Brief Reports

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Possibility of an ideal conductor at finite temperatures

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A conventional-size conductor with an extraordinary high resistance along one of the axes (z) may have at reasonable temperatures a vanishingly small resistance ρ_{xx}, ρ_{yy} in a magnetic field along z when the Fermi energy corresponds to a localized state.

Usually experimenters have aimed at high-temperature superconductors. However, one may also consider the possibility of a finite-temperature ideal conductor,[1] i.e., a zero resistance. Meanwhile, such a conductor, for all practical purposes except for the Meissner effect, would be indistinguishable from a superconductor. In fact, an ideal conductivity may have already been observed experimentally in quantized Hall-effect experiments. The quantized Hall effect is recognized as a method for the determination of the fine-structure constant[2] and for a resistance standard.[3,4] But the quantized Hall effect provides also[5] a zero resistance ρ (more accurately, an exponentially small resistance at low enough temperatures). The observed value[5(b)] of ρ was less than the experimental error and less than the resistance of any known nonsuperconductor. It would be very interesting to measure the actual value of ρ, and therefore[6] of the fine-structure constant determination, by the decay time of a dc current in a Corbino disk. But the conditions for an ideal conductivity are more general than those of the quantized Hall effect, and may be met for a conventional size of a conductor at reasonable temperatures, and magnetic fields, if: (i) A conductor is very anisotropic and has an extremely high resistance ρ_{zz} along one direction (say,z). More specifically, the energy change $\Delta\epsilon$, related to fixed quasimomentum projections p_x, p_y, and changing p_z, is small compared to the total bandwidth[6(b)] [and thus the fixed energy surfaces $\epsilon(\rho) = \text{const}$ of almost two-dimensional charges are close to cylinders, in a general case noncircular]. An example of such a material may be C_8AsF_5. (ii) A magnetic field $B = B_z$ is strong enough to provide the cyclotron frequency $\omega_c > \Delta\epsilon/\hbar$, k_BT (k_B is the Boltzmann constant, T is the temperature). (iii) The Fermi energy ϵ_F lies within the localized states and its distance $\delta\epsilon$ from the nearest extended state is $\delta\epsilon > k_bT$ (see Fig. 1).

If an electron effective mass is small, then $\Delta\epsilon$ and the temperature may be high (e.g., if $m^* \sim 10^{-29}$ g, then $B = 30$ kG allows for T, $\Delta\epsilon/k_B < 300$ K, while if $m^* \sim 10^{-27}$ g, then the same B implies T, $\Delta\epsilon/k_B < 3$ K).

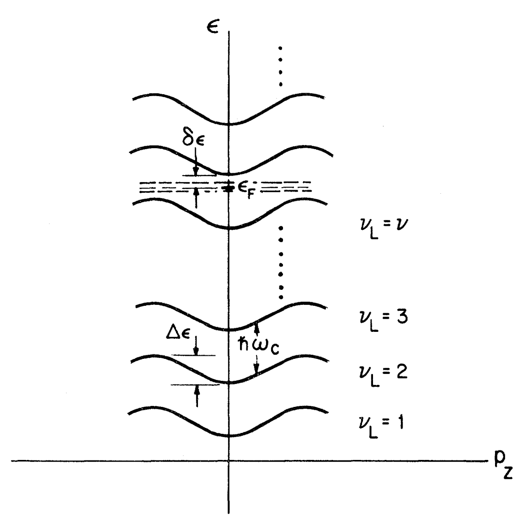

FIG. 1. Energy spectrum of a charged quasiparticle in an ideal conductor. (Such a spectrum is provided, e.g., by the dispersion law

$$\epsilon = D\cos\alpha_x + D\cos\alpha_y + \Delta\epsilon\cos\beta \ ;$$

in the absence of a magnetic field $\vec{\alpha} = \vec{p}a/\hbar$; $\beta = p_z b/\hbar$; $\Delta\epsilon << D$.) The ordinal number of a quantized extended level is ν_L; their total number is $\nu \sim \epsilon_F/\hbar\omega_c$; the number of localized states per unit volume is $\gtrsim N_t\hbar\omega_c/\epsilon_F$; N_t is the total charge density. A very weak (Ref. 7) dependence of ϵ on the y position of a classical orbit center is neglected. Note that all presented states are electron ones, since their $\omega_c \equiv \omega_c(\nu_L, p_z) > 0$.

