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Field, impurity, and other effects on cyclotron resonance in silicon inversion layers

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The density and field (frequency) dependences of the cyclotron effective mass of the electrons in Si(001) inversion layers measured recently by Wagner, Kennedy, McCombe, and Tsui are analyzed theoretically in consideration of finite thickness, plasmon, and impurity effects. When frequency is increased, the effective mass decreases several percent. Temperature, valley occupancy transition, and other effects on cyclotron resonance are discussed. The width function is evaluated explicitly as a function of impurity potentials.

I. INTRODUCTION

The carrier dependences of the effective mass and scattering time of electrons in inversion or accumulation layers of metal-oxide–semiconductor field-effect transistors (MOSFET's) have been determined effectively by cyclotron resonance or magnetoconductivity experiments.[1] In a recent paper,[2] hereafter called I, we treated the cyclotron resonance of a two-dimensional electron gas neglecting thickness and other effects. The analysis has supported the general view that the carrier dependences are largely due to many-body effects.[3] However, theoretical results based on an idealized model must be taken with caution since magnetic field,[4] temperature,[5] stress,[6] impurity,[7] and other effects exist in actual samples.

In view of these dependences and the recent data of Wagner, Kennedy, McCombe, and Tsui[3] on Si[001] inversion layers, we investigate in the present article those effects which we left out in I. These include thickness, singularity, impurity, and field effects. These effects are not nessarily separable from each other. However, we shall try to clarify their effects individually even though for numerical results some will be combined.

Although the present analysis shows good general agreement with the data of Wagner *et al.*, we remark that the data show some complications. Moreover, Fang, Fowler, and Hartstein[7] have reported that the effective mass of electrons determined by Shubnikov–de Haas (SdH) oscillations depends very strongly on oxide charges. They have revealed that the effective mass obtained by extrapolation to zero-oxide concentration is surprisingly a carrier-independent bulk effective mass. Also, they have found that the effective mass decreases with decreasing inversion layer width possibly due to increase in interface scattering. For convenience, we shall use the natural units in which $\hbar=1$ and $2m=1$, unless an explicit display is desirable.

II. THICKNESS EFFECT

In I we used an idealized two-dimensional model without thickness. Let us investigate the thickness

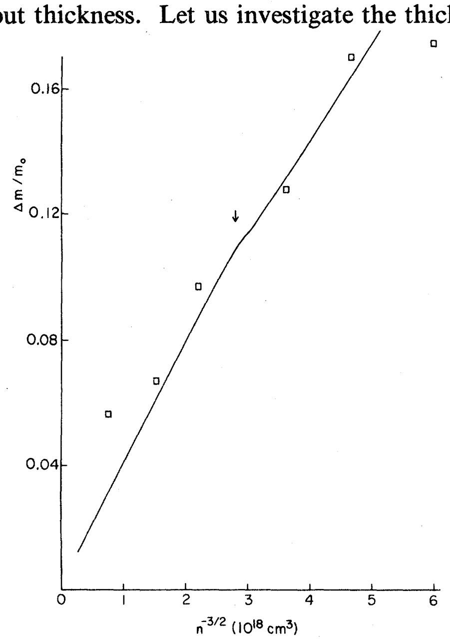

FIG. 1. Relative effective-mass shift against $n^{-3/2}$. The small bump represents the effect due to the singularity in the dielectric constant. The data are due to Wagner *et al.* (Ref. 3).

effects on cyclotron effective mass and scattering time based on Stern-Howard's formula[8]:

$$b=\frac{3}{\langle z\rangle}=\left[\frac{48\pi m_z e^2}{\epsilon_s \hbar^2}\left(\tfrac{11}{32}n_{\rm inv}+n_{\rm dep}\right)\right]^{1/3} . \tag{2.1}$$

Note in this formula that b is a function of electron densities in inversion and depletion layers. This parameter enters in the impurity-electron interaction as follows:

$$\begin{aligned} v(q)&=-\frac{2\pi e^2}{\epsilon_s q}I_i\left[\frac{q}{b}\right] ,\\ I_i(x)&=2(1+x)^{-3}\\ &\quad\times\epsilon_s(\epsilon_s+\epsilon_{ox}\coth Dq)^{-1} . \end{aligned} \tag{2.2}$$

