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Variable range hopping in one-dimensional metals

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Mott variable range hopping in one dimension is analyzed for a chain with a uniform distribution of localized states. We calculate the distribution of the largest hopping probabilities from states at the Fermi surface. The activated resistivity of Kurkijärvi is found to be an asymptotic limit valid for very long chains and low frequencies. An important anomalous hopping regime is found for intermediate lengths and times which crosses over into the quantum-mechanical regime. Experimental situations in anisotropic materials and for long wires and the relationship of the classical and quantum-mechanical frequency dependence are also discussed briefly.

The purpose of this paper is to study Mott variable range hopping[1] for a one-dimensional (1D) metal.[2] The problem is of interest for a number of reasons. One has to consider a finite-temperature variable range hopping regime if one wants to understand experiments relevant to the quantum-mechanical zero-temperature anomalies in the resistance of one-dimensional metals.[3–6] We are largely motivated by the feeling that one should also be able to understand the curious temperature and frequency dependence observed in anisotropic quasi-one-dimensional metals[7,8] from this point of view. The experimental results seem to agree very well with the predictions of a phenomenological model with a distribution of activation barriers.[9,10] There is, however, no obvious reason why a model of this type should be applicable. The problem is also of interest in its own right.

A random potential is expected to lead to the localization of all eigenstates in a 1D metal.[11] As a result the resistance is expected to increase exponentially with length[3] and to show large fluctuations.[5] Thoulless[6] and Abrikosov and Ryzhkin[4] have considered the finite-temperature effects in perturbation theory. This represents the onset of the effect of localization and should be followed by a hopping regime at lower temperatures. The standard Mott-type variable range hopping arguments are, however, not applicable in 1D. The conductance of a long one-dimensional system is dominated by fluctuations, i.e., by rare high-resistance regions along the chain. In essence the reason for this is that the percolation density in 1D is unity- and critical-path analysis[12] therefore does not apply. This was noticed by Kurkijärvi,[2] who was able to show that the zero-frequency resistivity of an infinite line should show a simple $1/T$ activated behavior. We want to calculate the time (or frequency) dependence and the finite-length effects in the hopping regime.

We shall follow Kurkijärvi[2] and assume a uniform distribution of localized levels in energy and along the chain. We also neglect quantum-mechanical fluctuations and assume that all eigenfunctions decay exponentially with the same "localization length" (l). We shall calculate the probability distribution for the most probable hop from a localized level at the Fermi surface. We then use the techniques developed recently to treat classical transport in 1D in the presence of randomness[10,13,14] to predict the frequency, length, and temperature dependence following from this distribution. An explicit effective medium calculation is being published elsewhere.[15] We emphasize that this is a purely classical calculation. In particular the frequency dependence we derive is that due to the distribution of hopping times as, e.g., in Ref. 14. Quantum effects for which the energy is supplied by the radiation field[4,16] are not considered.

We assume that the hopping probability from a state of energy E_n centered at n to a state at m (of energy E_m) is given by

$$W_{n\to m}=\exp\{-[(|n-m|-1)/l - (E_n-E_m)/T]\}\ , \tag{1}$$

where Boltzman's constant is taken equal to unity and distances (n,m,l) are measured in units of an elementary translation distance, and transition rates

(W) in units of the preexponential factor which should otherwise show up in Eq. (1). As noted, the localized eigenstates are all assumed to decay exponentially with decay constant $1/2l$. It is evident that Eq. (1) implicitly assumes long hops ($|n-m|/l \gg 1$).

We further assume that the localized states are uniformly distributed in energy and along the chain and their distribution at different sites is independent. The probability of finding a level of energy E centered at n is thus always given by the density of states $N(0)$. As a result there is a unique energy scale describing the level spacing,

$$\Theta=[N(0)l]^{-1} , \tag{2}$$

which determines the scale for the activation processes. We assume low temperatures,

$$T/\Theta \ll 1 , \tag{3}$$

and also, implicitly, that the mean free path for inelastic scattering[6,17] is large compared to l.

