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Temperature gradient along superfluid ^{4}He films in the presence of superflow

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We have measured the nonlinear thermal conductance of a thin film of ^{4}He as a function of temperature and heater power for a fixed film thickness. When heat transfer through the gas is included we obtain a qualitative understanding of the data in terms of Kosterlitz-Thouless theory as extended to finite superflow by Ambegaokar *et al.* The exponent in the power law relating conductance and heater power exhibits the universal jump at T_c predicted by its simple relationship to the areal superfluid density.

It is now rather well established that the superfluid transition in thin films of ^{4}He is associated with vortex-antivortex unbinding as discussed by Kosterlitz and Thouless.[1] The magnitude of the jump in the superfluid density at the transition temperature, as observed by Rudnick *et al.*,[2] Chester *et al.*,[3] and Bishop *et al.*,[4] agrees with universal behavior predicted by Nelson and Kosterlitz.[5] The detailed character of the dissipation of superflow at finite frequencies and in the presence of a finite superflow, has been discussed by Ambegaokar *et al.*[6] and Huberman *et al.*[7] Some of these predicted features have recently been confirmed by Maps and Hallock[8,9] and by Angolet *et al.*[10] They observed a predicted exponential dependence of the effective thermal conductance on reduced temperature for $T < T_c$. More recently,[9-11] a power-law dependence of the conductance on applied thermal power was observed for $T < T_c$.[12]

In this Communication we report measurements of the effective conductance in a superfluid film as a function of applied thermal power and of temperature for fixed film thicknesses. Contrary to what has been assumed,[13] the superflow velocity along the film is not uniform because of heat transport through the gas, and this nonuniformity leads to position dependence of the conductance. More importantly, the dependence of the conductance on applied heater power is, under accessible experimental conditions, quite different from that which had been expected. Only when gas conduction is included are we able to quantitatively explain the simple relationship observed between the exponent and the coefficient in the power law relating conductance and heater power. Finally, the measurements of the temperature dependence of the exponent are presented and it is shown that, except perhaps very close to T_c, the exponent (and therefore the superfluid density) varies linearly with temperature. Within 10 mK of T_c the exponent increases from one to two, consistent with the universal jump in σ_s. It is reasonable that the rise in the exponent is spread over this temperature range in that the correlation length becomes comparable with the size of the system within a few mK of T_c.

Our experimental arrangement is similar to that of Maps and Hallock. A thin glass strip 1 cm wide by 5 cm long is thermally anchored at one end and a resistance heater is placed at the other. The apparatus is placed within a sealed container inside of which is also placed a large quantity of Grafoil.[14] The effect of the Grafoil is to greatly reduce the temperature dependence of the film thickness. A fiberglass "wick" is used to couple the Grafoil to the thermally anchored end of the glass strip so as to complete the thermal anchoring of the film. Several carbon thermometers were placed at various positions on the glass strip and the resistors were moved about during the course of the experiment in order to make certain that the thermometers do not substantially affect the superflow.

In film conduction experiments of the type discussed in this paper, the conductivity of the helium gas plays an important role. In response to applied power at the heater, helium is evaporated away and the superfluid flows along the film. As a consequence, there will be a steady-state density of free vortices[6]

$$n_f \propto v_s^{[2+x(T)/2]} \quad , \tag{1}$$

where $x(T) = -4 + 2\pi\hbar^2 m^{-2} k_B^{-1} \sigma_s / T$ is zero at T_c. Because the vortice move in response to v_s, a temperature gradient is created along the film:

$$\frac{dT}{dz} \propto v_s n_f \propto v_s^{[3+x(T)/2]} \quad . \tag{2}$$

Now a temperature gradient along the film necessarily implies that there is thermal conduction from the film through the gas.[15] Of course, the film cannot transport entropy so the energy conducted from the film is obtained through a net condensation of gas onto the film. We neglect the very small entropy production rate associated with vortex dissipation in the film. The rate at which latent heat is released through condensation is then just equal to the rate of thermal conduction. This condensation causes a gradient in the superfluid velocity. It follows that

$$L\sigma_s \frac{dv_s}{dz} = \frac{k(T-T_b)}{l} , \quad (3)$$

where L is the latent heat, σ_s the areal superfluid density, k is the gas conductivity, T is the temperature of the film at the location z, T_b is the temperature of the surroundings, and l is some effective distance to those surroundings. The heat input by the heater must, of course, be included in the vicinity of the heater. We explicitly ignore any variation of the film thickness along the film, which would probably make the effect discussed here even more important. Solving for the temperature and superfluid velocity gradients self-consistently yields an effective conductance which depends on the location of the thermometer. In addition, the temperature at some position on the film depends on the heater power in a more complicated manner than $T \propto Q^{3+x/2}$.

