})}\ .$$

The FP equations are

$$v^*\left[-6\epsilon+x+48y-36y\frac{z^*}{v^*}+(1+\tilde{\mu}^2)^{-1}\left[-16x+24x\frac{z^*}{v^*}\right]\right]=0\ , \qquad (4.13)$$

$$w^*\left\{-6\epsilon+2x+21y+(1+\tilde{\mu}^2)^{-1}\times\left[-26x+24x\frac{z^*}{v^*}\left[2-\frac{z^*}{v^*}\right]\right]\right\}=0\ , \qquad (4.14)$$

$$\frac{z^*}{v^*}=4x(x+10y-2\epsilon)^{-1}\ . \qquad (4.15)$$

Besides the Gaussian FP that is unstable for $\epsilon>0$, the coupled nonlinear equations (4.13)–(4.15) have three nontrivial solutions. The stability of these solutions was analyzed by calculating the eigenvalues $\Theta(\tilde{\mu})$ of the 3×3 derivative matrix with elements

$$D_{\sigma\rho}=\left[\frac{\partial\beta_\sigma}{\partial\rho}\right]_{\rm FP},\quad \sigma,\rho=v,w,z,$$

from Eqs. (4.3)–(4.5). A fixed point is completely stable if all the eigenvalues have a positive real part.[10,15]

A. Asymmetric percolation fixed point

This corresponds to the solution

$$z_{\rm I}^*=v_{\rm I}^*=0\ , \qquad (4.16a)$$

$$y_{\rm I}^*=w_{\rm I}^{*2}/(1+\tilde{\mu}^2)=2\epsilon/7\ , \qquad (4.16b)$$

with three positive eigenvalues,

$$\Theta_1^{\rm I}=9\epsilon/10\ , \qquad (4.17a)$$

$$\Theta_2^{\rm I}=\epsilon\ , \qquad (4.17b)$$

$$\Theta_3^{\rm I}=3\epsilon/14\ . \qquad (4.17c)$$

The set of values in Eqs. (4.16) are a solution to Eqs. (4.13)–(4.15) only when $\omega_{\rm I}^*(\tilde{\mu})$ or $\tilde{\mu}^2$ are finite. In the limit $\tilde{\mu}\to\infty$ the only solution to Eq. (4.14) is $w^*=0$, and this permits a nonzero v^* and z^* which correspond to the fixed point that describes the critical line.

Note that at this fixed point the only nonzero coupling—normalized by $(1+\tilde{\mu}^2)$ — has the value for ordinary percolation. This stable fixed point is one of the new results of this work. It describes a new transition in the one-state Potts model with trilinear symmetry breaking even in the absence of quadratic anisotropy. We call this transition "asymmetric percolation" to distinguish it from ordinary percolation that corresponds to the fixed point which is symmetric when $\tilde{\mu}=0$ and that is discussed below. From Eqs. (4.10)–(4.12) the exponents for all finite $\tilde{\mu}$ are

$$\eta_{\rm I}=+\epsilon/21\ ,\qquad \nu_{\rm I}^{-1}-2=5\epsilon/21\ . \qquad (4.18)$$

We do not have, at present, further insight into the physical nature of the asymmetric percolation and we feel that it deserves a separate study to be considered in future work.

B. Ordinary percolation and thermally driven crossover

We are concerned here with the crossover that starts at the symmetric FP, Eq. (4.6), in the limit $\tilde{\mu}=0$. The FP for finite $\tilde{\mu}$ is the solution of the equations obtained by setting both brackets simultaneously equal to zero in Eqs. (4.13) and (4.14) together with Eq. (4.15). This leads to cubic and quadratic equations that were solved analytically for $y(\tilde{\mu})$ and that yield

$$y_{\rm II}(\tilde{\mu})=f_3(x_{\rm II}(\tilde{\mu}),\tilde{\mu})$$

and

$$y_{\rm II}(\tilde{\mu})=f_2(x_{\rm II}(\tilde{\mu}),\tilde{\mu}) .$$

The solutions satisfy the initial symmetric FP relation

$$y_{\rm II}(0)=f_3(x_{\rm II}(0),0)=f_2(x_{\rm II}(0),0) =x_{\rm II}(0)=2\epsilon/7 . \quad (4.19)$$