The assumptions for the distribution of levels are obviously an oversimplification. In particular the fact that the distance $|n-m|$ determines a minimum level separation is neglected. This is not important for the regime we are considering.

We make two further assumptions:

(a) We consider only activated hops *from* the Fermi surface ($E_m - E_n > 0$) and neglect the effect of the (rapid) downward jumps.

(b) For each site we only consider the largest of all available $W_{n\to m}$ (to the right and independently to the left)—for example, $\hat{W}_n$.

We shall calculate the probability distribution of the $\hat{W}_n$ and shall then consider the transport process determined by this distribution using the ideas and techniques of Ref. 14.

We feel that this is, at least qualitatively, correct. The one-dimensional conductance is dominated by those hops for which $\tilde{W}$ is very small because of fluctuations in the distribution of levels. This is precisely the effect which is emphasized, and described correctly by our approach.

We first calculate the probability distribution $\rho(\tilde{W})$ that the largest hopping rate from a site (say the origin) is $\tilde{W}$. This can be written:

$$\rho(\tilde{W})=\left[\prod_{n=1}^{\infty} Q_n(\tilde{W})\right]\sum_{n=1}^{\infty}\rho_n(\tilde{W}) , \tag{4}$$

where $Q_n(\tilde{W})$ is the probability that there is no level at n such that

$$W_{0\to n} > \tilde{W} \tag{5}$$

and $\rho_n(\tilde{W})=(d\ln Q)/(dW)$ is the probability that one can indeed hop to n with a rate W. This immediately implies a cutoff:

$$\rho_n(\tilde{W})\equiv 0, \quad Q_n(\tilde{W})\equiv 1, n/l > |\ln\tilde{W}| . \tag{6}$$

For smaller n one finds

$$\rho_n(\tilde{W})=TN(0)/\tilde{W} \tag{7}$$

and

$$Q_n(\tilde{W})=\exp\{-[N(0)\Delta(n,\tilde{W})]\} , \tag{8}$$

where we have defined the level spacing required to give $\tilde{W}$ for separation n:

$$\Delta(n,\tilde{W})=-T[\ln\tilde{W}+(n-1)/l]>0 , \tag{9}$$

remembering that $\ln\tilde{W}<0$.

Substituting the expressions (7) and (8) in Eq. (4) gives

$$\rho(\tilde{W})=\left[\frac{T}{\Theta}\right]\frac{-\ln\tilde{W}+1/l}{\tilde{W}} \times\exp\left[-\frac{T}{2\Theta}\left[(\ln\tilde{W})^2-2\frac{\ln\tilde{W}}{l}\right]\right] , \quad \ln\tilde{W}<0 . \tag{10}$$

We are interested in low temperature, where long hops dominate and can therefore assume

$$l\,|\ln\tilde{W}| \gg 1 , \tag{11}$$

so that

$$\rho(\tilde{W})\approx -\frac{T}{\Theta}\left[\frac{\ln\tilde{W}}{\tilde{W}}\right] \times\exp\left[-\left[\frac{T}{2\Theta}\right](\ln\tilde{W})^2\right] . \tag{12}$$

This is the expression we shall use in the following. One notes that $\rho(\tilde{W})$ has a maximum for

$$W_{\max}=\exp(-\Theta/T) . \tag{13}$$

The distribution in Eq. (12) [and Eq. (10)] has the nice property that the distribution of hopping times ($\tau=\tilde{W}^{-1}$) has exactly the same form as the distribution of rates.

It is fairly straightforward to generalize this calculation to initial states of arbitrary energy (E_n) and including the Fermi distribution of available

empty states. Using the complete results would considerably complicate our discussion of the transport process below without changing anything of substance. The important small $\tilde{W}$ are for jumps from the Fermi surface and are correctly described by $\rho(\tilde{W})$ [Eq. (12)].