When $\delta\epsilon >> k_B T$, the situation is similar to that of the quantized Hall effect: Extended states below ϵ_F are completely (with the exponential accuracy) filled, and thus have the exponentially small dissipation and exponentially large mean-free-path time τ, whereas localized states do not contribute into the current.

Thus, any conventional conductor of arbitrary dimensions with the "bandwidth along B" $\Delta\epsilon >> k_B T$ yields the ideal conductor conditions of the quantized Hall effect when $\hbar\omega_c > \Delta\epsilon > \delta\epsilon >> k_B T$. A highly anisotropic material with a small effective mass m^* (in the xy plane) may meet these conditions at reasonable temperatures and magnetic fields.

Since one knows how to evaluate m^* and $\Delta\epsilon$ for a given material, it may be an easier task to create a high-temperature quasi-ideal conductor than a high-temperature superconductor.

Electric and magnetic fields and a current must be orthogonal, so the topology requires to use a Corbino disk or to complement an ideal conductor by a common conductor or superconductor in a small region. A finite junction resistance is immaterial in a very long conductor.

Diamagnetic properties of a quasi-ideal conductor are also considered. The calculation of the current j is conventional—see, e.g., Ref. 7. An interesting point is its straightforward applicability to free electrons in an arbitrary (in particular, random) potential, independent of the coordinate along $E \times B$, and thus very special (cf. Refs. 9 and 10).

A strong magnetic field $\vec{B} \parallel z$, in both classical and quantum cases, provides[7] the conductivities $\sigma_{xx} \sim \sigma_{yy} \sim (\omega_c^2 \tau)^{-1}$ and the resistivities $\rho_{xx} \sim \rho_{yy} \propto \tau^{-1}$, if the difference N between electron and hole densities is $N \neq 0$. When $\tau \rightarrow \infty$, then conductivities and resistivities *simultaneously* tend to zero. Physically this is not surprising. In the absence of the dissipation, orthogonal electric E and magnetic fields provide the motion of all charges with the same average velocity $c\vec{E} \times \vec{B}/B^2$ (which is equivalent to the reference system moving with such a velocity), thus producing a current density

$$\vec{j} = eNc\vec{E} \times \vec{B}/B^2 , \tag{1}$$

and a nondissipative Hall current J_H related to the potential difference U_H:

$$J_H = eNcU_H/B \tag{2}$$

with the Hall constant[8] R_H,

$$R_H = B/(ceN) . \tag{3}$$

Since, by Eq. (1), $\vec{E}$, $\vec{B}$, and $\vec{j}$ are othogonal, a fixed $\vec{E}$ provides $j_E = 0$, i.e., the zero conductivity σ:

$$\sigma = j_E/E = 0 , \tag{4}$$

whereas a fixed $\vec{j}$ provides $E_j = 0$, i.e., the zero resistivity ρ:

$$\rho = E_j/j = 0 , \tag{5}$$

and thus an ideal conductor. [In Eqs. (4) and (5), $j_E/E = \vec{j} \cdot \vec{E}/E^2$; $E_j = \vec{E} \cdot \vec{j}/j^2$.]

All these results are probably independent of electron-electron interactions, crystal lattice imperfections, nonlinearity with E, experimental details, etc., and imply a resistance, exponentially decreasing with the temperature.