Also, the parameter modifies the electron-electron interaction in the following way:

$$\begin{aligned} u(q)&=\frac{2\pi e^2}{\epsilon_s q}I_e\left[\frac{q}{b}\right] ,\\ I_e(x)&=\frac{8+9x+3x^2}{8(1+x)^3}\\ &\quad+\frac{\epsilon_s-\epsilon_{0x}\coth qD}{\epsilon_s+\epsilon_{0x}\coth qD}\frac{1}{(1+x)^6} . \end{aligned} \tag{2.3}$$

In the literature, $u(q)$ is often denoted as $v(q)$ and vice versa. In addition to thickness, we have adopted an improved formula for the scattering time[9]:

$$\frac{1}{\tau}=\frac{r_s}{\sqrt{2}}\left[\frac{n_i}{n}\right]4p_F^2 J(t,r_s)\left[1+\frac{r_s}{\sqrt{2}}\frac{n_i}{n}I(t,r_s)\right]^{-1} , \tag{2.4}$$

$$\begin{aligned} J(t,r_s)&=\frac{1}{4\sqrt{2}p_F^2 r_s}\frac{1}{t}\int ds\, s^3\frac{v(s)^2}{u(s)}\frac{\epsilon_i(s,t)}{|\epsilon(s,t)|^2}+\Delta J_p(t,r_s) ,\\ I(t,r_s)&=\frac{1}{\sqrt{2}\pi r_s}\frac{1}{t^2}\int ds\, s^3\frac{v(s)^2}{u(s)}\left[\frac{1}{\epsilon_r(s,0)}-\frac{\epsilon_r(s,t)}{|\epsilon(s,t)|^2}\right] , \end{aligned} \tag{2.5}$$

The plasmon contribution is given for finite thickness as follows:

$$\Delta J_p(t,r_s)=-\frac{\pi s_p^2}{2t\left[\dfrac{d\epsilon_p(s,t)}{ds}\right]_{s=s_p}}\frac{[I_i(p_F/b)]^2}{I_e(p_F/b)}\left[\frac{\bar{\epsilon}}{\epsilon_s}\right] , \tag{2.6}$$

where p stands for a plasmon and

$$s=\frac{q}{p_F},\quad t=\frac{\omega}{p_F^2} . \tag{2.7}$$

Formula (2.4) should be compared with Eq. (1.2) of I. n_i is the impurity density.

For our numerical computation, we have chosen theoretical parameters as follows: $\epsilon_{0x}=11.7$, $\epsilon_s=3.9$, $\bar{\epsilon}=7.8$, $m_0=0.191m_e$, $m_z=0.98m_e$, $D=1000$ Å, $n_{\rm dep}=1.5\times10^{11}$ cm^{-2}, and the valley degeneracy is 2. We have found that although the general behavior of the function $J(t,r_s)$ is similar to that in I, its value and plasmon contribution are significantly larger. As a result the impurity concentration, which reproduces the data well, is found to be 1.4×10^{11} cm^{-2} in contrast to 0.334×10^{11} in I.

The thickness effects have been considered by Lee, Ting, and Quinn[9] and also by Ganguley and Ting.[10] We shall discuss their works in comparison with ours in the concluding section.

III. SINGULARITY EFFECT

The random-phase approximation (RPA) dielectric function such as used in I possesses a well-known singularity. In consideration of the thickness effect we have made a more precise numerical analysis than in I in order to find how the singularity appears in the final results.

A singularity effect on the effective mass is shown in Fig. 1. The arrow indicates the point where theoretically a change in slope takes place due to the singularity. With the "bump" in the curve due to the change, the theoretical curve comes somewhat closer to the data. This plot should be compared with Fig. 17 of Wagner *et al.*, who used a linear least-squares fit. Also, when the effective mass is plotted against $\omega\tau$, a small cusp appears at around $\omega\tau=4.85$.

A similar change in slope appears in the theoretical curve in the collision time also, but the change is small. Owing to experimental errors it seems difficult to detect such a small change.

IV. FIELD EFFECT

The data of Wagner *et al.* show that the effective mass decreases with magnetic field (frequency) considerably thoughout all samples. The effective mass is reduced from around $0.22m_e$ at 10 cm^{-1} to $0.195m_e$ at 60 cm^{-1}, i.e., some 10% decrease. Their data contrast those of Abstreiter *et al.*,[4] who reported in the range 3–30 cm^{-1} only a very small field dependence, which was within the experimental error.

