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Critical pair-breaking current in superconducting aluminum strips far below T_c

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Critical currents of narrow, thin aluminum strips have been measured as a function of temperature. For the smallest samples uniformity of the current density is obtained over a large temperature range. Hence the intrinsic limit on the current-carrying capacity of the superconductor was measured outside the Ginzburg-Landau regime. The experimental values are compared with recent theoretical predictions by Kupriyanov and Lukichev. An approximate method of solving their equations is given, the results of which agree with the exact solution to within 1%. Experimental data are in excellent agreement with theoretical predictions. The absolute values agree if one assumes a ρl value of $4\times 10^{-16}\ \Omega\,\mathrm{m}^2$ with $v_F = 1.3\times 10^6$ m/s. This value for ρl is the same as that found from measurements of the anomalous skin effect but differs from values extracted from size-effect-limited resistivity.

I. INTRODUCTION

The ultimate limit on the current-carrying capacity of a superconductor is determined by the pair-breaking mechanism. When the superfluid velocity reaches a value where a further increase leads to a rapid reduction of the number of pairs, the superconducting state collapses and a voltage is measured. It is generally accepted that close to T_c the critical value is given by the Ginzburg-Landau result with parameters derived from the Gorkov theory. Various experiments on tin samples have shown that the temperature dependence and absolute value are in good agreement with theoretical predictions.[1,2] At lower temperatures the validity of the Ginzburg-Landau equations breaks down, and more complex theories are required to describe the superconductor in the presence of gradients and fields. A particular useful result has been obtained recently by Kupriyanov and Lukichev,[3] who solved the Eilenberger equations numerically for the current-carrying state.

Experimentally, little is known about the critical current density at low temperatures. The difficulty lies in the fact that it has to be determined in samples with cross-section dimensions that are small compared to the characteristic lengths of the superconductor. First, if the sample is wider than the electromagnetic penetration depth, the current piles up at the edges and the current distribution is nonuniform. For a thin-film sample the relevant penetration depth is that for a magnetic field perpendicular to the film $\lambda_\perp$, which for films much thinner than the bulk penetration depth λ equals λ^2/d, where d is the thickness of the film. For a dirty superconductor at low temperatures λ is about $\lambda_L(0)\sqrt{\xi_0/l}$ with $\lambda_L(0)$ the London penetration depth, ξ_0 the Bardeen-Cooper-Schrieffer (BCS) coherence length, and l the mean free path for elastic scattering. As a typical example for aluminum one finds for a film of thickness $d=0.1\ \mu$m, $l=0.1\ \mu$m, $\xi_0=1.6\ \mu$m, and $\lambda_L(0)=16$ nm that $\lambda_\perp=40$ nm. Clearly one has to go to rather thin and narrow films to prevent nonuniformity of the current at low temperatures. This difficulty does not exist close to T_c since the penetration depth increases with increasing temperatures.

Secondly, the width must be compared with the coherence length. If the width is larger than the coherence length, vortex nucleation and vortex flow can be induced at high current densities. Likharev[4] predicts that ideally, vortex flow will occur at about $\frac{1}{2}$ of the pair-breaking current. A minimum width is found of 4.4ξ below which no vortices can exist. At low temperatures we expect $\xi\sim\sqrt{\xi_0 l}$, which for the same numbers given above equals 0.4 μm. Hence with a careful choice of thickness and using aluminum with its large BCS coherence length, an experimentally accessible range for meaningful measurements of the critical pair-breaking current appears to be available.

This paper, which extends earlier work,[5] reports on an investigation of critical currents in aluminum films in which a uniform current density was obtained at temperatures well below T_c. The result will be compared with calculations based on the Eilenberger equations following the approach taken

TABLE I. Normal state properties of the samples.

Sample	R_{295} (Ω)	$R_{4.2}$ (Ω)	L (mm)	W (μm)	d (μm)	$\mathscr{R}$
1	628.7	137.5	1.00	0.61	0.089	4.57
2	492.2	93.3	1.00	0.68	0.098	5.28
3	377.2	70.8	1.00	0.88	0.099	5.33
4	502.4	184.1	0.20	0.50	0.034	2.73
5	782.6	384.2	0.10	0.30	0.020	2.04
6	47.7	2.5	3.00	0.35/2.5	1.24	19.2

in Ref. 3. Some attention is paid to a simplified numerical calculation.

