Ellipticity and Faraday rotation due to a two-dimensional electron gas in a metal-oxide-semiconductor system

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We calculate the ellipticity and Faraday rotation due to the two-dimensional electron gas at the oxide-semiconductor interface of a metal-oxide-semiconductor system. The results obtained are radically different from those we previously obtained using the single-pass three-dimensional Drude model. To track down the difference, we extended the latter model to include boundary effects at the inversion-layer interfaces as well as multiple-reflection effects within the inversion layer, and we find that, as the inversion-layer thickness goes to zero (compared to the wavelength of the radiation), we reproduce the two-dimensional-model results.

Recently Chiu et al.[1] have calculated the transmission coefficients for a two-dimensional electron gas (2DEG), in the case where the directions of both the incident radiation of frequency ω and the external dc magnetic field $B > 0$ are oriented normal to the insulator-semiconductor interface containing the 2DEG in a metal-oxide-semiconductor (MOS) structure. This model of the inversion layer has been used[2] to interpret experimental cyclotron resonance data. It would also be appropriate for the interpretation of Faraday rotation and ellipticity data in the case where a wedge is used to eliminate multiple reflection effects at the outer semiconductor interface.

In this work we investigate the Faraday rotation and ellipticity due to the 2DEG and show that there is a striking difference between the predictions of this model and those of the single-pass Drude model investigated in Refs. 3 to 5.

The results of Chiu et al.[1] for the transmission coefficients of the right and left circularly polarized components of a linearly polarized wave are

$$t_{\pm} = \frac{2n_o}{n_o + n_s + (4\pi/c)\sigma_{\pm}} , \tag{1}$$

where n_o and n_s are the indices of refraction of the oxide and the semiconductor respectively, and

$$\sigma_{\pm} = \frac{iNe^2/m^*}{\omega \pm \omega_c + i\nu} \tag{2}$$

is the conductivity of the 2DEG, N is the electron

surface concentration, m^* the effective mass, $\omega_c = eB/m^*c$ the cyclotron frequency, and ν is the collision frequency (equal to τ^{-1}, where τ is the collision time).

It is convenient to rewrite Eq. (1) as

$$t_{\pm} = |t_{\pm}|e^{i\xi_{\pm}} , \tag{3}$$

where

$$\xi_{\pm} = -\tan^{-1}\frac{\omega_{ps}(\omega \pm \omega_c)}{(\omega \pm \omega_c)^2 + \nu(\nu + \omega_{ps})} , \tag{4}$$

where, in the notation of Ref. 1,

$$\omega_{ps} \equiv 4\pi Ne^2/m^*c\,(n_o + n_s) .$$

The Faraday rotation θ and the ellipticity δ are related[6] to the transmission coefficients by

$$\frac{t_-}{t_+} = \left(\frac{1-\delta}{1+\delta}\right)e^{-2i\theta} . \tag{5}$$

Thus

$$\theta = \tfrac{1}{2}(\xi_+ - \xi_-) \tag{6}$$

and

$$\delta = \frac{|t_+| - |t_-|}{|t_+| + |t_-|} . \tag{7}$$

From Eqs. (1)–(7) we obtain

$$\theta = \tfrac{1}{2}\tan^{-1}\left\{\frac{2\omega_{ps}\omega_c[\omega^2 - \omega_c^2 - \nu(\nu + \omega_{ps})]}{[(\omega + \omega_c)^2 + \nu(\nu + \omega_{ps})][(\omega - \omega_c)^2 + \nu(\nu + \omega_{ps})] + \omega_{ps}^2(\omega^2 - \omega_c^2)}\right\} , \tag{8}$$

and

$$\delta = \frac{4\omega\omega_c\omega_{ps}(2\nu + \omega_{ps})}{\{[(\omega + \omega_c)^2 + \nu^2]^{1/2}[(\omega - \omega_c)^2 + (\nu + \omega_{ps})^2]^{1/2} + [(\omega - \omega_c)^2 + \nu^2]^{1/2}[(\omega + \omega_c)^2 + (\nu + \omega_{ps})^2]^{1/2}\}^2} , \tag{9}$$

for the Faraday rotation and ellipticity, respectively, due to the 2DEG at the oxide-semiconductor interface.

