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Spin-glass and ferromagnetic states in amorphous solids

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The spatial correlation between the directions of magnetization is calculated phenomenologically for an amorphous solid with a random distribution of easy axes. It is found that, for large exchange and small anisotropy, ferromagnetism can exist only in more than four dimensions. For all dimensions $d > 2$, spin-glass states of different types are possible. The dependence of magnetic behavior on the intrinsic parameters of an amorphous solid is established.

A wide range of magnetically ordered amorphous solids has been found during the last decade.[1] There are two general models for amorphous magnetism which assume either random space distribution of exchange[2] or random distribution of anisotropy axes.[3] Considering the later model, it is possible to divide amorphous magnetics into two groups depending on the ratio λ of anisotropy to exchange.

When anisotropy is large compared with exchange, the local crystal field orients the atomic spins practically along easy axes at every site. Random distribution of easy axes gives way to a magnetically disordered state. It is evident, though, that a state with the atomic spins directed in a hemisphere is energetically favorable due to exchange interaction. This leads to the appearance of the local magnetization which rotates smoothly over the volume of magnetic. The scale of this rotation is defined by the prehistory of the sample. In particular, bulk ferromagnetism, when all the atomic spins of the sample are directed in a hemisphere, can be created by a strong magnetic field.[4]

The case of $\lambda << 1$ seems more subtle to us. On the one hand, the large exchange favors the uniform magnetization, while on the other hand, when moving along some path through a solid, the magnetization "feels" numerous pushes of random local anisotropy field, which is similar to the Brownian motion. One cannot predict beforehand whether these pushes destroy the long-range ferromagnetic order or not. This problem was studied by Pelcovits, Pytee, and Rudnic[5,6] by means of renormalization-group techniques. They have shown that long-range ferromagnetism exists in more than four spatial dimensions, and that in fewer than four dimensions the low-temperature phase is a spin-glass.[7]

We have used the macroscopic approach to the description of amorphous magnetism with random anisotropy field $\vec{n}(x)$ and $\lambda << 1$. The easy axes are considered correlated on the correlation length R_c, which includes some interatomic distances a. We find the correlation between the directions of the local magnetization $\vec{M}(x)$ in two spatially separated points. The characteristic scale R_m of the destruction of this correlation defines the type of the magnetic ordering. In two dimensions we find a completely disordered state for a very large system. In more than two dimensions we obtain a spin-glass state with $R_m \sim R_c$ for $\Lambda = \lambda (R_c/a)^2 \gtrsim 1$. For $\Lambda << 1$, long-range ferromagnetism is established in more than four dimensions, while the case of three and four dimensions give a spin-glass state with $R_m >> R_c$. The ratio R_m/R_c depends strongly on the dimensionality of space.

We shall describe an amorphous magnetic in the d-dimensional space by the energy functional

$$E = \int d^dx \left[\frac{1}{2}\alpha \left(\frac{\partial \vec{M}}{\partial x_i} \right)^2 - \frac{1}{2}\beta (\vec{M}\cdot\vec{n})^2 \right] , \qquad (1)$$

with random anisotropy field $\vec{n}(x)$, $\vec{n}^2(x) = 1$. For $\vec{n}(x) = \text{const}$, (1) is used for the description of domain structure in a crystalline ferromagnet. In (1), $\alpha > 0$ and $\beta > 0$ are the exchange and anisotropy constants, respectively. In terms of these constants, $\lambda << 1$ corresponds to $\lambda = \beta a^2/\alpha << 1$. The local magnetization is assumed to rotate smoothly over the volume so that $\vec{M}^2(x) = \text{const}$. To begin with, we shall consider for simplicity a two-dimensional field $\vec{n}(x)$, its components being equal to

$$n_1(x) = \cos\theta(x), \quad n_2(x) = \sin\theta(x) , \qquad (2)$$

where $\theta(x)$ is a given random scalar field. This makes the local magnetization also to be two-dimensional with the components

$$M_1(x) = M \cos\xi(x), \quad M_2(x) = M \sin\xi(x) . \qquad (3)$$

Substitution of (2) and (3) into (1) gives

$$E = M^2 \int d^dx \left[\frac{1}{2}\alpha \left(\frac{\partial \xi}{\partial x_i} \right)^2 - \frac{1}{2}\beta \cos^2(\xi - \theta) \right] . \qquad (4)$$

Minimization of (4) leads to the following equation:

$$\Delta_d \xi(x) = \frac{\lambda}{2a^2} \sin 2[\xi(x) - \theta(x)] , \qquad (5)$$

Δ_d being a d-dimensional Laplacian. The solution of (5) can be presented as

$$\xi(x)=\frac{\lambda}{2a^2}\int d^dx'\,G_d(x-x')\sin 2[\xi(x')-\theta(x')] \ , \tag{6}$$

where $\Delta_d G_d(x)=\delta_d(x)$; $G_d(x)$ and $\delta_d(x)$ are the d-dimensional Green function and the δ function. The Green function $G_d(x)$ is given by

$$G_d(x)=\begin{cases}(2\pi)^{-1}\ln|x|, & \text{for } d=2\\ -[\Gamma(d/2)/2\pi^{d/2}(d-2)]|x|^{2-d}, & \text{for } d\geqslant 3 \ ,\end{cases} \tag{7}$$

where Γ is the gamma function.