By making equal both expressions for $y_{\rm II}(x_{\rm II}(\tilde{\mu}))$, we derived an equation for $x_{\rm II}(\tilde{\mu})$ that was solved numerically in the crossover region. The flow of the couplings $v^*_{\rm II}(\tilde{\mu})$, $w^*_{\rm II}(\tilde{\mu})$, and $z^*_{\rm II}(\tilde{\mu})$ is shown in Fig. 5 together with the flow of $[y_{\rm II}(\tilde{\mu})]^{1/2}$ defined in Eq. (4.9). The flow of $\eta(\tilde{\mu})$ under crossover is shown in Fig. 6. The stability was studied analytically in the symmetric limit and we obtain for the eigenvalues when $\tilde{\mu}=0$,

$$\Theta^{\rm II}_1(0)=\Theta^{\rm II}_2(0)=\epsilon , \quad \Theta^{\rm II}_3(0)=0 . \quad (4.20)$$

Equation (4.20) indicates that the symmetric fixed point of Eq. (4.6) which describes the ordinary percolation transition[13,14] is stable in two directions and marginal along a third direction.

For finite anisotropy $\tilde{\mu}$ this FP is nonsymmetric. The flow of the couplings shown in Fig. 5 are a solution of the fixed-point equations for any large but finite anisotropy $\tilde{\mu}$. We point out, however, that this fixed point cannot drive asymptotically the crossover to the critical line because in the limit $\tilde{\mu}\to\infty$ the only solution of Eq. (4.14) is $w^*=0$.

The approach to the critical line is described by the third solution to Eqs. (4.13)–(4.15):

$$w^*_{\rm III}=0 , \quad (4.21a)$$

$$x^*_{\rm III}(\tilde{\mu})=[v^*_{\rm III}(\tilde{\mu})]^2 =2\epsilon\{2(\tilde{\mu}^2-3)+[\Delta(\tilde{\mu})]^{1/2}\} \times(81+\tilde{\mu}^2)^{-1} , \quad (4.21b)$$

$$z^*_{\rm III}(\tilde{\mu})=4[x^*_{\rm III}(\tilde{\mu})]^{3/2}[x^*_{\rm III}(\tilde{\mu})-2\epsilon]^{-1} , \quad (4.21c)$$

with

$$\Delta(\tilde{\mu})=4(\tilde{\mu}^2-3)^2-3(1+\tilde{\mu}^2)(81+\tilde{\mu}^2) \quad (4.22)$$

for $\tilde{\mu}\geq\tilde{\mu}_c,\tilde{\mu}_c^2\approx 270$. For $0\leq\tilde{\mu}\leq\tilde{\mu}_c$, $\Delta(\tilde{\mu})$ is negative and Eqs. (4.21) are not a real solution of the FP equations. For $\tilde{\mu}_c\leq\tilde{\mu}\leq\infty$ the couplings $v^*_{\rm III}(\tilde{\mu})$ and $z^*_{\rm III}(\tilde{\mu})$ are finite, with limiting values,

