PHYSICAL REVIEW B VOLUME 26, NUMBER 4 15 AUGUST 1982

Crossover behavior in a two-dimensional electron gas and quasiplasma oscillation

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(Received 4 May 1982)

The dynamic structure factor for a two-dimensional (2D) electron gas at $T=0$ is calculated by a method of recurrence relations. Our result indicates an interesting crossover behavior in going from interacting to noninteracting, which is unique to 2D electronic systems. The crossover behavior is characterized by a simple power law with a "classical" exponent, probably detectable by x rays. The validity of our result is tested against standard moment sum rules and also the static form-factor sum rule at high and low frequencies.

The metal oxide semiconductor (MOS) and other similar materials have stimulated considerable interest in the physics of two-dimensional (2D) electronic systems in recent years.[1] Early efforts have centered on elucidating their static behavior. There are now advances being made in understanding their time-dependent and dynamic behavior. We report here our calculations of the dynamic structure factor $S_k(\omega)$ for a 2D electronic system, in part, in hope of stimulating experimental work. To our knowledge there have been no measurements of the corresponding dynamic structure factor. These materials, in which 2D or quasi-2D electronic systems are realized, allow a considerable range of the electron density ρ or more commonly r_s.[2] It is well known that raising the density (i.e., $r_s \to 0$) is equivalent to turning off the electron-electron interaction.[3] Thus it appears that one can through these materials observe a crossover behavior, unique to 2D electronic systems, which, we predict, arises as the interaction is gradually removed. If the density can be smoothly varied, one need not obtain extremely high values to observe some of the effects of the crossover behavior. According to our calculations, this crossover behavior takes place in the *low-frequency* regime at small wave vectors. Inelastic x-ray and electron-scattering experiments[4] or possibly laser-optical methods[5] may be able to detect it.

Recently we have shown using a method of recurrence relations[6] that the dynamic structure factor for the 2D electron gas model of Sawada[7] at $T=0$ has the following form:

$$\frac{\pi S_k(\omega)}{\chi_k} = A_s \frac{\omega(\mu^2-\omega^2)^{1/2}}{\mu^2-\alpha\omega^2}, \quad 0<\omega<\mu \tag{1a}$$

$$= \frac{\pi}{2} A_p \omega[\delta(\omega-\omega_p)+\delta(\omega+\omega_p)], \tag{1b}$$

$$\mu < \omega < \infty ,$$

where wave vector k and frequency ω are measured in units of the Fermi wave vector k_F and the Fermi energy E_F, respectively. The other symbols are defined as follows: χ_k is the static density-density response function or susceptibility; $\mu = 2kE_f$; $\alpha = (x^2+\frac{1}{4})/(x^2+\frac{1}{2})^2$, where $x=\omega_p^{\text{cl}}/\mu$ and the 2D classical plasma frequency[8] $\omega_p^{\text{cl}} = (2\pi\rho e^2 k/m)^{1/2}$; $\omega_p = \alpha^{-1/2}\mu$, which represents the plasmon dispersion relation[9]; $A_s = 1-(1-\alpha)^{1/2}$, $A_p = [(1-\alpha)^{1/2}-(1-\alpha)]/\frac{1}{2}\alpha$. The above expression (1) is valid for $k \ll 1$. It is otherwise exact.[10]

It may be helpful to examine some of the parameters introduced here. One can write $x^2 = cr_s$, where c is a constant for a fixed k. Hence α, which turns out to be a natural parameter, may be expressed as a function of r_s. For example, $\max\alpha = 1$ represents the ideal gas limit[11] and $\min\alpha = 0$ the classical or mean-field-like limit.[12] The relationship between α and r_s for a fixed k is illustrated in Fig. 1(a). The plasmon frequency is bounded by $\mu \leq \omega_p \leq \omega_p^{\text{cl}}$, where the lower bound is attained at $\alpha = 1$ and the upper bound at $\alpha = 0$.