A small field effect on the effective mass was reported also by Ting, Ying, and Quinn.[9] We shall comment on their work shortly. In order to find the field effect, we have generalized the RPA dielectric function to the case in which the field energy is much less than the Fermi energy but larger than the thermal energy.[11] We have arrived at for $T=0$

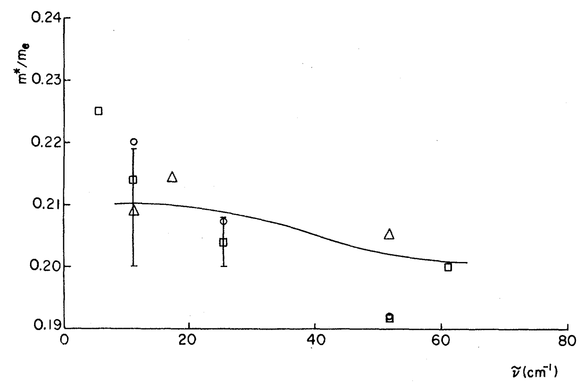

FIG. 2. Effective-mass ratio as a function of magnetic field (frequency). The data are due to Wagner *et al.* (Ref. 3). △, sample 1, $n_s=1.4\times10^{12}$ cm^{-2}. ○, sample 2, $n_s=1.49\times10^{12}$ cm^{-2}. □, sample 3, $n_s=1.59\times10^{12}$ cm^{-2}.

$$\epsilon(q,\omega)=1+u(q)\lambda(q,\omega)\ ,$$

$$\lambda(q,\omega)=\frac{1}{4\pi}\left[1+\frac{i\omega_0}{\pi}\sum_s\frac{1}{\omega-s\omega_0\Sigma_s}\right]\int_0^{2\pi}dx\,\exp(i\omega x/\omega)\sin(Q\sin x)G_s(x)\ , \tag{4.1}$$

where

$$G_s(x)=e^{-\gamma y}g_s(y),\quad g_s(y)=\theta L_s(2\gamma y)+2L_{s-1}^1(2\gamma y),\quad y=(Q/\gamma)(1-\cos x)\ , \tag{4.2}$$

$$\theta=\begin{cases}1 & \text{if } p_F^2/\omega_0-\frac{1}{2}=\text{integer}\\ 2 & \text{if } p_F^2/\omega_0-\frac{1}{2}\neq\text{integer}\end{cases}\qquad Q=q^2/\omega_0,\quad \gamma=\omega_0/2p_F^2,\quad \omega_0=2eH/c \tag{4.3}$$

$$\Sigma_s(\omega)=\tfrac{1}{2}\{\omega-s\omega_0\pm[(\omega-s\omega_0)^2-4W_{ss}]^{1/2}\},\quad (\omega-s\omega_0)^2\geq 4W_{ss},\quad +\text{ for } s>0,\quad -\text{ for } s\leq 0$$

$$\Sigma_s(\omega)=\tfrac{1}{2}\{\,\omega-s\omega_0-i[4W_{ss}-(\omega-s\omega_0)^2]^{1/2}\,\},\quad (\omega-s\omega_0)^2<4W_{ss}\ . \tag{4.4}$$

Here, $W_{ss}=\Gamma_s^2$, Γ_s being the width function, will be evaluated shortly, and Q and γ are dimensionless.

Figure 2 compares our theoretical curve with the data of Wagner *et al.* for sample 3. As in the next section, explicit results are obtained for the impurity potential:

$$v(r)=\frac{V_0}{2\pi d^2}e^{-r^2/2d^2}\ . \tag{4.5}$$

The parameters are $d = 17.7$ Å, $n_i v_0^2 = 6.9 \times 10^{-42}$ erg cm^2, and $n = 1.59 \times 10^{12}$ cm^{-2}. These parameters may be compared with those used by Heuser and Hajdu[12] for the three-dimensional case with

$$v(r) = \frac{v_0}{(2\pi d^2)^{3/2}} e^{-r^2/2d^2} . \tag{4.6}$$

Their potential value for $r = 0$ and $d = 73$ Å is 16 meV while our value corresponding to $d = 18$ Å and $v_0 = 6.83 \times 10^{-27}$ erg cm^2 for $n_i = 1.5 \times 10^{11}$ cm^{-2} is 21 meV. Therefore, the order of magnitude is the same.