Inspection of Eq. (8) shows that null Faraday rotation[3] is achieved when the photon, cyclotron, collision, and plasma frequencies are related by

$$\omega^2 = \omega_c^2 + \nu(\nu + \omega_{ps}) \tag{10}$$

or, using the notation of Refs. 3 and 4,

$$\omega^2 = \Omega^2 + \nu\omega_{ps} , \tag{11}$$

where

$$\Omega \equiv (\omega_c^2 + \nu^2)^{1/2} . \tag{12}$$

In Figs. 1 and 2 we present a plot of θ versus B and ω, respectively. The existence of both positive and negative values for θ is in striking contrast to what was obtained using the single-pass Drude model. In the latter case θ is always negative for the same range of parameters.[5]

Evaluation of θ at $\omega = \omega_c$ from Eq. (4) gives

$$\theta|_{\omega=\omega_c} = -\frac{1}{2}\tan^{-1}\left[\frac{2\omega\omega_{ps}}{4\omega^2 + \nu(\nu + \omega_{ps})}\right] . \tag{13}$$
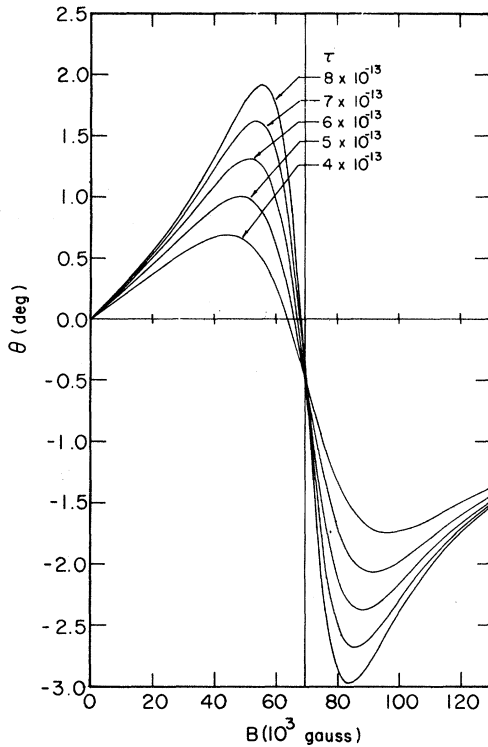
Thus $\theta|_{\omega=\omega_c}$ is ν (or τ) dependent. The apparent ν (or τ) independence of $\theta|_{\omega=\omega_c}$ in Figs. 1 and 2 is due to the fact that for each plot we have the conditions $\nu \ll 2\omega$ and $\omega_{ps} \ll \omega$ satisfied.

Similar differences occur in the case of ellipticity predictions. Inspection of Eq. (9) shows that $\delta > 0$ for all choices of the parameters. In particular, null ellipticity[4] is never achieved in this model. Thus we have a very striking difference between the present model [defined by Eqs. (1) and (2)] and the model investigated in Refs. 3 to 5 where it was shown that null ellipticity is obtained for a photon frequency ω satisfying $0 < \omega < \Omega$.

In Fig. 3 we present a plot of δ versus B and Fig. 4 is the corresponding plot of δ versus ω. The similarity in shape of the curves in Figs. 3 and 4 is due to the fact that δ remains unchanged under the replacement $\omega \rightleftharpoons \omega_c$.

