Area dependence of exchange stiffness energy in a spin-glass

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The area dependence of the exchange stiffness energy is investigated for a $d=3$ Heisenberg spin-glass with Gaussian couplings. The results agree with the linear area dependence postulated by Halperin and Saslow.

The presence of random and conflicting exchange interactions in a spin-glass introduces frustration in the system. This results in a lack of any long-range spatial ordering of the spins and leads to a multiplicity of ground states. The physics of a spin-glass at low temperatures may be understood in terms of two kinds of phenomena: (1) spin-wave-like excitations within the vicinity of a given ground state, and (2) transitions between different ground states. At very low temperatures, due to the presence of barriers between ground states, it is possible that the spin-glass remains close to a ground state for a long period of time.

Huber and Ching,[1] using the method of equations of motion, and Halperin and Saslow,[2] by constructing a hydrodynamic theory, predicted the existence of spin-wave excitations with a linear dispersion relation in the spin-glass. The occurrence of such excitations relies upon a nonzero value of the exchange stiffness for the system.

Following Halperin and Saslow,[2] the stiffness constant ρ_s is defined by the increase in free energy, ΔE, above its equilibrium value given by

$$\Delta E = \tfrac{1}{2}\rho_s \int d^3r\,|\nabla\theta(\vec{r})|^2 , \tag{1}$$

where $\theta(\vec{r})$ measures the local rotation angle in the spin-glass about the equilibrium state. It is assumed that the spins are relaxed to equilibrium (a local minimum in the free energy is obtained), while a net rotation is maintained across the sample.

For an overall twist of $\Delta\theta$ across the sample, ΔE scales as

$$\Delta E \sim \rho_s(\Delta\theta)^2 A/L . \tag{2}$$

Here A refers to the area of a hypercube and L to its length. The twist $\Delta\theta$ is assumed to be imposed across the length of the hypercube.

Recently Reed[3] and Walstedt[4] have carried out numerical calculations of the exchange stiffness constant for a nearest-neighbor Heisenberg and a Ruderman-Kittel-Kasuya-Yosida (RKKY) spin-glass, respectively, in three dimensions. In both cases the magnitude of ρ_s was found to be consistent with that predicted by Halperin and Saslow. Their calculations, however, assumed the validity of Eq. (2) with respect to the area dependence. In both calculations the length dependence was studied and found to be in accord with Eq. (2).

In this note we report results of an investigation of the area dependence of ΔE. Our results confirm the predicted linear dependence on the area and lend further support for the possible existence of spin-wave-like excitations in spin-glasses. It is important to note that we have not considered effects of damping which may play an important role in the question of whether these excitations are propagating or not.

We considered classical Heisenberg spins on a simple cubic lattice coupled by nearest-neighbor exchange interactions and at $T=0$. The distribution of exchange couplings was Gaussian characterized by unit variance and zero mean value. We investigated systems with $L=8$ and $A=4\times 4$, 8×8, 12×12, and 16×16. Periodic boundary conditions were applied in the transverse directions (across the ends of the planes of area A).

In the first stage of the calculations we applied free boundary conditions in the longitudinal direction and we relaxed the system to five different "ground states" by starting from five random configurations of spins. The process of relaxation was done as suggested by Walker and Walstedt,[5] i.e., by aligning the spins sequentially in the direction of their instantaneous local fields. If we define one iteration as AL spin alignments then the "ground state" was reached within 800–5000 iterations, depending on the size of the system.

In the second stage of the calculations we applied a twist of 5° about a fixed axis to the spins on one of the (initially free) boundaries. Thereafter, the spins at both boundaries were kept frozen. The bulk spins were then relaxed within several hundred iterations. The final energy of the system was higher than the "ground-state" energy by an amount ΔE. For a given sample an average of ΔE over the five "ground states" was taken. The individual ΔE's differed by less than 50%.

The procedure was repeated for seven other samples with the same probability distribution of the exchange couplings and the average $\overline{\Delta E}$ over the eight samples was calculated.

We determined $\overline{\Delta E}$ for four values of A and the results of our findings are presented in Fig. 1. The error bars indicate the size of the standard deviation. Our results agree with the linear dependence on the area, as suggested by Eq. (2). The straight line in Fig. 1 represents the linear law. The value of $\overline{\Delta E}$ for the smallest system, i.e., the one of area 4×4, is a little above the straight line, presumably due to the poor statistics of the small system.

It should be noted that our numerical procedure is somewhat simpler than the one adopted by Walstedt.[4] Checks of the length dependence yielded results in agreement with those of Walstedt[4] and Reed.[3]

A spin-glass may be visualized as being made up of many statistically similar blocks of length L and area A. The strength of the coupling between neighboring blocks is related to the free-energy sensitivity of a block to a change in the boundary conditions. Recently it has been suggested[6,7] that the sensitivity of the system to changes in the boundary conditions may yield information about the nature of ordering in spin-glasses. It is interesting to note that both attempts[6,7] at determining this sensitivity yielded an $A^{1/2}$ dependence.

In contrast to the exchange stiffness problem, when estimating the block-coupling energy one considers two initially separated blocks of spins. If each of the blocks is in its ground state one asks for the total energy change on coupling the blocks together. In spin-glasses, this change could be positive or negative and is proportional to $A^{1/2}$, as shown by Anderson and Pond.[6]

On the other hand, in Ref. 7, the effects on interacting blocks are mimicked in a single block by imposing periodic or antiperiodic boundary conditions on the system. If the corresponding free energies are denoted by F_P and F_{AP} respectively, then $\Delta F = F_{AP} - F_P$ for a spin-glass can again be of either sign. It

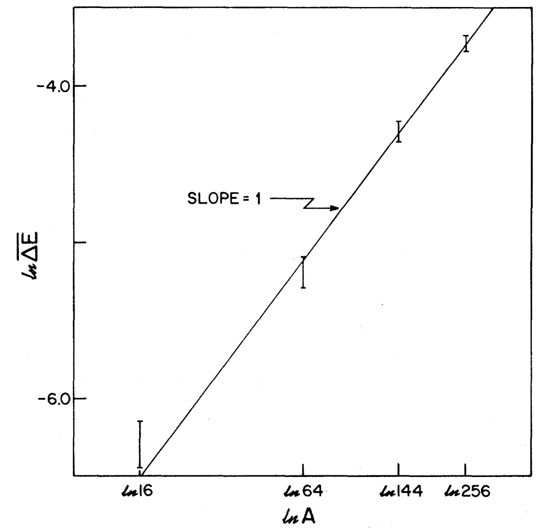

FIG. 1. Plot of $\ln\overline{\Delta E}$ vs $\ln A$ for $L = 8$.

follows that the configurational average of ΔF, $\langle \Delta F \rangle_c$, over the distribution of the exchange constants, vanishes. This, in turn, suggests that the appropriate measure of the sensitivity of the system to a change in boundary conditions is

$$\delta F = [\langle (\Delta F)^2 \rangle_c]^{1/2} \ . \tag{3}$$

The $A^{1/2}$ law for δF arises primarily as a result of adopting the second moment of ΔF in the definition of δF.

Unlike ΔF, the exchange stiffness energy is necessarily positive. Once a system is in its local free energy minimum, it always cost energy to impose a twist by a small angle in the vicinity of the minimum. Our numerical results show that this exchange stiffness energy, as postulated by Halperin and Saslow,[2] is indeed proportional to the area.

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