II. EXPERIMENTS

The samples were made by cutting a long, narrow filament out of a metal film. Deposition of the film was performed by evaporation of aluminum from an electron gun onto a room-temperature glass substrate at a rate of about 30 Å/s in a vacuum of 10^{-6} Torr. Thickness was determined by a quartz crystal oscillator. If necessary, a more accurate value was determined from resistance measurements, which turned out to agree with optical interference results. For cutting, a diamond tool was used mounted in a well-balanced and accurate scratching-apparatus[6] to perform the final step in the fabrication process. This apparatus enables one to limit the variations in the width to less than 0.1 μm. Optical and electron microscopic inspection reveals a well-defined edge. In Table I the data of six samples are shown. As indicated, strips as narrow as 0.3 μm could be made by this simple technique. One strip (no. 6) was made rather thick. The shape of the diamond tool leads to a trapezoidal cross section which is of importance in thick films.

The measurements were performed in a ^{3}He cryostat. The sample, mounted in a lead-plated vacuum can, was connected with the ^{3}He chamber via a thermal resistance. The temperature was raised by resistive heating of the copper block on which the sample was mounted. Temperature control provided a stability of better than 1 mK. At lower temperatures, the additional heat generated by a constant measuring current led to a significant rise in temperature of the sample. Therefore we recorded the voltage response to a one-period current ramp (60 ms) fast enough to prevent any significant rise in temperature. Improved sensitivity was obtained by using a signal averager. At the critical current a steep rise in voltage was observed providing a clear-cut indication of the critical value. With a voltage resolution of 1 μV, no precursor of voltage could be observed before the discontinuous rise in voltage occurred.

In Fig. 1 the critical current data are plotted as a function of temperature. Close to T_c the classical Ginzburg-Landau result is found:

$$j_c = j_c(0)\left[1-\frac{T}{T_c}\right]^{3/2} .$$

All data are normalized to the coefficient $j_c(0)$. Since thickness and width are not accurately known the measured currents are converted into values for the current density by using the expression

$$j = \frac{IR_{4.2}}{L\rho_{\rm ph}}(\mathscr{R}-1) ,$$

where the cross section has been expressed in the normal-state resistance and the resistivity at 4.2 K is determined from the measured resistance ratio $\mathscr{R}=R_{295}/R_{4.2}$. Obviously this method fails when width and/or thickness are nonuniform. $\rho_{\rm ph}$ is the

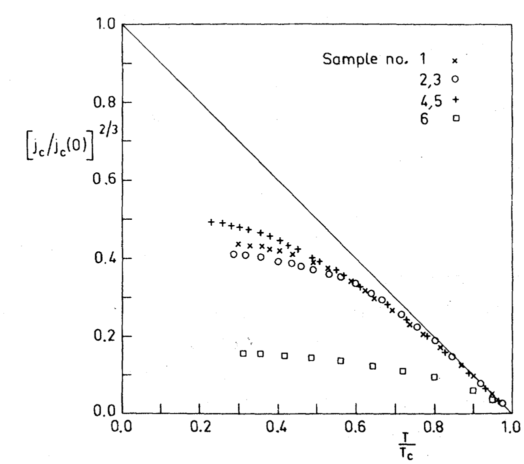

FIG. 1. Experimental results for the critical current in reduced units as a function of temperature for different samples.

TABLE II. Superconducting properties.

Sample	T_c (K)	l (nm)	ξ_0 (μm)	l/ξ_0	$\sqrt{\xi_0 l}$ (μm)	$\lambda_\perp$ (μm)	$j_c(0)$ (GA/m^2) Expt.	Theor.
1	1.196	53.5	1.50	0.035	0.28	0.081	153	159
2	1.203	64.1	1.49	0.043	0.31	0.061	130	175
3	1.203	64.8	1.49	0.043	0.31	0.059	156	172
4	1.267	25.9	1.41	0.018	0.19	0.41	111	120
5	1.356	15.5	1.32	0.012	0.14	1.1	107	103
6	1.154	272	1.55	0.175	0.65	0.0012	257	299

phonon-limited resistivity at room temperature of 2.67×10^{-8} Ω m.[7] For clarity, in Fig. 1 some of the data are taken together. Samples 2 and 3, fabricated on the same substrate, were almost identical. As a result the critical current data could not be distinguished from each other. Samples 4 and 5, although having different parameters, show practically the same critical current data. At lower temperatures most of the samples follow a universal curve down to $T/T_c=0.6$. The only strong deviation occurs in sample 6, which was exceptionally thick.