Now I present the solution for charged quasiparticles with an arbitrary dispersion law $\epsilon_0(p)$ in an electric field $E = E_y$ (with the electrostatic potential $\phi = -eEy$) and a magnetic field $B = B_z$ (with the vector potential in the Landau gauge $A = A_x = -By$). The crucial point of the further considerations is a completely nondissipative equilibrium (in given external fields) situation, with exact (i.e., nonsmeared by forbidden inelastic collisions) electron energy levels and equilibrium Fermi distribution function f_F. All considerations are immediately generalized to a Fermi-liquid interaction (which just makes the energy spectrum and the distribution function inter-related).

The current j equals

$$j = e\,\mathrm{Tr}[v f_F(H)] , \tag{6}$$

where v, e, and H are the electron velocity, charge, and Hamiltonian. The Hamiltonian, neglecting interband transitions, equals

$$H = \epsilon_0(p) + eEy - \mu_0 \sigma B ; \quad p = (\hbar/i)\nabla - eA/c ; \quad A = A_x = -B_y , \tag{7}$$

μ_0 is an electron magnetic moment; the energy $\epsilon = \epsilon_X \equiv \epsilon_{n\kappa}$ depends on an integer n (n represents the electron spin projection σ; the number of the band and the number of the diamagnetic Landau level) and continuous κ (in a periodic lattice, κ represents quasimomentum projections P_x, p_z; sample surfaces make some κ discrete). The surfaces make the motion along y finite, and thus, e.g.,

$$(v_y)_{XX} = (\dot{y})_{XX} = (i/\hbar)[H,y]_{XX} = 0 \tag{8}$$

(square brackets denote a commutator). So,

$$j_y = eh^{-2} \int d\kappa \sum_n (v_y)_{n\kappa} f_F(\epsilon_{n\kappa}) = 0 . \tag{9}$$

Similarly,

$$j_z = 0 . \tag{10}$$

To evaluate the Hall current j_x, note that, by Eq. (7),

$$\dot{p}_y = (i/\hbar)[H,p_y] = -(eB/c)(i/\hbar)[H,x] - eE = -(eB/c)v_x - eE , \tag{11}$$

and therefore

$$v_x = -(c\dot{p}_y/eB) - (cE/B) \ . \tag{12}$$

Thus,[9] according to Eqs. (6) and (11),

$$j_x = -(ecE/B)\mathrm{Tr} f_F = -NecE/B \ . \tag{13}$$

This reasoning is straightforwardly applicable to free electrons ($\epsilon_0 = p^2/2m$) in an arbitrary y-independent potential $U = U(x,z)$—cf. Ref. 10.

The calculations of the de Haas–van Alphen susceptibility χ (per unit volume) are similar to those of Ref. 11 and lead to

$$\chi \sim \nu^2 e^2 \Delta p_z / h m^* c^2 \tag{14}$$

(Δp_z is the period of p_z, $\nu \sim \epsilon_F/\hbar\omega_c$ is the number of occupied levels); χ oscillates with $1/B$ and may be of any sign. The value of the integer ν increases at certain values of B, in the region ΔB which corresponds to $\hbar\Delta\omega_c \sim k_B T$, and then provides the extra factor $\hbar\omega_c/k_B T$ for χ in Eq. (14). A large enough value of χ implies magnetic moment density wave—see Ref. 11 and references therein.

Any theoretical proof of a *zero* resistance is handicapped by its being based on a certain idealized model. However, the physics of the suggested ideal conductor is essentially the same as that of the quantized Hall effect. Recent quantized Hall-effect experiments[5(b)] on GaAs-$Al_xGa_{1-x}As$ heterojunctions indicate an incredible *lower* limit for an electron scattering time: $\tau > 1.5.10^{-3}$ sec, i.e., *at least* 5×10^8 times higher than τ at $B = 0$. These figures (which I learned after the paper was completed) are probably the most convincing demonstration of the possibility of the suggested ideal conductor.