The effective mass under a relatively strong magnetic field is expected to show oscillations. However, in view of the data of Wagner *et al.*, which do not show such oscillations we have simplified our numerical analysis by choosing the approximate expression $p_F^2 = 2\pi n$. More importantly the stronger field dependence in our result is due to the use of the field dependences of the dielectric function and width function. Since the latter is determined by the impurity potential, we shall discuss it in the next section. We note, however, that the small field dependence of Ting, Ying, and Quinn is probably due to their replacement of the width function by a limiting $\Gamma = 2\omega_c/\pi\tau$, which is expected when the cyclotron radius l is much larger than the range of the impurity potential, i.e., relatively weak field. In our case, we have used Γ_s which is evaluated as a function of field. The form of the dielectric function is also probably different. Our dielectric function corresponds to the quantum-mechanically degenerate case.

V. IMPURITY EFFECTS

In our present work we have used the potential form of Eq. (4.5) for impurity scattering. The linewidth can be obtained from[9]

$$\Gamma_s^2 = n_i \int \frac{d\vec{q}}{(2\pi)^2} |v(q)|^2 |\rho_{ss}|^2 , \tag{5.1}$$

where ρ_{ss} is evaluated from the harmonic oscillator eigenfunctions as follows:

$$\rho_{ss} = \exp(-q^2 l^2/4) L_s^0(q^2 l^2/2) , \tag{5.2}$$

where

$$l^2 = \frac{c}{eH} . \tag{5.3}$$

We arrive at

$$\Gamma_s^2 = \frac{a(\xi-1)^s}{(\xi+1)^{s+1}} P_s \left[\frac{\xi^2+1}{\xi^2-1} \right] , \tag{5.4}$$

where

$$\xi = \frac{2d^2}{l^2}, \quad a = \frac{n_i v_0^2}{2\pi l^2} . \tag{5.5}$$

We have evaluated the linewidth as a function of s for $\xi = 0.1$ and 2. The results are illustrated in Fig. 3. The upper curve corresponds to $\xi = 0.1$ and the lower to $\xi = 2$. As we see, the ratio Γ_s/Γ_0 decreases with increasing s for a given ξ. Although the ratio decreases as a function of ξ for small values of ξ, it actually increases gradually for larger ξ values starting around $\xi = 1$.

In particular when $\xi \ll 1$, i.e., $l \gg d$, the impurity potential is short ranged, and the field is weak. The square of the width function Γ_s^2 will be proportional to $n_i v_0^2/l^2$. That is,

$$\Gamma_s \propto n_i^{1/2} v_0 H^{1/2} , \tag{5.6}$$

which corresponds to the limiting expression used by Ting, Ying, and Quinn. If d is small and the cyclotron orbits are large, the electrons will feel the field broadly during their entire cyclotron motion.

In the opposite limit of $\xi \gg 1$, the potential is long ranged, $P_s \to 1$ and $\Gamma_s^2 \propto a/\xi$, so that

$$\Gamma_s \propto n_i^{1/2} v_0/d . \tag{5.7}$$

In this case the impurity effect is broad, while the field localizes the electrons. The probability that the electrons are under the influence of impurities may be given by n_i/n, leading to the above square-root dependence.

The linewidth depends on the functional form of the impurity potential. We remark that the width function for the screening potential,

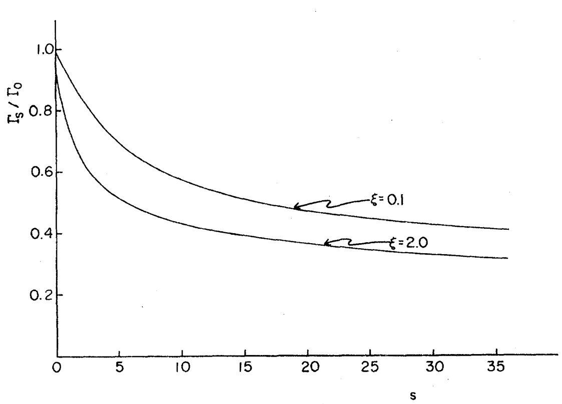

FIG. 3. Relative width Γ_s/Γ_0 for two ξ values as a function of integer s.

$$v(r)=\frac{e^2}{\bar{\kappa}r}e^{-\kappa r}\,, \tag{5.8}$$

increases with field strength in accordance with

$$\Gamma_s^2=\frac{n_i}{2\pi}\left[\frac{2\pi e^2}{\bar{\kappa}}\right]^2 \times\int_0^\infty \frac{e^{-q^2l^2/2}}{q^2+\kappa^2}\left[L_s\left[\frac{q^2l^2}{2}\right]\right]^2 q\,dq\ . \tag{5.9}$$

Also, as the screening constant increases, Γ_s decreases. If this screening constant is proportional to the Fermi momentum as in the case of the Thomas-Fermi screening, the increase in density will reduce Γ_s.