In Table II specific data about the samples are summarized. The critical temperature increases, as usual for aluminum thin films, with decreasing thickness. The mean free path for elastic scattering is determined from the resistance ratio $R_{295}/R_{4.2}$, the phonon-limited resistivity, and an appropriate value for the ρl product of aluminum. For reasons given below we have used $\rho l = 4\times10^{-16}$ $\Omega\,\mathrm{m}^2$. Using the BCS value for $\xi_0=\hbar v_F/\pi\Delta(0)$ with $v_F=1.3\times10^6$ m/s and $\Delta(0)=1.76k_BT_c$, one finds that all samples are in the dirty limit $l \ll \xi_0$ which is inevitable with thin films. In Table II we have also given typical values for the coherence length and penetration depth at $T=0$ K [with $\lambda_L(0)=16$ nm]. As one observes, only the thinnest samples approach the required limits for uniform current density and exclusion of vortex formation.

III. THEORY

The critical pair-breaking current at arbitrary temperatures has been calculated by a number of authors. For arbitrary mean free paths one must rely on the Gorkov equations or their simplified version, the Eilenberger equations. In the following, we will recall briefly the calculation given by Kupriyanov and Lukichev,[3] which is based on the Eilenberger equations and, to our knowledge, provides the most general results presented so far. Other results are obtained by Bardeen,[8] Maki,[9] and Ovchinnikov.[10]

The Eilenberger equations involve the functions f and g, which are closely related to the anomalous and normal Green's functions of a superconductor. For a current-carrying state in a one-dimensional situation where the vector potential can be neglected, solutions can be looked for in the form

$$\tilde{f}=fe^{iux},\quad \tilde{\Delta}=\Delta e^{iux} .$$

$\tilde{\Delta}$ is the gap parameter and x is the coordinate along the current. u is the phase gradient, proportional to the velocity of the superconducting condensate. The values of f and Δ have to satisfy the following set of equations:

$$(2\hbar\omega+i\hbar v_F u\cos\nu)f = 2\Delta g+\frac{\hbar}{\tau}(g\langle f\rangle - f\langle g\rangle), \quad (1)$$

$$g=(1-f^2)^{1/2} , \quad (2)$$

$$\Delta\ln\frac{T}{T_c}+2\pi k_B T\sum_{\omega>0}\left[\frac{\Delta}{\hbar\omega}-\langle f\rangle\right]=0 . \quad (3)$$

Here $\hbar\omega=(2n+1)\pi k_BT$ are the Matsubara frequencies and v_F is the modulus of the Fermi velocity. It is assumed that scattering by impurities can be approximated by a relaxation-time model with τ the elastic scattering time. ν is the angle between the direction of $\vec{v}_F$ and the direction of the supercurrent. The angular brackets around f and g indicate averaging over all velocity directions of the vector $\vec{v}_F$.

For a given value of u this system of equations has a solution for Δ, $\langle f\rangle$, and $\langle g\rangle$ at each temperature T. The supercurrent carried at these values is given by

$$j=-4ieN(0)\pi k_B T v_F\sum_{\omega>0}\langle g\cos\nu\rangle , \quad (4)$$

where $N(0)$ is the single spin density of states per unit volume at the Fermi surface. A solution of this set of equations involves calculation of f and g and their averaged values. As shown by Kupriyanov and Lukichev a major simplification is obtained by introducing the function Φ with the substitution

$$f=\frac{\Phi}{[(\hbar\omega)^2+\Phi^2]^{1/2}}\ ,\quad g=\frac{\hbar\omega}{[(\hbar\omega)^2+\Phi^2]^{1/2}}\ . \tag{5}$$