In summary, the current in the quantized Hall effect is like that in an ideal conductor—to exponential accuracy, it is nondissipative. The conditions for quantized Hall effect are generalized from a "pure" two-dimensional case to a quasi-two-dimensional case (in the sense of the corresponding dispersion law) of a conductor, highly anisotropic in one direction.

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[1]At zero temperature Heeger *et al.* [D. K. Chakraborty, R. Spal, A. M. Denenstein, K.-B. Lee, A. J. Heeger, and M. Ya. Azbel, Phys. Rev. Lett. 43, 1832 (1979)] suggested a zero residual resistance in one-dimensional metals, and T. M. Rice [*Physics in One Dimension* (Springer-Verlag, Berlin, 1981), p. 229] proved such a possibility theoretically.

[2]K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. 45, 494 (1980).

[3]E. Braun, E. Staben, and K. von Klitzing, PTB Mitt. 90, 350 (1980).

[4]D. C. Tsui and A. C. Gossard, Appl. Phys. Lett. 38, 550 (1981).

[5](a) T. Ando, Y. Matsumoto, and Y. Uemura, J. Phys. Soc. Jpn. 39, 279 (1975); (b) D. C. Tsui, H. L. Stormer, and A. C. Gossard (unpublished).

[6](a) K. von Klitzing, in *Proceedings of the Fourth International Conference on Electronic Properties of Two-Dimensional Systems, New London, New Hampshire* (North-Holland, Amsterdam, 1982), p. 1. (b) In a strong-coupling case the characteristic relation is $(\rho_{xx}/\rho_{zz})^{1/2}$.

[7]I. M. Lifshitz, M. Ya. Azbel, and I. M. Kaganov, *Electron Theory of Metals* (Consultants Bureau, New York, 1973). If $E = E_l + E_t$, where $\vec{E}_l \parallel \vec{B}$, $\vec{E}_t \perp \vec{B}$, then $v = (eE_l t/m) + c\vec{E}_t \times \vec{B}/B^2 + \tilde{v}$ (v being an electron velocity, t is time) provides $m\dot{\tilde{v}} = (ec)\tilde{v} \times \vec{B}$ with the time average of the energy change being zero only when $E_l = 0$, i.e., $\vec{E} \perp \vec{B}$.

[8]A magnetic field $\vec{B} \parallel z$ does not change the number of states. Thus the quantization provides $dp_x\, dp_y/h^2 \equiv dS/h^2 \rightarrow \Delta S/h^2$, where ΔS is the momentum area quantization: $\Delta S = eBh/c$. So the number of states per level is eB/ch [see Ref. 7 and L. D. Landau and I. M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory,* 3rd ed. (Pergamon, New York, 1958)], and ν completely filled levels provide (Ref. 5) the quantized Hall constant $R_H = h/e^2\nu$.

[9]Different signs of electron and hole densities are related to ϵ increasing with the diamagnetic level number for electrons and decreasing with it for holes. In a more explicit classical case (Ref. 7),

$$j_x \propto \int v_x f_F(\epsilon_0 + e\phi)\, d\vec{p} \propto -E \int f_F(\epsilon)\, d\epsilon\, dp_z\, dt \ ,$$

where ϵ is the total energy; t is the time; the time period $2\pi/\omega_c \propto \partial S/\partial\epsilon$; the orbit area S increases with ϵ for electrons and decreases for holes. The fact that impurities do not affect Eq. (13) was accurately proven by R. E. Prange, Phys. Rev. B 23, 4802 (1981); R. E. Prange and R. Joynt (unpublished).

[10]H. Aoki and T. Ando, Solid State Commun. 38, 1079 (1981).

[11]M. Ya. Azbel, Zh. Eksp. Teor. Fiz. 53, 1751, 2131 (1967) [Sov. Phys. JETP 26, 1003, 1203 (1968)]; I. A. Privorotskii and M. Ya. Azbel, Zh. Eksp. Teor. Fiz. 56, 388 (1969) [Sov. Phys. JETP 29, 214 (1969)].