We turn now to a discussion as to why the present model gives results which are strikingly different from the results obtained from the single-pass three-dimensional Drude model. At first glance this might even be surprising since the choice of the two-dimensional conductivity, given by Eq. (2), is itself of the Drude type—in the sense that it only differs
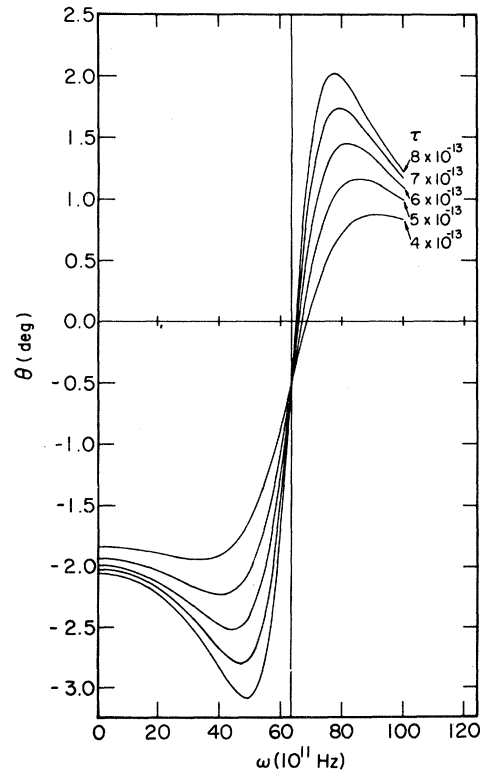

FIG. 1. Plot of the Faraday rotation θ vs the magnetic field B. The parameters used are $N = 2.3 \times 10^{12}$ cm^{-2}, $m^* = 0.19 m_e$, where m_e is the rest electron mass, $n_o = 1.95$, $n_s = 3.44$, $\omega = 6.455 \times 10^{12}$ s^{-1}, and for the τ values indicated on the curves. The vertical line corresponds to the value $\omega = \omega_c$ or $B = 6.97 \times 10^4$ G.

FIG. 2. Plot of the Faraday rotation θ vs the photon frequency ω using the same parameters as in Fig. 1 and $B = 6.89 \times 10^4$ G.
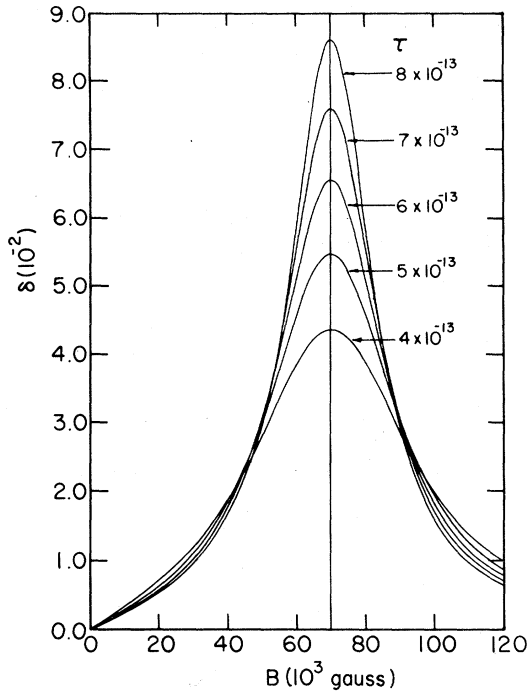
FIG. 3. Plot of the ellipticity δ vs the magnetic field B using the same parameters as in Fig. 1.
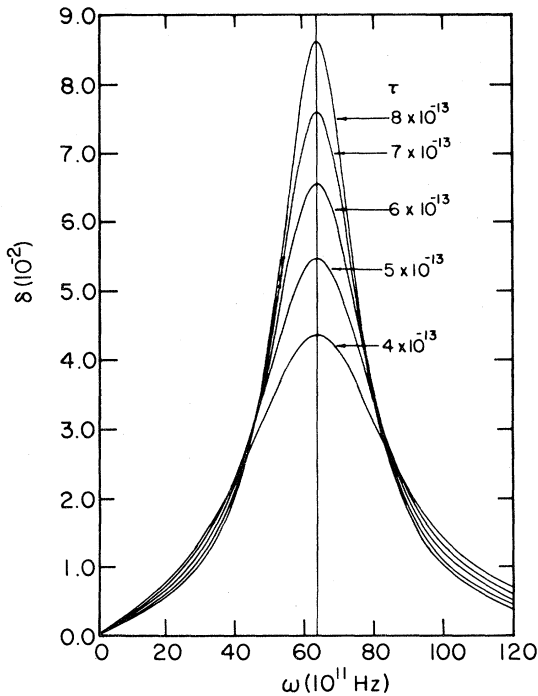
from the conventional three-dimensional Drude model by the replacement of the three-dimensional electron concentration by the electron surface concentration. Actually, the reason is because the present model contains essential features not found in the single-pass model. In fact, we have extended the single-pass three-dimensional model to include boundary effects at the inversion-layer interfaces as well as multiple-reflection effects within the inversion layer (see Fig. 5) and we find that as the inversion layer thickness goes to zero (compared to the wavelength of the radiation) we reproduce the results quoted above for the two-dimensional model.