Then f and g can be eliminated, and a new set of equations is found that involves only $\langle f\rangle$ and $\langle g\rangle$. For convenience we define $X=\langle g\rangle$ and $Y=\Delta/\hbar+\langle f\rangle/2\tau$, which leads to

$$\begin{aligned} Y&=\frac{v_F u}{2Z}(1-X^2)(1+Z^2)^{1/2}\ ,\\ Y&=\frac{\Delta}{\hbar}[1-(v_F u\tau)^{-1}\arctan Z]^{-1}\ , \end{aligned} \tag{6}$$

where

$$Z=v_F u\tau\left[1+\frac{2\omega\tau}{X}\right]^{-1}\ .$$

Supplemented with the gap equation and the expression for the current

$$\Delta\ln\frac{T}{T_c}+2\pi k_B T\sum_{\omega>0}\left[\frac{\Delta}{\hbar\omega}-\frac{2Y}{v_F u}\arctan Z\right]=0\ , \tag{7}$$

$$\begin{aligned} j=&\ 8\pi e N(0)v_F k_B T\\ &\times\sum_{\omega>0}\left[\frac{Y}{v_F u}\right]^2[\arctan Z-(Z+Z^{-1})^{-1}]\ , \end{aligned} \tag{8}$$

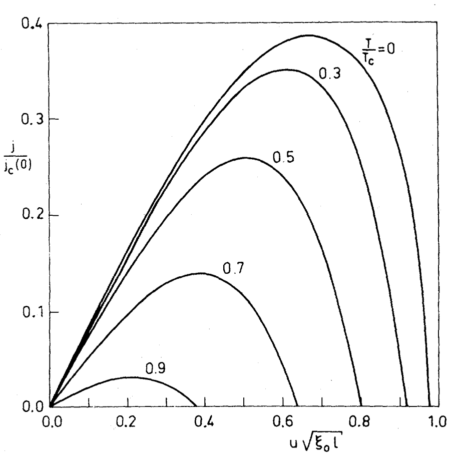

FIG. 2. Supercurrent as a function of phase gradient for dirty superconductors as given by the microscopic theory at various temperatures. $j_c(0)$ is defined by Eq. (10).

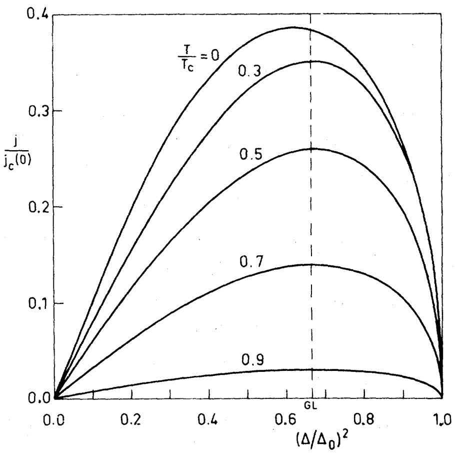

FIG. 3. Theoretical dependence of the supercurrent on the order parameter at various temperatures. Δ_0 is the value of the order parameter for $u=0$. GL indicates the critical value of Ginzburg-Landau theory.

Eqs. (6)–(8) constitute the final result of Ref. 3, which allows a straightforward numerical calculation of j and Δ as a function of u and T. [Note the misprint in the first line of Eq. (7) of Ref. 3 to be compared with our Eq. (6)]. A numerical computation runs as follows: At fixed temperature T one chooses values for u and Δ and computes for each ω value the quantities X, Y, and Z from Eq. (6). A new value for Δ is then derived from Eq. (7), whereupon the calculation is repeated. This iteration continues until self-consistency in Eq. (7) is achieved. In Fig. 2 the current density as a function of the phase gradient u is shown for parameters of sample 5. For the same parameters Fig. 3 shows the current density as a function of the order

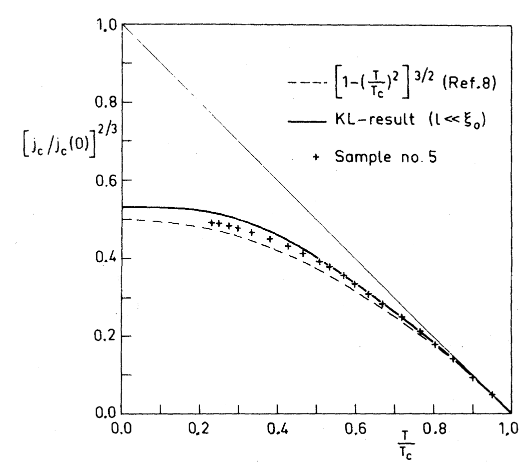

FIG. 4. Experimental results of sample 5 compared to theoretical predictions. Thick solid and dashed lines are predictions of Refs. 3 and 8, respectively.

parameter. No dramatic changes of behavior compared to the Ginzburg-Landau result are observed. The critical current density is given by the maximum shown in both figures. Over the whole temperature range the critical current has been determined and given in Fig. 4 as a thick solid line. For different mean free paths, compared to the BCS coherence length, small differences are predicted. However, the experimental values for l/ξ_0 are so close to each other that different theoretical curves are indistinguishable on this scale. For very short mean free paths the computation time for the determination of the critical value can be reduced considerably by using an analytic approximation of Eqs. (6)–(8) (Appendix).