The temperature gradient is given by[6]

$$\frac{dT}{dz} \approx \frac{h}{s} \frac{Dh\sigma}{mk_BT} v_0 a_0^{-2} x(T) \left(\frac{v_s}{v_0}\right)^{3+x(T)/2}$$

$$\equiv \frac{1}{2} Fx(T) \left(\frac{v_s}{v_0}\right)^{3+x(T)/2} , \quad (4)$$

where s is the specific entropy, D is the vortex diffusivity, σ is the "background" superfluid areal density, m is the mass of a helium atom, a_0 is the vortex core radius, and $v_0 = \hbar/ma_0$. Differentiating Eq. (3) yields

$$\frac{d^2v_s}{dz^2} = \frac{kFx}{2L\sigma_s l} \left(\frac{v_s}{v_0}\right)^{3+x/2} . \quad (5)$$

Therefore

$$\frac{dv_s}{dz} = \left(\frac{kFx}{L\sigma_s l(4+x/2)}\right)^{1/2} \times \left[\left(\frac{v_s(z)}{v_0}\right)^{4+x/2} - \left(\frac{v_s(0)}{v_0}\right)^{4+x/2}\right]^{1/2} . \quad (6)$$

One can easily show that for $v_s(\text{heater}) >> v_s(0)$,

$$\frac{\Delta T(\text{heater})}{\Delta T_0} = \left(\frac{Q}{Q_0}\right)^{2+x/4} , \quad (7)$$

where

$$\Delta T_0 \sim \frac{L\sigma_s l}{k} \left(\frac{Fx}{4+x/2}\right)^{1/2} , \quad Q_0 \sim \frac{L\sigma_s w\hbar}{ma_0} ,$$

and w is the width of the film.

In Fig. 1, we show a plot of the temperature at a point about 1 cm from the heater as a function of heater power for various temperatures. Clearly, the temperature varies as a power of the applied heater power over a substantial range before saturating at high powers. The temperature difference, at which non-power-law behavior is apparent, increases as the temperature is decreased below T_c. In the power-law region where $\Delta T = A(T)Q^{B(T)}$, Eq. (7) implies that A and B are related to each other by the expression

$$\ln A(T) = \ln(\Delta T_0) - B(T)\ln Q_0 , \quad (8)$$

where $\ln(\Delta T_0)$ is only a weak function of temperature as long as T is not too close to T_c.

Figure 2 is a plot of the measured values of $\ln(\Delta T)$ vs $B(T)$, confirming the linear relationship for temperatures not too close to T_c. Note that this relationship implies that the power-law curves in Fig. 1 radiate from a common point on the plot. Furthermore, the values of Q_0 (120 μW) and ΔT_0 (240 K) obtained from the data are in reasonable agreement with the expected values. Taking $a_0 \sim 4$ Å yields $Q_0 \sim 400$ μW. Taking $k \sim 30$ μW/cm K,

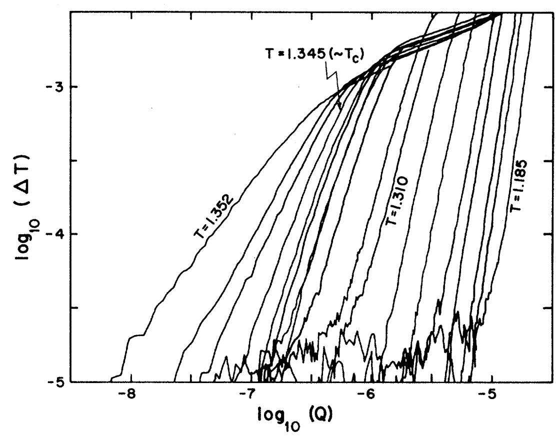

FIG. 1. Plot of the change in temperature at a point in the film as a function of applied heater power for various temperatures.