$$v^*_{\rm III}(\infty)=\sqrt{6\epsilon} , \quad (4.23a)$$

$$z^*_{\rm III}(\infty)=6\sqrt{6\epsilon} \quad (4.23b)$$

The fixed point is attractive and it has three

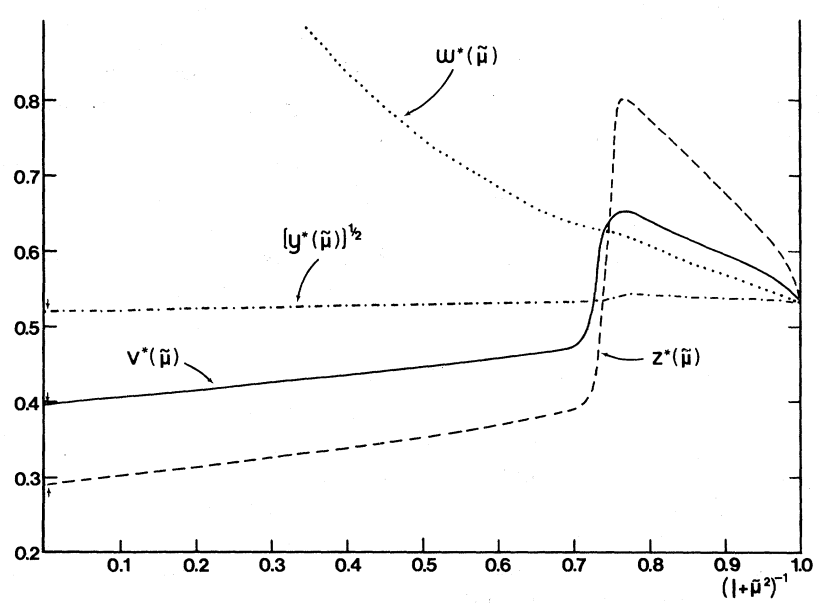

FIG. 5. Flow of the trilinear FP couplings from the percolation point $\tilde{\mu}=0$ as functions of $(1+\tilde{\mu}^2)^{-1}$, where $\tilde{\mu}=\tilde{m}/\kappa$ is the scaled anisotropy parameter. $v^*_{\rm II}(\tilde{\mu})/\sqrt{\epsilon}$: solid line; $w^*_{\rm II}(\tilde{\mu})/\sqrt{\epsilon}$: dotted line; $z^*_{\rm II}(\tilde{\mu})/\sqrt{\epsilon}$: dashed line. The dash-dot line represents the effective coupling $[y^*_{\rm II}(\tilde{\mu})/\epsilon]^{1/2}$ in Eq. (4.9). The arrows indicate the onset of the fixed point in Eq. (4.21) on the critical line.

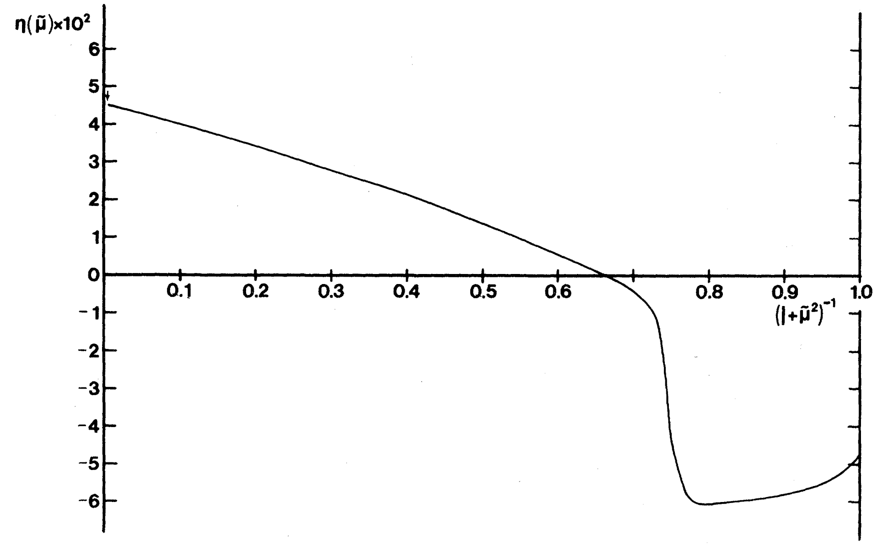

FIG. 6. Flow under crossover from the percolation point $\tilde{\mu}=0$ of the effective exponent $\eta(\tilde{\mu})=[\nu^{-1}(\tilde{\mu})-2]/5$, as function of $(1+\tilde{\mu}^2)^{-1}$. $\tilde{\mu}=\tilde{m}/\kappa$ is the scaled anisotropy parameter. The arrow is as in Fig. 5.