In Fig. 2, the dynamic structure factor is illustrated as a function of the frequency for three different values of r_s or α at $k=0.2$. For $r_s = 1$ ($\alpha = 0.2324$) and $r_s = 0.5$ ($\alpha = 0.3924$), the dynamic structure factor [see Figs. 2(a) and 2(b)] shows a low-frequency broad spectrum due to single-particle scattering and a high-frequency sharp peak due to the plasmon mode. These features are superficially familiar from the dynamic structure factor for 3D electronic systems.[13] Observe, however, that in both cases the low-frequency spectrum terminates at $\omega = \mu = 0.4$, which shall be referred to as the upper terminus. As $\alpha \to 1$, the frequency for $\max S_k(\omega)$ due to single-particle scattering (indicated by a small arrow) increases. We shall denote this frequency by ω_m. The amplitude of $\max S_k(\omega)$, hence $S_k(\omega_m)$, also increases. As α reaches its maximum value, both ω_m and $S_k(\omega_m)$ attain the ideal gas values, respectively, $\omega_m = \mu$ and $S_k(\omega_m) = \infty$, indicated in Fig. 2(c). In 3D electronic systems, $S_k(\omega_m)$ remains finite as $\alpha \to 1$.[13]

In Figs. 2(a) and 2(b) one observes a gap between

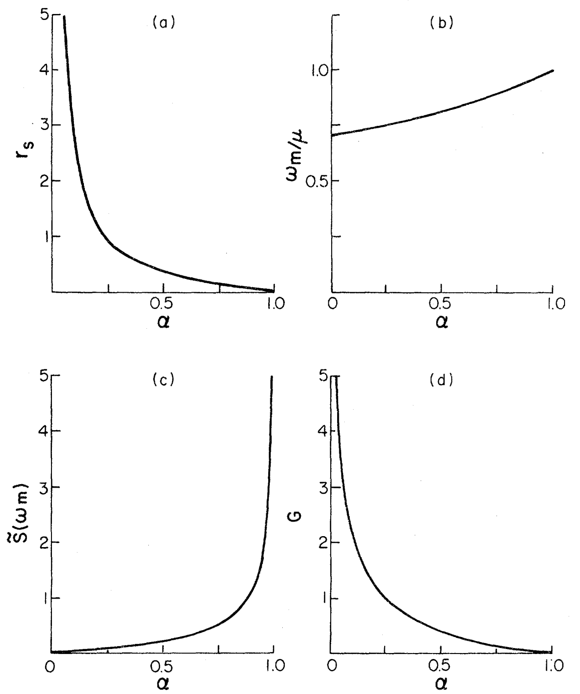

FIG. 1. (a) r_s vs α. Physically allowed values of α can range from 0 to 1 and also from $-\infty$ to 1. We consider here only the first branch. The second branch gives a mathematical mechanism for the disappearance of the plasmon mode. (b) ω_m vs α. ω_m is the frequency at which Eq. (1a) is maximum. Observe that ω_m/μ is bounded by 0.7071 and 1. (c) $\tilde{S}_k(\omega_m)$ vs α. $\tilde{S}_k(\omega_m)$ diverges as a power law as $\alpha \rightarrow 1$. It gives rise to a quasiplasma oscillation. (d) G vs α. The gap G is the distance between the plasmon frequency ω_p and the upper terminus μ. The gap must vanish at $\alpha=1$ since the ideal electron gas cannot support normal plasma oscillations.