We shall assume random field $\theta(x)$ to satisfy the following conditions:

$$\langle \sin 2\theta(x)\rangle =0 \ ,$$
$$\langle \sin[2\theta(x)]\sin[2\theta(x')]\rangle=\tfrac{1}{2}\exp\left(-\frac{|x-x'|}{R_c}\right) \ . \tag{8}$$

The magnetic disorder in our model is characterized by the correlation function $C_d(|x|)=\langle \xi(x)-\xi(0)^2\rangle$. It can be calculated assuming $R_m >> R_c$ which corresponds to $\xi(x)\approx\xi(x')$ for $|x-x'|\sim R_c$. In this case, with the help of (6) and (8), we have for C_d,

$$C_d=\frac{\lambda}{8a^4}\int\int d^dx'\,d^dx''\exp(-|x'-x''|/R_c)[G_d(x')-G_d(x-x')][G_d(x'')-G_d(x-x'')] \ . \tag{9}$$

Integration in (9) gives a logarithmic divergence of $C_2(|x|)$ for a very large system

$$C_2\sim\Lambda^2\left(\frac{|x|}{R_c}\right)^2\ln\frac{L}{|x|} \ , \tag{10}$$

where L is the size of a system. For $D\geqslant 3$ we obtain

$$C_3=\tfrac{1}{4}\Lambda^2\frac{|x|}{R_c} \ , \tag{11}$$

$$C_4=\tfrac{3}{8}\Lambda^2\ln\frac{|x|}{R_c} \ , \tag{12}$$

$$C_d\sim\Lambda^2, \quad \text{for } d\geqslant 5 \ . \tag{13}$$

Condition $C_d << 1$ defines the spatial region with ferromagnetic ordering. For small λ and not too large R_c, so that $\Lambda << 1$, we have from (13) a long-range ferromagnetic order for $d\geqslant 5$. In three and four dimensions we obtain from (11) and (12) a spin-glass behavior. The condition $C_d << 1$ gives the characteristic scale of the destruction of the ferromagnetic order

$$R_m\sim\begin{cases}\Lambda^{-2}R_c, & \text{for } d=3 \quad (14)\\ R_c\exp\Lambda^{-2}, & \text{for } d=4 \ . \quad (15)\end{cases}$$

Relation (14) was obtained by Alben, Becker, and Chi in Ref. 8.

For all $d\geqslant 3$, our assumption $R_m >> R_c$ is valid when $\Lambda << 1$. It is evident that $R_m\sim R_c$ for $\Lambda \gtrsim 1$. This state is the Edwards-Anderson spin-glass while the state with $R_m >> R_c$ could be called a correlated spin-glass. We would like to emphasize that these two types of magnetic behavior depend not on the ratio λ of anisotropy to exchange but on the parameter $\Lambda=\lambda(R_c/a)^2$. This means that the cases of large and small anisotropy discussed before correspond to $\lambda > (a/R_c)^2$ and $\lambda < (a/R_c)^2$, respectively. In the latter case the magnetic behavior of amorphous solid does not depend on its prehistory.

It should be noted that all our results would not change if we use in (8) any other correlation function which rapidly goes to zero for $|x-x'| >> R_c$.

It is of great interest to consider a physical case, i.e., three-component fields $\vec{n}(x)$, $\vec{M}(x)$ in three-dimensional space

$$\vec{n}(x)=(\cos\theta,\ \sin\theta\sin\phi,\ \sin\theta\cos\phi) \ ,$$
$$\vec{M}(x)=M(\cos\xi,\ \sin\xi\sin\psi,\ \sin\xi\cos\psi) \ , \tag{16}$$

where (θ,ϕ) and (ξ,ψ) are the angles of $\vec{n}(x)$ and $\vec{M}(x)$ in a spherical coordinate system. In this case the energy functional (1) takes the form

$$E=M^2\int d^3x\left\{\tfrac{1}{2}\alpha\left[\left(\frac{\partial\xi^2}{\partial\vec{x}}\right)+\sin^2\xi\left(\frac{\partial\psi}{\partial\vec{x}}\right)^2\right]-\tfrac{1}{2}\beta[\sin\xi\sin\theta\cos(\psi-\phi)+\cos\xi\cos\theta]^2\right\} \ . \tag{17}$$

We will calculate the correlation function $\langle[\vec{M}(x)-\vec{M}(0)]^2\rangle$. For $|x| << R_m$ it reduces to the form

$$\langle[\vec{M}(x)-\vec{M}(0)]^2\rangle=M^2\langle[\xi(x)-\xi(0)]^2+\sin\xi(x)\sin\xi(0)[\psi(x)-\psi(0)]^2\rangle \ . \tag{18}$$

Minimizing the functional (17) we have in the lowest order in λ

$$\Delta\xi=\frac{\lambda}{a^2}[\sin\xi\cos\theta-\cos\xi\sin\theta\cos(\psi-\phi)][\sin\xi\sin\theta\cos(\psi-\phi)+\cos\xi\cos\theta] \ ,$$
$$\Delta(\psi\sin\xi)=\frac{\lambda}{a^2}\sin\theta\sin(\psi-\phi)[\sin\xi\sin\theta\cos(\psi-\phi)+\cos\xi\cos\theta] \ . \tag{19}$$

Calculations analogous to those of $C_d(|x|)$ give the following expression for the correlation function of magnetization for $R_c << |x| << R_m$:

$$\langle [\vec{M}(x) - \vec{M}(0)]^2 \rangle = \frac{4}{15} M^2 \Lambda^2 \frac{|x|}{R_c} \quad . \qquad (20)$$

It gives the same estimation for R_m as Eqs. (11) and (14). Thus we see that the correlation function practically does not depend on the number of components of the random field $\vec{n}(x)$, but depends strongly on the dimensionality of space.

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