directions of stability with limiting eigenvalues:

$$\Theta_v^{\rm III}(\infty)=\epsilon\ ,$$
$$\Theta_w^{\rm III}(\infty)=\frac{\epsilon}{2}\ , \qquad (4.24)$$
$$\Theta_z^{\rm III}(\infty)=\epsilon\ .$$

For $\tilde{\mu}>\tilde{\mu}_c$ the flow under crossover of the effective exponents is obtained by introducing Eqs. (4.21) in Eqs. (4.10)–(4.12). This yields

$$\eta(\tilde{\mu})=-\tfrac{1}{3}x_{\rm III}^*(\tilde{\mu})/(1+\tilde{\mu}^2)\ ,$$
$$\nu^{-1}(\tilde{\mu})-2=5\eta(\tilde{\mu})\ , \qquad (4.25)$$

and the transition to the critical line has asymptotic mean-field exponents $\eta(\infty)=0$, $\nu(\infty)=\frac{1}{2}$ from Eq. (4.23) as expected for a random ferromagnet in $d>4$.

V. SUMMARY AND DISCUSSION

In accordance with previous works,[4,6] we studied here the crossover from the percolation threshold to the critical line in random bond-diluted ferromagnets by means of quadratic symmetry breaking along the components of the fields in a continuous ϕ^3 field theory in dimension $d=6-\epsilon$. Assuming that the trilinear coupling can be taken at the percolation fixed-point value, it has been shown before that there is a crossover to classical mean-field exponents[4] near $d=6$. However, it is difficult to understand why the trilinear percolation fixed point should be used under a crossover away from percolation. Taking the crossover in an n-component ϕ^4 theory with quadratic symmetry breaking as a guide, where an isotropic quartic coupling is justified for small n, it is known that the correct asymptotic critical behavior is not obtained by keeping the quartic coupling at the fixed-point value of the primary critical point.[16]

In an attempt to understand this point, the work described in I was carried out. We found there, with a quadratic symmetry breaking that favors the ordering in directions associated with the single-spin components of the Potts vectors e_i^α, indications of a relevant break in trilinear symmetry that may not allow to keep the trilinear coupling at the percolation fixed point in the course of the crossover to the critical line. In the work reported in the present paper we carried out the detailed RG calculations to study the effects of quadratic symmetry breaking on the trilinear coupling of a Potts ϕ^3 field theory. This is a theory in which the trilinear couplings $d_{ijk}\ \phi_i\phi_j\phi_k$, not necessarily the same for all field components i, j, and k, involve a tensorial coefficient defined exclusively in terms of the Potts vector e_i^α, that is, $d_{ijk}=e_i^\alpha e_j^\alpha e_k^\alpha$. The work reported in this paper is the first study, as far as we know, of the combined effects of quadratic and trilinear symmetry breaking in a ϕ_3 field theory.

The detailed calculations of this work show that, for any nonzero quadratic anisotropy, there are only nonsymmetric sets of trilinear fixed point couplings

v^*, w^*, and z^*, in addition to the trivial, fully unstable, fixed point $v^*=w^*=z^*=0$, because the renormalization of the three-point vertex functions is not symmetric under quadratic symmetry breaking. This is an important point which had not been fully realized before. It also turns out from our work that the multicritical percolation point is the symmetric fixed point in the three Hamiltonian-parameter space (v,w,z) that favor certain trilinear couplings between field components. We also find that in the crossover from the percolation point to the critical line there is a wide region where the effective critical exponents are nonclassical in $d=6-\epsilon$ dimensions and that their asymptotic mean-field values are only reached very close to the critical line. We cannot attach any particular significance to the special value in between where $\eta(\tilde{\mu})$ and $\nu^{-1}(\tilde{\mu})-2$ pass through zero, as shown in Fig. 6.