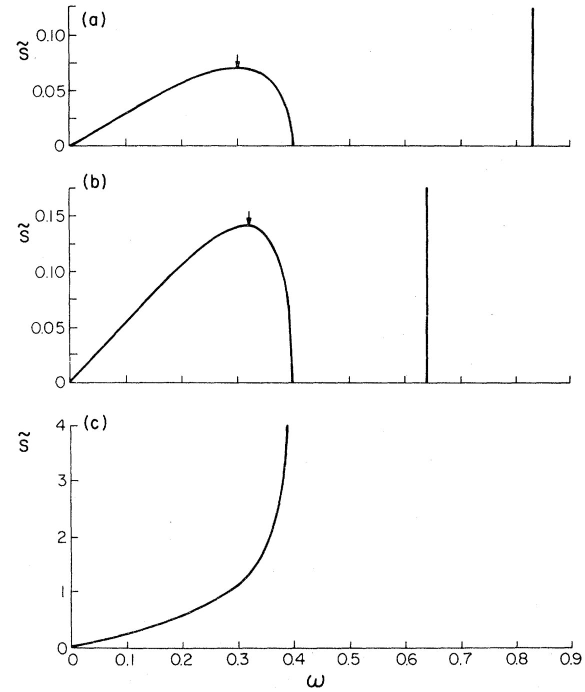

FIG. 2. Normalized dynamic structure vs frequency as a function of the density. The normalized dynamic structure $\tilde{S}_k(\omega) \equiv \pi S_k(\omega)/\chi_k$ is plotted against the frequency ω at $k=0.2$ for three different values of r_s and α: (a) $r_s=1$, $\alpha=0.2324$; (b) $r_s=0.5$, $\alpha=0.3924$; (c) $r_s=0$, $\alpha=1$. In all cases $\mu=0.4$. Small arrows in (a) and (b) indicate the positions where the amplitude of $\tilde{S}_k(\omega)$ is maximum, denoted by ω_m. $\omega_m=0.3009$ in (a), 0.3155 in (b), and $\omega_m=\mu=0.4$ in (c). The plasmon peaks are found at $\omega_p=0.8297$ in (a) and 0.6386 in (b). The wave vector and frequency are measured in units of k_F and E_F, respectively.

the upper terminus μ and the plasmon frequency ω_p, which we define by $G=(\omega_p-\mu)/\mu$. As $\alpha \rightarrow 1$, $G \rightarrow 0$. The gap disappears at $\alpha=1$. The disappearance of the gap is not special, since the plasmon mode cannot exist when the interaction is turned off. But the single-particle scattering at $\omega=\mu$ now suddenly behaves like a long-lived excitation, which we shall term a quasiplasma oscillation.[14] Hence as $\alpha \rightarrow 1$, the gap disappears but not the long-lived excitation. In 3D electronic systems, both disappear.

The above observations can be made quantitative. One can readily obtain from Eq. (1a)

$$\omega_m/\mu=(2-\alpha)^{-1/2} \ . \tag{2}$$

Hence

$$\tilde{S}_k(\omega_m) \equiv \pi S_k(\omega_m)/\chi_k = \tfrac{1}{2}(1-\alpha)^{-1/2} \ . \tag{3}$$

The gap G follows directly from the plasmon dispersion relation $\omega_p=\alpha^{-1/2}\mu$,

$$G=\alpha^{-1/2}-1 \ . \tag{4}$$

In Fig. 1, ω_m, $\tilde{S}_k(\omega_m)$, and G are illustrated as a function of α for $k=0.2$. Perhaps most remarkable is the behavior of $\tilde{S}_k(\omega_m)$, shown in Fig. 1(c). As $\alpha \rightarrow 1$, it diverges as a simple power law. This crossover behavior manifested in going from interacting to noninteracting clearly is unique to 2D electronic systems. It would appear that one can detect the onset of the divergence long before the density enters into the "ideal gas region."

Finally, sum rules are almost always of theoretical interest. The validity of our result for the dynamic structure, for example, may be tested against them. In this case they amount to adding up the areas illustrated in Fig. 2 with appropriate weighting factors. For this purpose we introduce the following quantity:

$$\bar{\omega}_{2n}(k)=\int_0^\infty d\omega\, \omega^{2n-1}\tilde{S}_k(\omega) \tag{5}$$

with $n \geqslant 0$. Those with integer n represent standard moment sum rules, e.g., the susceptibility sum rule ($n=0$), the f-sum rule ($n=1$).[15] Those with halfinteger n also represent sum rules, but they are not necessarily exact sum rules, e.g., the static form-factor sum rule ($n=\frac{1}{2}$).[16]

By using Eq. (1), one can show that all the moment sum rules are exactly satisfied independently of α. For $n=\frac{1}{2}$, we find

$$\bar{\omega}_1(k)/\mu=[1-(1-\alpha)^{1/2}]\times[1-(\alpha^{-1/2}-1)(\sin^{-1}\alpha-\pi\alpha^{-1})] \quad . \qquad (6)$$

For $\alpha\rightarrow 1$,
Eq. (6) may be simplified to

$$\bar{\omega}_1/\mu\approx 1+(\tfrac{1}{2}\pi-1)(1-\alpha)^{1/2} \quad . \qquad (7a)$$