We first calculate the resistivity of an infinite chain with this distribution of (independent) hopping rates. This seems reasonable. Implicitly one is considering hopping between sites at the Fermi level and assumes that the nearest-neighbor (NN) hopping rates are controlled by the activated process described. It follows from the definition of the $\tilde{W}$ that one can never "skip" a nearest neighbor. (There are, however, delicate problems with detailed balance which we shall disregard.) We assume that a chain with a probability distribution $\rho(\tilde{W})$ for nearest-neighbor hopping rates will describe the transport properties correctly. To calculate the transport properties we need averages of the type[14]:

$$\langle 1/\tilde{W}^n\rangle_T=-\frac{T}{\Theta}\int_0^1\frac{d\tilde{W}}{\tilde{W}}(\ln\tilde{W}/\tilde{W}^n)\times\exp[-(T/2\Theta)(\ln\tilde{W})^2]\ . \tag{14}$$

The integrals are immediate and the result can be expressed in terms of exponentials and error functions. To leading order in the assymptotic expansion of the error function one has

$$\langle 1/\tilde{W}^n\rangle_T\approx n(2\pi\Theta/T)^{1/2}\exp(n^2\Theta/2T) \tag{15}$$

with corrections of order $T/n^2\Theta$. In the rest of this paper we will mostly evaluate similar integrals to this order without stating this explicitly.

Thus the average resistance *per step* is

$$R_T\propto\langle 1/\tilde{W}\rangle_T=(2\pi\Theta/T)^{1/2}e^{\Theta/2T}\ . \tag{16}$$

To calculate a resistivity we have to remember that there is a distribution of hopping distances. The average distance for a given $\tilde{W}$ is

$$S(\tilde{W})=(-l\ln\tilde{W}+1)/2\ , \tag{17}$$

using the fact that all $\rho_n(\tilde{W})$ are equal [Eq. (7)]. Combining (17) and (12) leads to an average step length:

$$S_T=l(2\pi\Theta/T)^{1/2}\ . \tag{18}$$

Thus the resistivity becomes

$$\rho_T=R_T/S_T=l\,e^{\Theta/2T}\ . \tag{19}$$

We have only included the most probable hops and neglected the contribution to the transport of downwards jumps. Thus Eq. (19) should be regarded as an upper bound on the resistivity. It should be compared with Kurkijärvi's lower bound[2]

$$\rho_T>\exp(\Theta/8T)\ . \tag{20}$$

While the average resistivity is well defined and activated one must worry about statistical fluctuations. From the expression for the moments [Eq. (15)] it is evident that these can be large, and will only average out for sufficiently long chains. One can define a step number

$$N_T=\langle 1/\tilde{W}^2\rangle/\langle 1/\tilde{W}\rangle^2=(2T/\pi\Theta)^{1/2}e^{\Theta/T}\ . \tag{21}$$

One needs more than N_T steps to have a small variance for the resistance of a finite chain. Using Eq. (18) this can be expressed as a length

$$L_T=S_TN_T=2le^{\Theta/T}\ . \tag{22}$$

One expects to observe the average resistivity only for chains longer than L_T. For shorter chains fluctuations are important and one has to consider the predictions more carefully. Even the condition $L>L_T$ is not really sufficient. It assures a small variance but, because of the factor n^2 in the exponent of Eq. (15) sufficiently high moments are always large, for any length. While this is intriguing and somewhat similar to the problems one has in the quantum-mechanical problem,[5] one does not expect important experimental implications. For chains long enough to assure a small variance the probability of finding a chain with a resistance deviating from the average value (ρ_T) becomes very small.[18]