Finally, we return to our exact results and note that if $\omega_{ps} \ll$ all the other frequencies (which is true for the parameters chosen in Figs. 1–4) a considerable simplification occurs, viz.,

$$\theta \approx \frac{\omega_{ps}\omega_c(\omega^2 - \Omega^2)}{[(\omega + \omega_c)^2 + \nu^2][(\omega - \omega_c)^2 + \nu^2]} \quad , \tag{14}$$

and

$$\delta \approx \frac{2\omega\omega_c\nu\omega_{ps}}{[(\omega + \omega_c)^2 + \nu^2][(\omega - \omega_c)^2 + \nu^2]} \quad . \tag{15}$$

For added insight and also as a check on the above, we note that if in Eq. (1) we assumed that $(4\pi/c)|\sigma_\pm| \ll (n_o + n_s)$, which is equivalent to the assumption $\omega_{ps} \ll [(\omega \pm \omega_c)^2 + \nu^2]^{1/2}$, then it follows immediately that

$$\theta \approx \frac{2\pi}{c(n_o + n_s)}(\sigma_-'' - \sigma_+'') \quad , \tag{16}$$

FIG. 4. Plot of the ellipticity δ vs the photon frequency ω using the same parameters as in Fig. 2.
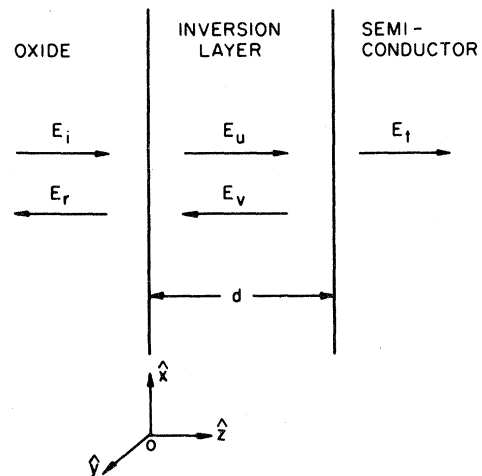

FIG. 5. Geometry for the propagation of electromagnetic radiation through an inversion layer of finite thickness d in a MOS structure.

and

$$\delta = \frac{2\pi}{c\,(n_o + n_s)}(\sigma'_- - \sigma'_+) \; , \qquad (17)$$

where $\sigma'_\pm$ and $\sigma''_\pm$ refer to the real and imaginary parts of $\sigma_\pm$, respectively. Hence, using (2) we again obtain Eqs. (14) and (15).

In conclusion, we have calculated the ellipticity and Faraday rotation produced by the electron gas in the inversion layer of a MOS system by extending our previous work beyond the realm of the so-called "single-pass" results and we found that significant changes occurred as a consequence.

ACKNOWLEDGMENTS

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