The above is contributed largely by the low-frequency (i.e., single-particle-scattering) portion of $\tilde{S}_k(\omega)$. Equation (7a) evidently does not satisfy the static form-factor sum rule even at $\alpha=1$.[17] For $\alpha\rightarrow 0$, Eq. (6) may be reduced to

$$\bar{\omega}_1/\mu\approx\tfrac{1}{2}\pi\alpha^{-1/2}(1-\alpha)^{1/2} \quad , \qquad (7b)$$

which is now contributed largely by the high-frequency (i.e., plasmon) portion of $\tilde{S}_k(\omega)$. When the plasmon mode is dominant, there is a simple relationship between the susceptibility and the static form factor.[12] Using it one can prove that as $\alpha\rightarrow 0$, Eq. (7b) indeed satisfies the static form factor sum rule.

We are grateful to Professor A. K. Rajagopal of Louisiana State University and Professor K. S. Singwi of Northwestern University for explaining their work to us. We thank Mr. S. K. Oh of the University of Georgia for assisting us with the evaluation of sum rules. Our work is supported in part by the U.S. Department of Energy Office of Basic Energy Sciences under Contract No. DE-AS09-77ER01023.

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[9] A. K. Rajagopal [Phys. Rev. B 15, 4264 (1977)] obtained a plasmon dispersion relation. This work includes exchange energy, which contributes to a higher order in k. Excepting this exchange contribution, our plasmon dispersion relation is identical to Rajagopal's. See also our comment in Ref. 10. A. Czachor, A. Holas, S. R. Sharma, and K. S. Singwi [Phys. Rev. B 25, 2144 (1982)] have obtained a similar dispersion relation, which additionally includes self-energy corrections. They also contribute to a higher order in k. We thank Professor Singwi for informing us of this work prior to publication.

[10] Since $\pi S_k(\omega)/\chi_k=-\text{Im}\chi_k(\omega)$, where $\chi_k(\omega)$ is the frequency-dependent response function, $\text{Re}\chi_k(\omega)$ can be obtained from Eq. (1). Rajagopal (Ref. 9) also obtained an expression for the polarizability. His is a formal expression [see Eq. (22) of Ref. 9], obtained by a variational method. To illustrate the effects of single-particle scattering and others he computed the plasmon dispersion relation as a power series in $k_Fk/m\omega$ to the leading order only, i.e., valid for $k_Fk/m\omega<<1$ and $k/k_F<<1$.

[11] For $\alpha=1$, our result can be shown to agree identically with the celebrated result of Stern (Ref. 8).

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[14] We have shown (Ref. 6) that for $t\rightarrow\infty$, the relaxation function for the ideal electron gas behaves as $t^{-1/2}\cos(\mu t-\frac{1}{4}\pi)$, whereas the relaxation function for an interacting electron gas (the Sawada model) due to single-particle scattering behaves as $t^{-3/2}\cos(\mu t-3\pi/4)$. The relaxation function for the 3D ideal electron gas can be shown to behave as $t^{-1}\sin\mu t$.

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[16]P. Nozieres, *The Theory of Interacting Fermi Systems* (W. A. Benjamin, New York, 1964), p. 53. Also Ref. 13, p. 89. The static form-factor sum rule is not satisfied when the dispersion relation is operative. At high frequencies, ω usually becomes independent of k. As a result, the sum rule can hold.

[17]This arises from the well-established fact that, except in some special regions, the susceptibility and fluctuation (static form factor here) are generally not simply related. See H. Falk and L. W. Bruch, Phys. Rev. 180, 442 (1969); M. H. Lee, Phys. Rev. B 8, 1203 (1973); J. Naudts and A. Verbeure, J. Math. Phys. 17, 419 (1976); G. Roepstorff, Commun. Math Phys. 46, 253 (1976); M. Fannes and A. Verbeure, *ibid.* 57, 165 (1977); F. Dyson, E. H. Lieb, and B. Simon, J. Stat. Phys. 18, 335 (1978); I. M. Kim and M. H. Lee, Phys. Rev. B 24, 3961 (1981).