Equation (21) also implies that one needs very long times (or low frequencies) to observe the activated average behavior. We can estimate the time scale by considering the time required to diffuse a distance L_T. We write

$$D_T\tau_T\propto N_T^2\ , \tag{23}$$

where D_T is a diffusion constant and is given by

$$D_T\propto R_T^{-1}\approx(\langle 1/\tilde{W}\rangle_T)^{-1}\ , \tag{24}$$

because for segments of this length the averages are adequate. Thus using Eqs. (16) and (21) in Eq. (23)

$$\tau_T\propto R_TN_T^2\approx(8T/\pi\Theta)^{1/2}\exp(5\Theta/2T)\ . \tag{25}$$

For times shorter than τ_T the fluctuations become important. The conditions we have found for observing the average activated Kurkijärvi behavior:

$$\omega^{-1} \approx t \gg \tau_T , \quad L \gg L_T \tag{26}$$

become quite stringent when Θ/T becomes large—i.e., at low temperatures. For shorter chains ($L < L_T$) or at higher frequencies ($\omega\tau_T > 1$) the averages do not give an adequate description. In the following we shall try to determine the finite length and frequency properties under these conditions. We do this using the heuristic scaling arguments suggested in Ref. 13 and discussed in more detail in Ref. 10 and in Sec. IX of Ref. 14. While these arguments are certainly not rigorous they should give a fairly accurate description of the qualitative properties.

Consider first the zero-frequency conductance of a one-dimensional system of finite length. At zero temperature one would have

$$\ln\langle R \rangle_L \approx L/l \tag{27}$$

for the model we are using, consistent with the Landauer[3] result. We want to compare this with the hopping result at finite temperatures. For a finite chain there is an intrinsic cutoff on $\tilde{W}$:

$$\tilde{W} \geq W_L \approx e^{-L/l} , \tag{28}$$

i.e., one can always hop directly to the end of the chain with a rate of order W_L. Since the probability distribution of hopping rates for short hops is not modified, we can write for such chains

$$\begin{aligned} \rho_L(\tilde{W}) &= \rho(\tilde{W}) , \quad W > W_L \\ \rho_L(\tilde{W}) &= p_L \delta(W - W_L), \quad W \leq W_L \end{aligned} \tag{29}$$

where $\rho(\tilde{W})$ is given in Eq. (12) and

$$p_L = \int_0^{W_L} \rho(\tilde{W})\, d\tilde{W} = \exp[-(T/2\Theta)(L/l)^2] \tag{30}$$

is the total probability of finding $W < W_L$ in the original distribution.

There are three distinct regimes as a function of L or T. For relatively long chains

$$L_T > L > l\Theta/T, \quad W_L < R_T^{-1} , \tag{31}$$

where L_T and R_T are given in Eq. (16) and (22), respectively. Averages of the type $\langle 1/\tilde{W}^n \rangle_L$ computed with the truncated distribution $\rho_L(\tilde{W})$ are then not sensitive to the truncation. They are still, roughly, represented by Eq. (15). In particular,

$$\langle 1/\tilde{W} \rangle_L \lesssim R_T , \tag{32}$$

so that the average resistance is not modified drastically but fluctuations (in an ensemble of chains) become large.

The results are quite different when W_L becomes smaller than the value of $\tilde{W}$ at the maximum of $\rho(\tilde{W})$ [$W_{\max}$ in Eq. (13)]. This happens when

$$L/l < \Theta/T . \tag{33}$$

The integrals are straightforward and one finds

$$\begin{aligned} \langle 1/\tilde{W}^n \rangle_L &\approx C P_L / W_L^n \\ &= C \exp\{-[(T/2\Theta)(L/l)^2 - nL/l]\} \end{aligned} \tag{34}$$

again to leading order (in $[n(\Theta/T) - (L/l)]^{-1}$) in the asymptotic expansion of the error functions. The constant C is of order unity.