VI. TEMPERATURE EFFECT

At low but finite temperatures the dielectric function can be obtained in a Sommerfeld fashion. At finite temperatures the oscillations of the polarizability function is somewhat smooth. We find the function $G_s(x)$ of Eq. (4.2) is given by

$$G_s(x)=\frac{2J_1(x)}{\gamma x} +\frac{2\pi}{\alpha}J_0(X)\sum_k \frac{\sin(\pi k/\gamma)}{\sinh(\pi^2 k/\alpha)}\ . \tag{6.1}$$

Here, we have assumed for simplicity that $g=2$, g being the Lande factor. If $g\neq 2$, we need only a factor $(-1)^k\cos(g\pi k/2)$ in the sum. The first term on the right-hand side yields precisely the polarizability in the absence of field. $X=2(\gamma y)^{1/2}$, y being defined by Eq. (4.2), and $\alpha=\omega_0/2kT$.

Equation (6.1) shows characteristic oscillations. The appearance of $\sinh(\pi^2 s/\alpha)$ in the amplitude is familiar. Owing to this factor $G_s(x)$ decreases when the temperature is increased. Hence, we expect that the effective mass and relaxation time will be reduced.

VII. VALLEY OCCUPANCY EFFECT

So far we have assumed that the two valleys in the [001] direction of silicon inversion layers are equally populated. However, as the density of electrons decreases, their correlations increase, and as a result a one-valley state may be preferred.[13] The transition from two-valley to one-valley states have been estimated to be around 3×10^{11} cm^{-2}.

Associated with this transition is a sudden increase in the Fermi momentum. For two-valley states it is $(\pi n)^{1/2}$, while for one-valley states it is $(2\pi n)^{1/2}$. If impurity scatterings are represented by a screened Coulomb potential, Eq. (5.9) suggests that a sudden narrowing in cyclotron resonance takes place. In Eq. (5.8) we have used different screening constants κ and $\bar{\kappa}$. If $\bar{\kappa}$ is replaced by κ, which is determined by the Fermi momentum, then Eq. (5.9) indicates that narrowing of a factor 3 may take place.

Recently, Wilson, Allen, and Tsui have reported a remarkable narrowing and shift if only the lowest Landau level is partially occupied.[14] Their data are consistent with those reported by Kennedy, Wagner, McCombe, and Tsui.[15] From a plot of the peak frequency versus magnetic field, Wilson *et al.* concluded a shifted cyclotron resonance[15] rather than a reduced effective mass. Their observed narrowing is dramatic: Below 0.5 for the filling factor, narrowing by more than a factor of 10 has been observed.

A similar anomaly has been observed in GaAs-GaAlAs by at least two groups.[16] These experiments show that the important parameter is the filling factor and a certain many-body treatment is needed. Although the valley occupancy transition is a many-body effect, it is still unclear whether or not and how the symmetry-breaking mechanism is related to a coupling of the electrons to the lattice. Further theoretical investigations are definitely needed.

VIII. CONCLUDING REMARKS

We have analyzed theoretically the cyclotron data of Wagner *et al.* and achieved some good agreements. However, the data are more complicated. For instance the effective mass reaches a maximum at a low density and a minimum at a high density. The relaxation time exhibits a maximum which depends on magnetic field. The present theory does not explain these effects.

The thickness effect has been considered earlier by Lee, Ting, and Quinn,[9] who chose the parameter b appearing in Eq. (2.1) as a constant independent of carrier density. Later, Ganguley and Ting[10] took into consideration the density dependence of b based on a simple expression. We have used the Stern and Howard formula which gives b as a function of the electron densities in the inversion and depletion layers. The constant D which we chose is very close to what Lee, Ting, and Quinn used.

With the singularity effect the theoretical effective mass comes somewhat close to the data. On the other hand, Wagner *et al.* used in Fig. 17 straight-line fits for three frequencies. When plotted against $\omega\tau$, the effective mass is expected to show a small cusp at around 4.85. However, such a singularity effect may be rather small for experimental detection.

Our analysis shows that the field effect on the effective mass could reach 10%. However, in view of the previous work of Abstreiter *et al.*,[10] further experimental studies seem to be worthwhile.

The linewidth is directly determined by the impurity potential. Hence, the effect of impurity potentials on linewidth is important. In particular we have analyzed the effects based on a short-range potential and a screened Coulomb potential. In the latter case, the screening constant could depend on electron density, suggesting the density dependence of the width function.

ACKNOWLEDGMENT

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