A second and interesting set of results apply even in the absence of quadratic symmetry breaking. First, we have shown that the percolation point itself is *not* completely stable under trilinear symmetry breaking in the components of the fields of our Potts ϕ^3 field theory. We found that there is a zero eigenvalue of the stability matrix in the (v,w,z) space and this corresponds to a marginal eigenvector. Then we showed that a trilinear symmetry-breaking perturbation can yield a more stable fixed point, with new critical exponents, than the percolation point. This is what we called asymmetric percolation, in which the only nonzero trilinear coupling has the percolation fixed-point value. Further work is clearly needed to elucidate the nature of this transition. We can anticipate here, however, that a nonsymmetric trilinear fixed point is not a unique feature of the one-state Potts ϕ^3 field theory. Indeed, further work in progress shows that it also appears in the $(n+1)$-state model, for nonzero n.[17]

In studying the crossover from the percolation point to the critical line in random bond-diluted ferromagnets we test the stability of the multicritical percolation point to the combined effects of quadratic and trilinear symmetry breaking in the field component. As pointed out above, we had to allow for a break in trilinear symmetry in the presence of quadratic symmetry breaking, although the inverse is not the case. In the works of Harris, Lubensky, and Isaacson[12] that describe crossover phenomena in branched polymers near percolation, the multicritical point is subject to different and either to quadratic or trilinear symmetry-breaking perturbations.

Quadratic and linear (in an external field) symmetry breaking in the states of the Potts vectors has been shown to describe the crossover to lattice-animal behavior in dimension $d=8-\epsilon$ in a Potts ϕ^3 field theory, assuming a symmetric trilinear interaction.[12(b)] It is possible that a break of quadratic symmetry in the states, in place of the components, of the Potts vectors e_i^α has different implications on the symmetry of the trilinear fixed point than what we found in our work and that a symmetric trilinear coupling is all that is needed in the crossover to lattice-animal behavior. Although this point has not been raised before, there seems to be an open question that may be important to study.

A somewhat different ϕ^3 field theory with an additional field component associated to the component $a_0^\nu=n^{-1/2}(1,1,\ldots,1)$ of an orthogonal set of n n-dimensional unit vectors $a_l^\nu,\nu=1,\ldots,n$; $l=0,1,\ldots,n-1$, that satisfy relationships similar to Eq. (2.3), has been proposed to describe the crossover from the percolation threshold to the behavior of θ solvent branched polymers.[12(a),(c)] With a trilinear symmetry breaking that always involves the additional field component, the percolation point appears to be given by the next most stable fixed point, in dimension $d=6-\epsilon$, in a space of three trilinear couplings. However, the specific trilinear symmetry breaking in the Potts vectors of the enlarged set of a_l^ν has not been considered, apparently, so far and it is quite possible that this would yield a more complex critical behavior in branched polymers that may be worthwhile exploring.

Further work that needs to be done, in addition to the points referred to above, for the crossover in random bond-diluted ferromagnets near the percolation threshold is a study of the ordered phase and of the effect of an external field. We expect to report on these points in future work.

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APPENDIX A

We illustrate here how the tensorial sums are evaluated corresponding to the diagrams in Figs. 1–4. We give as an example the detailed calculation of

$$S_{\mu\nu p}=\sum_{r,s,\eta} d_{\mu\eta r}d_{\nu rs}d_{ps\eta} \tag{A1}$$

that appears as a factor of diagram 3(c). By using the explicit expression for d_{ijk} in Eq. (2.6a) together with Eq. (2.3b) in Eq. (A1), we obtain

$$S_{\mu\nu\rho}=\sum_{\alpha\beta\gamma}^{n+1}\sum_{\eta}e_\mu^\alpha e_\eta^\alpha e_\nu^\beta e_\rho^\gamma e_\eta^\gamma[(n+1)\delta_{\alpha\beta}-1-\sum_{\theta}e_\theta^\alpha e_\theta^\beta][(n+1)\delta_{\gamma\beta}-1\sum_{\zeta}e_\zeta^\gamma e_\zeta^\beta] \ . \tag{A2}$$