This is a somewhat strange result. The averages (34) are computed with the distribution $\rho_L(\tilde{W})$ [Eq. (29)]. They include the contribution of those chains for which there is no proper hopping. The fraction of such chains is p_L. At sufficiently low temperatures

$$T/\Theta < (l/L)^2 . \tag{35}$$

p_L becomes of order unity [Eq. (30)] and one finds the quantum-mechanical "zero-temperature" behavior [Eq. (27)]. Since quantum-mechanical fluctuations were not included in the initial model we find no fluctuations in this limit. There is, however, an intermediate situation:

$$(l/L)^2 < T/\Theta < l/L \tag{36}$$

for which only the inequality (33) holds. Since p_L is small, transport for most chains is by hopping. The average resistance per hop is then given by

$$R_L \propto \langle 1/\tilde{W} \rangle_L \approx (1/W_L) \exp[-(T/2\Theta)(L/l)^2] \tag{37}$$

much smaller than W_L^{-1}. One thus has, for most chains, a proper hopping regime with a resistance decreasing with T.

The average step length is still given by S_T [Eq. (18)] with exponentially small corrections [in $(T/2\Theta)(L/l)^2$]. Thus the average step number is

$$N_L \approx (L/S_T) \approx [(L/l)^2 (T/\Theta)]^{1/2} \tag{38}$$

and fluctuations are large

$$\langle 1/\tilde{W}^2 \rangle_L / \langle 1/\tilde{W} \rangle_2^2 \approx \exp(T/2\Theta)(L/l)^2 \gg N_L \tag{39}$$

for most of the range (36).

To obtain the distribution of resistances of lines of length L one would have to supplement these considerations by the requirement that the sum of all hopping distances cannot exceed L. This gets fairly complicated because there is a wide distribution in the number of hops required to traverse the line, which is correlated to the $\tilde{W}$ of the separate hops. We only note that even the average resistance is not necessarily proportional to $N_L\langle 1/\tilde{W}\rangle_L$. The calculation of the frequency dependence on the infinite line, which we describe below, avoids most of these difficulties.

The distribution we have derived [$\rho(\tilde{W})$, Eq. (12)] is similar to that obtained from a truncated activation barrier distribution[10] which has been used successfully in interpreting experimental results in anisotropic quasi-one-dimensional metals.[8] We shall use a self-consistent scaling argument to determine the frequency dependence in the present case. Similar arguments were presented in Ref. 13, in Sec. IX of Ref. 14, and in Ref. 10.

We consider a diffusion process on the chain controlled by the distribution $\rho(\tilde{W})$. One notes that the small $\tilde{W}$ (for example, smaller than W_c) are rare. They constitute a fraction

$$p_c=\int_0^{W_c}\rho(\tilde{W})d\tilde{W}=\exp[-(T/2\Theta)(\ln W_c)^2] \tag{40}$$

of all hops. Thus a diffusing particle must, on the average, transverse

$$\xi_1(W_c)=p_c^{-1}\approx\exp(T/2\Theta)(\ln W_c)^2 \tag{41}$$

successive steps along the chain before meeting such a small $\tilde{W}$ ($<W_c$). We note that ξ_1 is defined as a number of steps and not as a distance. On the other hand it is evident that a diffusion process will only spread out over some finite number of steps (ξ_t) in time t. If ξ_t is smaller than $\xi_1(W_c)$ most particles will not encounter any sites with $\tilde{W}$ smaller than W_c. One can therefore disregard these small $\tilde{W}$ in describing the diffusion process for short times [such that $\xi_t\lesssim\xi_1(W_c)$]. In general this will imply a time-dependent diffusion constant. We write

$$D_t t=\xi_t^2 \tag{42}$$

and try to make the argument self-consistent. When

$$\xi_t\lesssim\xi_1(W_c)\ , \tag{43}$$

we can disregard the small $\tilde{W}$ and compute D_t from the truncated distribution

$$\begin{aligned}&\rho_c(\tilde{W})=\rho(\tilde{W}),\quad \tilde{W}>W_c\\&\rho_c(\tilde{W})\equiv 0,\quad \tilde{W}<W_c\ .\end{aligned} \tag{44}$$