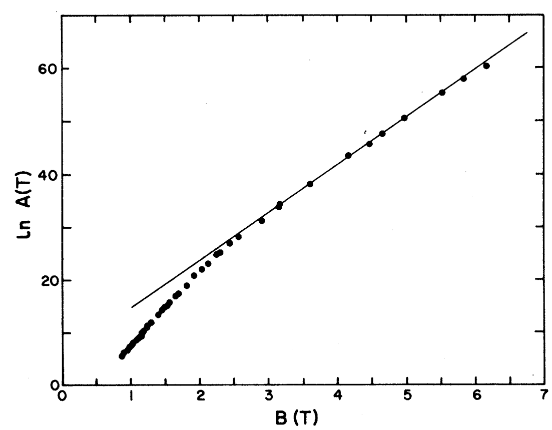

FIG. 2. Plot of the logarithm of the power-law coefficient vs the exponent. The straight line is our fit to the data below T_c.

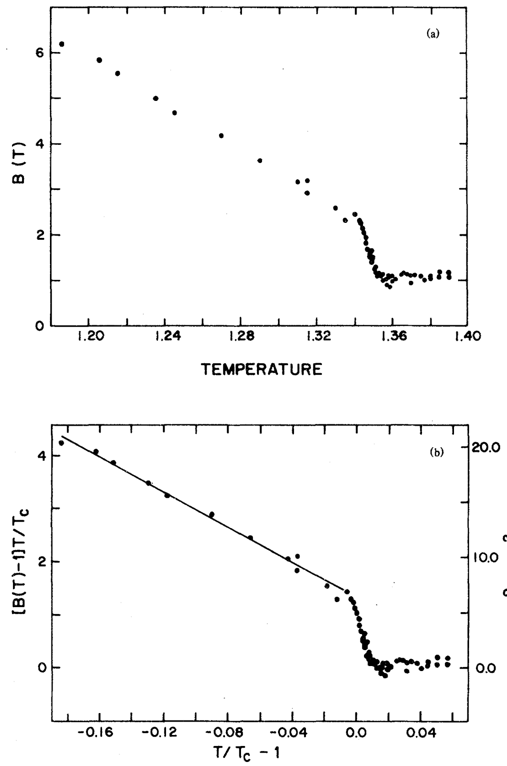

FIG. 3. (a) Plot of the power-law exponent $B(T)$ vs temperature. (b) Plot of the reduced exponent vs reduced temperature. In calculating reduced temperature, account is taken of the slow variation of film thicknesses with temperature. The scale on the right is the corresponding superfluid density. The straight line is for reference only.

$Dh\sigma/mk_BT \sim 0.2$, and $l \sim 1$ cm yields $\Delta T_0 \sim 170$ K. Note that ignoring the effect of thermal conduction from the film yields the same value for Q_0, but a value for ΔT_0 of 2.5×10^9 K, seven orders of magnitude larger.

Figure 3(a) shows a plot of the exponent $B(T)$ as a function of temperature. This exponent should be related to σ_s by the expression

$$B(T) = 1 + [(\pi/2)\hbar^2 m^{-2} k_B^{-1}]\sigma_s/T \quad .$$

To obtain σ_s we plot $[B(T)-1]T/T_c$ versus reduced temperature in Fig. 3(b). In spite of the large Grafoil surface area, the film thickness changes by about 7% over the range of temperatures shown. We therefore take T_c as a weak function of temperature in calculating the reduced temperature $(T/T_c - 1)$. The transition temperature is arbitrarily taken as the temperature where $B(T) = 2$. Except very close to T_c, the data are well characterized by a linear dependence on temperature.

The analysis presented in this paper suggests that one must be very careful in interpreting heat conductance experiments in thin superfluid films. The exponent characterizing the power-law dependence of temperature gradient on heater power is significantly affected by the conduction of heat from the film through the gas and the consequent condensation onto the film. The exponent observed is not the zero-power value $3 + x/2$ but rather the quantity $2 + x/4$. We have not observed crossover from one regime to the other and we expect this to occur at powers well below those limited by considerations of temperature resolution in our apparatus.

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[12]We note that R. A. Joseph and F. M. Gasparini [Physica (Utrecht) (in press)] observed non-power-law behavior on a stainless-steel surface.

[13]S. Teitel, J. Low Temp. Phys. 46, 77 (1982). We note that the effects considered in our paper are not relevant for the geometry used in the experiment of Agnolet *et al.* (Ref. 10).

[14]Grafoil is a tradename of Union Carbide Corporation.

[15]The very small Kapitza resistance at the liquid-gas interface [see, e.g., H. Wiechert, J. Phys. C 9, 553 (1976)] is ignored.