After splitting the sums and by using Eqs. (2.3a)—(2.3c) together with Eq. (2.6b) we find that $S_{\mu\nu\rho}$ reduces to

$$S_{\mu\nu\rho}=(n+1)^2\sum_{\eta}\sum_{\alpha}e_\mu^\alpha e_\nu^\alpha e_\rho^\alpha-(n+1)^2\sum_{\eta}\delta_{\mu\eta}\sum_{\beta}e_\nu^\beta e_\rho^\beta e_\eta^\beta$$
$$+(n+1)^2\sum_{\eta,\zeta}\delta_{\mu\eta}\delta_{\nu\zeta}\sum_{\gamma}e_\rho^\gamma e_\eta^\gamma e_\zeta^\gamma-(n+1)\sum_{\eta,\zeta}T_{\mu\nu\eta\zeta}\sum_{\gamma}e_\rho^\gamma e_\eta^\gamma e_\zeta^\gamma \ , \tag{A3}$$

where from the definition of e_μ^α in Eq. (2.1),

$$T_{\mu\nu\eta\zeta}=\sum_{\alpha}e_\mu^\alpha e_\nu^\alpha e_\eta^\alpha e_\zeta^\alpha$$
$$=(n+1)(\delta_{\mu\eta}\delta_{\nu\zeta}+\delta_{\nu\zeta}\delta_{\nu\eta}) \ . \tag{A4}$$

By using again Eq. (2.6a) we finally obtain

$$S_{\mu\nu\rho}=(n+1)^2 d_{\mu\nu\rho}(m-2) \ . \tag{A5}$$

The evaluation of any other tensorial sum follows the same lines leading to the vertex functions in Eqs. (3.12)—(3.16).

APPENDIX B

We show here that the results of this paper also follow from a cutoff-dependent version of RPT in terms of the bare parameters of the original Hamiltonian[10,15] which has been extended by one of us to describe crossover in anisotropic spin systems.[9]

The renormalized vertex functions $\Gamma_{R\mu\nu}^{(2)}$ and $\Gamma_{Ri_1i_2i_3}^{(3)}$ defined in Eqs. (3.17) and (3.19) satisfy the renormalization-group differential equation:

$$\left[\kappa\frac{\partial}{\partial\kappa}+\sum_j\beta_j\frac{\partial}{\partial j}-\tfrac{1}{2}(\gamma_\phi^{(i_1)}+\cdots+\gamma_\phi^{(i_N)})\right]\Gamma_{Ri_1\cdots i_N}^{(N)}=0 \ , \tag{B1}$$

where $j=v,w,z$ and the Wilson functions β_j and $\gamma_\phi^{(i)}=\gamma_\phi^{(1)}(\gamma_\phi^{(2)})$ if $i=\mu(p)$ are defined in Eqs. (4.2) and (4.7). This equation is the expression in differential form of the independence of the bare $\Gamma_{i_1,\ldots,i_N}^{(N)}$ with respect to κ for fixed dimensional couplings.

In the bare theory regularized with a cutoff Λ the renormalized $\Gamma_{Ri_1,\ldots,i_N}^{(N)}$ are asymptotically independent of Λ, and this is expressed in the differential RG equation[10,15] for the bare vertex functions $\Gamma_{\mu\nu}^{(2)}$ and $\Gamma_{ijk}^{(3)}$:

$$\left[\Lambda\frac{\partial}{\partial\Lambda}+\sum_j\beta_j\frac{\partial}{\partial j_0}-\tfrac{1}{2}(\gamma_\phi^{(i_1)}+\cdots+\gamma_\phi^{(i_N)})\right]\Gamma_{i_1\cdots i_N}^{(N)}\simeq 0 \ , \tag{B2}$$

with the new definitions

$$\gamma_\phi^{(r)}\left[\{j_0\},\frac{\tilde m}{\Lambda}\right]=-\left[\Lambda\frac{d}{d\Lambda}\ln Z_\phi^r\right]_{\{j\},\kappa} \ ,$$
$$r=1(2) \ \text{ if } i=\mu(p) \ , \tag{B3}$$

$$\beta_j\left[\{j_0\},\frac{\tilde m}{\Lambda}\right]=\left[\Lambda\frac{d}{d\Lambda}j_0\right]_{\{j\},\kappa} \ , \quad j=v,w, \text{ or } z \ . \tag{B4}$$