Thus,

$$D_t^{-1}\approx\langle 1/\tilde{W}\rangle_c=\int\frac{\rho_c(\tilde{W})}{\tilde{W}}d\tilde{W}\ . \tag{45}$$

Explicitly, one can choose a cutoff value W_c. Using (41) and (45) in Eq. (42) then determines a relationship between t and W_c,

$$t(W_c)\approx\xi_1^2(W_c)/D_t\approx\xi_1^2(W_c)\langle 1/\tilde{W}\rangle_c\ . \tag{46}$$

This can be used to eliminate W_c and express D_t explicitly in terms of the diffusion time t. For the distributions of hopping rates considered in Ref. 10 this led to anomalous power-law behavior. In the present case the explicit expressions are cumbersome and not very illuminating. We therefore present the results in a different form.

For sufficiently long t the cutoff (W_c) becomes small and $\langle 1/\tilde{W}\rangle_c$ becomes essentially independent of W_c. One then has regular diffusive behavior

$$\ln t\approx(\Theta/2T)+2\ln\xi\ , \tag{47}$$

to leading order in the expansion of the logarithms. This is certainly the case when $\xi>N_T$ [Eq. (21)] but qualitatively also as long as $\xi>N_T^{1/2}$. This is equivalent to the requirement

$$|\ln W_c|>\Theta/T\ . \tag{48}$$

The argument is essentially identical to that we presented for finite-length chains when the inequalities (31) hold. The range in t can be determined by substituting N_T (or $N_T^{1/2}$) for ξ in Eq. (47):

$$\ln t>\tfrac{5}{2}(\Theta/T)\quad(\text{or }\tfrac{3}{2}(\Theta/T)) \tag{49}$$

essentially equivalent to the estimate we had in Eq. (25).

Consider now the intermediate range of times given by

$$\Theta/T>|\ln W_c|>(\Theta/T)^{1/2}\ . \tag{50}$$

This is analogous to the intermediate lengths described by Eq. (36). One finds

$$\langle 1/\tilde{W}^n\rangle_c=C\exp\{-[(T/2\Theta)(\ln W_c)^2+n\ln W_c]\}\ , \tag{51}$$

where

$$C=1+\left[1+\frac{T\ln W_c}{n\Theta}\right]^{-1}\approx 2\ . \tag{52}$$

Using (51) and (41) in Eq. (42) gives

$$\ln t\approx(T/2\Theta)(\ln W_c)^2-\ln W_c \tag{53}$$

and explicitly from the expression for ξ_1 [Eq. (41)]

$$\ln\xi=(T/2\Theta)(\ln W_c)^2\ . \tag{54}$$

This should be used to eliminate $(\ln W_c)$ from Eq. (53). Now, from (50)

$$|(T/2\Theta)\ln W_c|<1\ , \tag{55}$$

and the leading term on the right-hand side (rhs) of (53) is $|\ln W_c|$. To this order:

$$\ln\xi\approx(T/2\Theta)(\ln t)^2\ . \tag{56}$$

This can also be written

$$\xi(t)\approx t^{\nu(t)}\ , \tag{57}$$

where

$$\nu(t)\approx(T/2\Theta)\ln t<\tfrac{1}{2}\ . \tag{58}$$

The inequality on the rhs of (58) follows from (50). One thus predicts anomalous diffusive behavior in this time domain. To match to the long-time diffusion described by Eq. (47) it is convenient to invert Eq. (56)

$$\ln t\approx[(2\Theta/T)\ln\xi]^{1/2},\quad \ln\xi\ll\Theta/T\ . \tag{59}$$

This crosses over smoothly into the long-time diffusion regime (47).

We note that the argument is self-consistent also with respect to fluctuations. From Eq. (41) and (51) one finds

$$[\langle 1/\tilde{W}^2\rangle_c/\langle 1/\tilde{W}\rangle_c^2]/\xi_1(W_c)\approx 1\ . \tag{60}$$

Thus the number of steps $\xi_1(W_c)$ also assures that fluctuations in the resistance will not be too large.