An important scaling parameter is $j_c(0)$ which is the critical current that is found experimentally by extrapolating the result found close to T_c to $T=0$. Theory predicts that

$$j_c(0)=\frac{16\pi^2\sqrt{2\pi}}{63\zeta(3)}eN(0)v_F k_B T_c\left[\frac{k_B T_c l}{\hbar v_F}\right]^{1/2} \tag{9}$$

for a superconductor in the dirty limit. It is advantageous to rewrite Eq. (9) in such a way that readily available experimental quantities can be inserted. By using the free-electron result

$$\sigma=\rho^{-1}=[2e^2 v_F N(0)l]/3\ ,$$

one finds

$$j_c(0)=\frac{8\pi^2\sqrt{2\pi}}{21\zeta(3)e}\left[\frac{(k_B T_c)^3}{\hbar v_F\rho(\rho l)}\right]^{1/2} . \tag{10}$$

This expression can be used for a quantitative comparison with experiments. The only quantity that is poorly known is the ρl product. In the last column of Table II theoretical values for $j_c(0)$ are given for $\rho l=4\times10^{-16}\ \Omega\,\mathrm{m}^2$.

IV. DISCUSSION OF THE RESULTS

From Fig. 4 one observes that the agreement between KL theory and the critical current of the smallest samples (Nos. 4 and 5) is satisfactory. A slight deviation of only a few percent remains at the lowest temperatures. Comparison of the various parameters given in Table II reveals that the different samples have approximately the same coherence length. They differ strongly with respect to their electromagnetic penetration depth. The best agreement is obtained for samples with penetration depths comparable to the width of the strips. Increasing $\lambda_\perp$ by a factor of 3 (sample 5 relative to 4) did not lead to a further increase of reduced critical current. Therefore we believe that we have reached the limit of a homogeneous current density down to relatively low temperatures. Samples 1–3 appear to develop some nonuniformity at the lower temperatures, leading to a reduction in the critical current as discussed in Refs. 1 and 2. All samples studied were small compared to the Likharev criterion for vortex formation.

It is interesting to compare the results also with the phenomenological expression given by Bardeen.[8] For a dirty superconductor he suggests

$$j_c=\frac{j_c(0)}{2\sqrt{2}}\left[1-\left[\frac{T}{T_c}\right]^2\right]^{3/2}, \tag{11}$$

where the proportionality constant is chosen to reproduce the Ginzburg-Landau result close to T_c. This dependence is shown in Fig. 4 as a dashed line. Although reasonably good as a first estimate, this phenomenological result is clearly inferior to the KL theory in fitting the data.

A deviation of a few percent between the measurements and KL theory remains at the lowest temperatures. No improvement was observed when $\lambda_\perp$ was increased. We believe that the main cause of these deviations is a fluctuation in width of the samples. On a length scale of a few micrometers or less a variation in width of about 10% on a total width of 0.3 μm occurs easily. At higher temperatures where the coherence length is longer than the length of such a variation no effect will be observed and an average width can be assumed. At lower temperature, however, the critical current will be determined by the minimum width. In other words, at lower temperatures the effective cross section will be smaller than close to T_c, leading to a smaller critical current.

As shown by Skocpol[2] and Andratskii *et al.*[1] the absolute values of the critical current of tin samples close to T_c are in excellent agreement with theoretical predictions. A similar quantitative comparison has not yet been made for aluminum. We find that the absolute values agree very well with the predictions of Eq. (10) assuming that $v_F=1.3\times10^6$ m/s and ρl is about $4\times10^{-16}\ \Omega\,\mathrm{m}^2$. It is interesting to compare the latter value with those found from other sources. Fickett[7] discusses a number of ρl values extracted from measurements on size-effect-limited resistivity. Although differences of a factor of 4 have been found, for polycrystalline samples at low temperatures a value of $9\times10^{-16}\ \Omega\,\mathrm{m}^2$ is recommended. On the other hand, from anomalous skin-effect studies on aluminum Fawcett[11] finds more or less the same values as indicated by the present experiments. Obviously, characterizing a metal with a complicated Fermi surface with a single parameter derived from a free-electron model is an oversimplification. Nevertheless, in many experiments the ρl product serves as a useful guide for quantitative evaluation, for example, in nonequilibrium superconductivity. The agreement between this result and that of the anomalous skin effect, combined with the uncertainties in the results given by Fickett,[7] suggests that one preferably should use $\rho l \simeq 4\times 10^{-16}\ \Omega\,\mathrm{m}^2$ for aluminum.