The bare dimensional coupling constants are now $\lambda_j=j_0\Lambda^{\epsilon/2}$. $j_0=v_0$, w_0, z_0, and κ is a renormalization point. Here $\tilde m^2$ is the noncritical bare mass or transverse-inverse susceptibility at criticality for the longitudinal components,

$$\delta_{pq}\tilde m^2=\Gamma_{pq}^{(2)}, \ (k=0) \ . \tag{B5}$$

The field renormalization constant $Z_\phi^{(2)}$ is introduced in Eq. (3.17b) to ensure that $\Gamma_{Rpq}^{(2)}-\tilde m^2\delta_{pq}$ remains finite in the limit $\Lambda\to\infty$, then for the transverse two-point vertex it is $\Gamma_{pq}^{(2)}-\tilde m^2\delta_{pq}$ that satisfies the RG differential equation (B2).

The coefficient functions $\gamma_\phi^{(r)}$ and β_j can be directly evaluated from the partial differential equation (B2) and the explicit expression for $\Gamma^{(N)}_{i_1,\ldots,i_N}(\{j_0\},\tilde{m}/\Lambda,\kappa/\Lambda)$ obtained to leading order from the perturbation expansions of Figs. 1–4. Momentum-space integrations are now done at $d=6$ with a sharp cutoff Λ and the contribution to the mass term $\Sigma_{1(2)}(k=0)$ should be explicitly subtracted in Eq. (3.1b).

The essential result we obtained here is that the asymptotic leading contributions to every diagram in a given bare-vertex function is the same if the diagram contains at least one transverse propagator. The integrals for diagrams *a*, *d* and *b*, *e* of Fig. 1 are as in Eqs. (3.5a) and (3.5b), respectively, with a subtracted mass term and $\kappa=1$. The asymptotic behavior in both cases is given by

$$[A_1(k)-A_1(0)]_{\text{asymp}}\simeq I(k)\ ,$$
$$[A_2(k)-A_2(0)]_{\text{asymp}}\simeq I(k)\ , \tag{B6}$$
$$I(k)=-k^2\frac{1}{6}\left[\frac{11}{6}+\ln\frac{1+\tilde{\mu}^2}{\tilde{\mu}^2}\right],$$

while we obtain for diagram *c* with all longitudinal propagators in Fig. 1,

$$[A_0(k)-A_0(0)]_{\text{asymp}}\simeq I_0(k)=-k^2\frac{1}{6}\left[\frac{11}{6}+\ln\frac{\Lambda^2}{k^2}\right], \tag{B7}$$

where now

$$\tilde{\mu}=\frac{\tilde{m}}{\Lambda}\ . \tag{B8}$$

Equations (B6)–(B8) are the counterpart in the bare theory of Eqs. (3.7)–(3.9) in the renormalized theory. Similarly, we find

$$[L]_{\text{asymp}}\simeq\frac{1}{2}\ln\frac{1+\tilde{\mu}^2}{\tilde{\mu}^2}\ , \tag{B9}$$

for the asymptotic behavior of the integrals with three internal propagators in diagrams $a-c$ and $e-g$ in Fig. 2, all diagrams in Fig. 3, and diagrams $b-d$ in Fig. 4, while

$$[L_0]_{\text{asymp}}\simeq\frac{1}{2}\ln\frac{\Lambda^2}{\kappa^2} \tag{B10}$$

for the integral of diagrams *d* and *a* in Fig. 2 and Fig. 4, respectively. In Eq. (B10) κ is an arbitrary renormalization point and the coefficient functions are independent of κ. Equations (B9) and (B10) should also be compared with Eqs. (3.10) and (3.11) in the renormalized theory.

It is now straightforward to prove by direct evaluation that the coefficient functions $\gamma_\phi^{(1)}$, $\gamma_\phi^{(2)}$, and β_j, $j=v$, w, or z, are expressed in terms of the parameters v_0, w_0, z_0, and $\tilde{\mu}$ of the bare theory by exactly the same functional relations that give the Wilson functions in terms of the renormalized parameters in Eqs. (4.3)–(4.5) and (4.10).

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