An alternative description of the results is in terms of the time (t) or distance (ξ) dependence of the diffusion constant:

$$\ln D\approx-\ln\langle 1/\tilde{W}\rangle_c\approx(T/2\Theta)(\ln W_c)^2+\ln W_c\ , \tag{61}$$

so that using Eq. (53),

$$\ln D\approx-\ln t[1-(T/2\Theta)(\ln t)]\leq 0\ , \tag{62}$$

or equivalently from Eq. (54),

$$\ln D\approx\ln\xi-[(2\Theta/T)\ln\xi]^{1/2}\ . \tag{63}$$

Finally we want to comment on the very-short-time behavior, i.e.,

$$\Theta/T>(\ln W_c)^2\ . \tag{64}$$

It can be seen from Eq. (40) that this implies $p_c\simeq 1$ or [Eq. (41)] $\xi_1(W_c)\simeq 1$. The statistical self-consistent approach becomes meaningless. It can be seen [e.g., from Eq. (53)] that this corresponds to

$$(\ln t)^2<\Theta/T\ . \tag{65}$$

The dynamics for this short-time regime is controlled by the average hopping rate ($\langle\tilde{W}\rangle$).

It would be fairly straightforward to present the time-dependent regime in more detail. One can, for example, translate from the number of hops (ξ) into real length using the proper averages of the hopping distances $s(\tilde{W})$ [Eq. (17)]. We present elsewhere[15] a numerical determination of the frequency- and temperature-dependent conductivity using an effective medium approach and the distribution $\rho(\tilde{W})$ [Eq. (12)].

Our main results are the following:

We have found that the activated Kurkijärvi resistivity[2] must be regarded as an asymptotic limit. It is valid only for sufficiently long chains and long time [Eq. (26)].

One predicts an anomalous intermediate time regime with a time-dependent diffusion constant and a corresponding intermediate length regime. Both can still be regarded as proper variable range hopping.

Comparison with experiments requires some further comment. For quasi-one-dimensional chain structures the average spacing Θ is found to be fairly large and, as noted in the introduction, one seems to observe an anomalous hopping regime similar to the one predicted. Detailed comparison is difficult because of crossover to three-dimensional behavior and the interference of the Peierls transition, but also because it is not obvious that the barriers observed are actually of the type we discussed and are not, e.g., due to interruptions in the one-dimensional chain structure.

For long wires the difficulties are of a different origin. Taking account of the finite cross section of the wires (S) one must replace Eq. (2) for the level spacing by

$$\Theta\approx[lN(0)S]^{-1}\ . \tag{66}$$

This tends to make Θ very small. Discussions of these experiments[17] usually assume that the Thouless length $[L\simeq v_f(\tau_i\tau_e)^{1/2}]$ is short compared to the localization length (l). Implicitly they also as-

sume that T/Θ is large so that localization has no effect on the inelastic scattering time (τ_i). Both assumptions seem justified for the actual experimental situations. To observe variable range hopping one would presumably need even thinner wires (to increase S) or much lower temperatures.

We have neglected quantum-mechanical effects in two ways. Near the crossover to the exponential regime [Eq. (35)] quantum-mechanical fluctuations must become important. Probably more interesting is the competition between the quantum-mechanical frequency dependence (e.g., Refs. 4 and 16) and the classical anomalies we predict. The latter must dominate at sufficiently low frequencies but the detailed comparison is model dependent and involves parameters which we did not specify. The classical frequency scale was factored out in Eq. (1). Our model also omitted correlations which are crucial for the quantum-mechanical problem. Thus a careful calculation on a more specific model is required for this comparison.

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[18]Using the fact that the distribution of $\ln\tilde{W}$ is well behaved a proof is straightforward.