In conclusion, we have found that a theory based on the Eilenberger equations provides an accurate prediction for the critical pair-breaking current at low temperatures in dirty superconductors. In performing such measurements careful attention must be paid to the uniformity of the current density. This limitation prevented us from measuring pair-breaking currents in superconductors with arbitrary mean free path.

ACKNOWLEDGMENT

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APPENDIX

Analytic expressions for the critical current can be found from Eqs. (6)–(8) only for $T \simeq T_c$ with arbitrary value for l/ξ_0, and for $T \simeq 0$ K in the clean and dirty limit (cf. Ref. 3). Intermediate cases are to be dealt with numerically. However, because of the small ratios l/ξ_0 realized in the present experiment, the equations can be simplified substantially. It will be shown that the results are identical with those obtained by Maki.[9] Subsequently, an approximation will be derived from these simplified equations that allows determination of the critical current in closed form over the entire temperature range.

From the analytic expressions valid near $T=0$ K and near $T=T_c$ it follows that in the dirty limit at the critical current the phase gradient u is proportional to $\tau^{-1/2}$. This means that even at the critical current

$$Z = v_F u \tau X (X+2\omega\tau)^{-1}$$

approaches zero when τ goes to zero. Expanding arctanZ, which occurs in Eqs. (6)–(8) up to third order, we get

$$\begin{aligned} Y &\simeq \frac{\Delta(X+2\omega\tau)}{2\hbar\omega\tau}\left[1+\frac{(v_F u\tau)^2 X}{6\omega\tau}\right]^{-1}, \\ 1-X^2 &\simeq \left[\frac{2YZ}{v_F u}\right]^2 \\ &\simeq \left[\frac{\Delta}{\hbar\omega}\right]^2 X^2 \left[1+\frac{(v_F u\tau)^2 X}{6\omega\tau}\right]^{-2}. \end{aligned} \tag{A1}$$

Substitution of these results in the expressions for the gap (for which the approximation $\arctan Z \simeq Z$ is sufficient) and the current, Eqs. (7) and (8), yields

$$\Delta \ln\frac{T}{T_c} \simeq 2\pi k_B T \sum_{\omega>0}\left[(1-X^2)^{1/2}-\frac{\Delta}{\hbar\omega}\right],$$

$$j \simeq \frac{4\pi}{3} e N(0) k_B T v_F^2 u\tau \sum_{\omega>0}(1-X^2).$$

These equations are exact in the dirty limit $\tau\to 0$. By means of the transformation $1-X^2=(1+u_n^2)^{-1}$ Maki's equations (17)–(19) (Ref. 10) are recovered (allowing for a factor of 2 in the definition of τ, as already noticed in Ref. 3).

The numerical evaluation of the full set of coupled nonlinear equations (6)–(8) requires as input parameter a value for u; to find the critical current one must repeat the computation for various values of this parameter. Therefore, it is profitable to choose these values as close as possible to the critical value u_c. In the following we develop a method to obtain an approximation for u_c. It turns out that this approximation provides to within 1% an accurate value for the critical current.

We consider the solution X of (A1) as a function of Δ and

$$\epsilon \equiv (v_F u)^2 \hbar\tau / 6 k_B T_c .$$

The connection between ϵ and the parameter ζ occurring in Ref. 3 is $\epsilon=(\Delta/k_B T_c)\zeta$. For u fixed, $\epsilon\to 0$ when $\tau\to 0$; however, when u assumes the critical value u_c, ϵ approaches a temperature-dependent constant when $\tau\to 0$. From the analytical results in the two temperature limits we know that this constant is small: 0.418 for $T=0$ K and 0 for $T=T_c$. Hence we expand X to first order in ϵ to find

$$X^2 \simeq X_0^2(1+a\epsilon) \tag{A2}$$

with

$$a = \frac{2k_B T_c}{\hbar\omega}\left[\frac{\Delta}{\hbar\omega}\right]^2 X_0^3$$

and

$$X_0^2=[1+(\Delta/\hbar\omega)^2]^{-1}\ .$$

Similarly expanding the equations for the gap and for the current to first order in ϵ, we get

$$\Delta \ln\frac{T}{T_c}\simeq 2\pi k_B T\left[\sum_{\omega>0}\frac{\Delta}{\hbar\omega}(X_0-1)\ -\epsilon\sum_{\omega>0}\frac{\Delta k_B T_c}{(\hbar\omega)^2}X_0^4\right],$$

$$j\simeq\frac{4\pi}{3}eN(0)k_B T v_F^2 u\tau\left[\sum_{\omega>0}\left[\frac{\Delta}{\hbar\omega}\right]^2 X_0^2\ -2\epsilon\sum_{\omega>0}\frac{k_B T_c}{\hbar\omega}\left[\frac{\Delta}{\hbar\omega}\right]^2 X_0^5\right].$$

It should be noticed that besides the explicitly shown dependence on ϵ in (A2) X also depends on ϵ via Δ and the gap equation. In the limits $T\to 0$ K and $T\to T_c$ these approximations still give the correct analytical expressions pertinent to the dirty limit.

Let us now introduce scaled variables $\delta\equiv\Delta/\pi k_B T$ and $t\equiv T/T_c$, and the functions

$$\varphi_1(\delta)\equiv\sum_{n=0}^{\infty}\left[\frac{1}{2n+1}-\frac{1}{[(2n+1)^2+\delta^2]^{1/2}}\right],\quad \varphi_2(\delta)\equiv\sum_{n=0}^{\infty}\frac{(2n+1)^2}{[(2n+1)^2+\delta^2]^2}\ ,$$

$$\varphi_3(\delta)\equiv\delta^2\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2+\delta^2},\quad \varphi_4(\delta)\equiv\delta^2\sum_{n=0}^{\infty}\frac{(2n+1)^2}{[(2n+1)^2+\delta^2]^{5/2}}\ ,$$

where the summation over ω is reexpressed in summation over n. In terms of these quantities the equations for the gap and the current read

$$t\ln t=-2t\varphi_1(\delta)-\frac{2\epsilon}{\pi}\varphi_2(\delta),\quad j=Cu\left[t\varphi_3(\delta)-\frac{2\epsilon}{\pi}\varphi_4(\delta)\right], \tag{A3}$$

where

$$C\equiv\frac{4\pi}{3}eN(0)k_B T_c v_F^2\tau\ .$$

From these equations we are able to determine the derivative of j with respect to u at constant temperature:

$$\left[\frac{dj}{du}\right]_t=\left[\frac{\partial j}{\partial u}\right]_{t,\delta}+\left[\frac{\partial j}{\partial\delta}\right]_{t,u}\left[\frac{d\delta}{du}\right]_t\ .$$

Setting $(dj/du)_t$ equal to zero leads to a quadratic equation in ϵ with the solution $[\epsilon_c\equiv\epsilon(u_c)]$

$$\frac{2}{\pi}\epsilon_c=tf(\delta)\ ,$$
$$f(\delta)\equiv\frac{\varphi_2'\varphi_3-2\varphi_2\varphi_3'-6\varphi_1'\varphi_4+[(\varphi_2'\varphi_3-2\varphi_2\varphi_3'-6\varphi_1'\varphi_4)^2+8\varphi_1'\varphi_3(3\varphi_2'\varphi_4-2\varphi_2\varphi_4')]^{1/2}}{2(3\varphi_2'\varphi_4-2\varphi_2\varphi_4')}\ . \tag{A4}$$

The prime indicates differentiation with respect to δ; the sign in front of the square root has been chosen so as to reproduce the correct result when $T\to T_c$. Substitution of this result for ϵ in (A3) then yields expressions for the temperature and critical current at the given value of δ:

$$T\equiv T_c t=T_c\exp[-(2\varphi_1+\varphi_2 f)]\ ,$$

$$\frac{j_c(t)}{j_c(0)}=\frac{21\zeta(3)}{4\pi}\left[\frac{3}{2}\right]^{1/2}t\sqrt{tf}\,(\varphi_3-\varphi_4 f)$$

with $j_c(0)$ as in Eq. (9).

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